Square ABCD Has A Diagonal AC With Vertices A(-2, 1) And C(2, 4). Find The Coordinates Of The Remaining

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Understanding the properties of a square and how to find its vertices when given specific points is a fundamental aspect of coordinate geometry. In this article, we will explore how to determine the remaining vertices of square ABCD, given that the diagonal AC has endpoints at A(-2, 1) and C(2, 4). This problem involves concepts such as the midpoint of a segment, the properties of squares, and vector analysis. By following a systematic approach, you will learn how to find the coordinates of points B and D, completing the square's vertices.

Understanding the Problem and Relevant Concepts

Before diving into calculations, it’s essential to understand the key concepts involved in the problem.

Properties of a Square

- All sides are equal in length.
- Adjacent sides are perpendicular (at 90 degrees).
- The diagonals are equal in length and bisect each other at right angles.

Given Data

- Vertices A and C are given as A(-2, 1) and C(2, 4).
- These points form the diagonal AC of the square.

Objective

- Find the coordinates of vertices B and D, which complete the square ABCD.

Step-by-Step Solution Approach

To find the remaining vertices, we need to analyze the given diagonal and utilize geometric properties.

Step 1: Find the Midpoint of Diagonal AC

Since the diagonal AC connects points A(-2, 1) and C(2, 4), the midpoint M of AC can be calculated as:

$$
M = \left(\frac{x_A + x_C}{2}, \frac{y_A + y_C}{2} \right)
$$

Calculating:

$$
x_M = \frac{-2 + 2}{2} = 0
$$
$$
y_M = \frac{1 + 4}{2} = 2.5
$$

So, the midpoint M is at (0, 2.5).

This midpoint is also the intersection point of the diagonals of the square.

Step 2: Determine the Length of Diagonal AC

The length of the diagonal AC is:

$$
|AC| = \sqrt{(x_C - x_A)^2 + (y_C - y_A)^2}
$$

Calculating:

$$
|AC| = \sqrt{(2 - (-2))^2 + (4 - 1)^2} = \sqrt{(4)^2 + (3)^2} = \sqrt{16 + 9} = \sqrt{25} = 5
$$

The diagonal length is 5 units.

Step 3: Find the Coordinates of Points B and D

Since ABCD is a square, points B and D are located such that:

- The diagonals bisect each other at M.
- The diagonals are perpendicular.
- The length of each side can be derived from the diagonal length, as:

$$
\text{Side length} = \frac{|AC|}{\sqrt{2}} = \frac{5}{\sqrt{2}} = \frac{5 \sqrt{2}}{2}
$$

However, for coordinates, it’s more practical to use vector rotation.

Step 4: Use Vector Rotation to Find B and D

The vector from M to A is:

$$
\vec{MA} = (x_A - x_M, y_A - y_M) = (-2 - 0, 1 - 2.5) = (-2, -1.5)
$$

Similarly, the vector from M to C is:

$$
\vec{MC} = (2 - 0, 4 - 2.5) = (2, 1.5)
$$

Note that $\vec{MC} = -\vec{MA}$, confirming the symmetry about M.

To find points B and D, we rotate the vector $\vec{MA}$ by +90° and -90°, respectively, because the sides are perpendicular.

- Rotation by +90°:

$$
(x, y) \to (-y, x)
$$

- Rotation by -90°:

$$
(x, y) \to (y, -x)
$$

Applying these:

- For B (rotation of $\vec{MA}$ by +90°):

$$
\vec{MB} = (-(-1.5), -2) = (1.5, -2)
$$

- For D (rotation of $\vec{MA}$ by -90°):

$$

$$
\vec{MD} = (-1.5, 2)
$$

Now, the coordinates of B and D are:

$$
x_B = x_M + x_{MB} = 0 + 1.5 = 1.5
$$
$$
y_B = y_M + y_{MB} = 2.5 - 2 = 0.5
$$

$$
x_D = x_M + x_{MD} = 0 - 1.5 = -1.5
$$
$$
y_D = y_M + y_{MD} = 2.5 + 2 = 4.5
$$

Thus, the remaining vertices are:

- B at (1.5, 0.5)
- D at (-1.5, 4.5)

Final Coordinates of the Square ABCD

Vertex	Coordinates
A	(-2, 1)
B	(1.5, 0.5)
C	(2, 4)
D	(-1.5, 4.5)

These four points define the vertices of square ABCD, with diagonal AC at the given positions.

Summary and Key Takeaways

- The midpoint of the diagonal helps locate the center of the square.
- Vector rotation by ±90° allows us to find perpendicular points relative to the diagonal.
- The symmetry of the square and properties of the diagonals facilitate the calculation of missing vertices.
- Coordinate geometry techniques are powerful tools for solving geometric problems involving polygons.

Additional Tips for Solving Similar Problems

- Always verify your calculations by checking distances and slopes to ensure perpendicularity and equal side lengths.
- Use vector analysis for efficient calculations involving rotation and translation.
- When given diagonals, focus on their properties — midpoint, length, and perpendicularity — as they are key to reconstructing the square.

By mastering these techniques, you can confidently find missing vertices of squares and other polygons when given partial information, enhancing your skills in coordinate geometry and geometric problem solving.

Frequently Asked Questions

How do you find the length of the diagonal AC of square ABCD with vertices A(-2, 1) and C(2, 4)?

Use the distance formula: length = $\sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2]}$. Plugging in, $\sqrt{[(2 - (-2))^2 + (4 - 1)^2]} = \sqrt{[(4)^2 + (3)^2]} = \sqrt{(16 + 9)} = \sqrt{25} = 5$.

What is the slope of the diagonal AC of the square?

Slope = (4 - 1) / (2 - (-2)) = 3 / 4 = 0.75.

Given the diagonal AC, how can we find the center point of the square?

The center point is the midpoint of AC: x = (-2 + 2)/2 = 0, y = (1 + 4)/2 = 2.5, so the center is at (0, 2.5).

How do we determine the length of the sides of the square from the diagonal length?

Since the diagonal of a square relates to its side length s by $d = s\sqrt{2}$, then $s = d / \sqrt{2}$. For d = 5, $s = 5 / \sqrt{2} \approx 3.535$.

What are the directional vectors for the sides of the square perpendicular to the diagonal AC?

The diagonal vector from A to C is (4, 3). Perpendicular vectors are obtained by swapping components and changing signs: (−3, 4) and (3, −4).

How do we find the coordinates of the remaining vertices B and D of the square?

From the center at (0, 2.5), move half the side length along the perpendicular directions to find B and D. For side $s \approx 3.535$, half is about 1.768. Therefore, B ≈ (0 + 1.768, 2.5 + 1.768), D ≈ (0 - 1.768, 2.5 - 1.768). Adjusting for the correct direction gives approximate coordinates for B and D.

What are the approximate coordinates of vertices B and D of the square?

Using the perpendicular vectors, B ≈ (1.768, 4.268) and D ≈ (−1.768, 0.732). Exact positions depend on orientation, but these are approximate calculations.

How can we verify the correctness of the remaining vertices B and D in the square?

Verify that the distances between B and D, B and C, D and A are equal to the side length, and check that the diagonals are perpendicular. Ensuring these conditions confirms the vertices form a square.

Additional Resources

Square ABCD Has a Diagonal AC with Vertices A(-2, 1) and C(2, 4). Find the Coordinates of the Remaining Vertices

Understanding the geometry of squares is fundamental in coordinate geometry, and working through problems such as determining the remaining vertices given specific points offers valuable insights into the properties of these quadrilaterals. In this detailed review, we will explore the problem step-by-step, covering the foundational concepts, mathematical methods, and practical techniques necessary to find the remaining vertices of the square ABCD when given the diagonal AC.

Understanding the Problem Context

At the core, the problem involves a square ABCD in the coordinate plane, where:

- The diagonal AC is known, with vertices at A(-2, 1) and C(2, 4).
- The goal is to find the coordinates of the remaining vertices B and D.

This problem is typical in coordinate geometry, combining knowledge of the properties of squares with vector operations, distance calculations, and geometric transformations.

Key Properties of a Square in Coordinate Geometry

Before delving into calculations, it is essential to recall the defining characteristics of a square:

- Equal sides: All four sides are of equal length.
- Right angles: Each interior angle measures 90 degrees.
- Diagonals: The diagonals are equal in length and bisect each other at right angles (perpendicular bisectors).

Specifically, for a square:

- The diagonals are perpendicular to each other.
- The diagonals bisect each other, meaning they intersect at their midpoints, and each diagonal's midpoint is the same point.

Analyzing the Known Data: Diagonal AC

Given:

- A(-2, 1)
- C(2, 4)

Let's analyze the known diagonal:

1. Length of diagonal AC

Using the distance formula:

$$
|AC| = \sqrt{(x_C - x_A)^2 + (y_C - y_A)^2}
$$

Plugging in the values:

$$
|AC| = \sqrt{(2 - (-2))^2 + (4 - 1)^2} = \sqrt{(4)^2 + (3)^2} = \sqrt{16 + 9}
$$

$$
= \sqrt{25} = 5
$$

So, the diagonal AC has length 5.

2. Midpoint of AC

The midpoint M_{AC} is:

$$
M_{AC} = \left(\frac{x_A + x_C}{2}, \frac{y_A + y_C}{2} \right) = \left(\frac{-2 + 2}{2}, \frac{1 + 4}{2} \right) = (0, 2.5)
$$

This midpoint is crucial because, in a square, the diagonals bisect each other at the same point, which will also be the intersection point of the other diagonal BD.

Determining the Orientation of the Square

Since ABCD is a square with diagonal AC known, the remaining vertices B and D must be positioned such that:

- The diagonals intersect at the same midpoint M.
- The diagonals are perpendicular to each other.
- The length of the other diagonal BD is equal to AC, i.e., 5.

Key observations:

- The vector from A to C is:

$$
\vec{AC} = (x_C - x_A, y_C - y_A) = (4, 3)
$$

- The length of $\vec{AC}$ is 5, as calculated.

- The other diagonal BD will be centered at the same midpoint $M(0, 2.5)$.

- The vector representing BD will be perpendicular to $\vec{AC}$.

Finding the Coordinates of Vertices B and D

Step 1: Find the vector representing the other diagonal BD

Since the diagonals are perpendicular, the vector $\vec{BD}$ must be perpendicular to $\vec{AC}$.

- $\vec{AC} = (4, 3)$
- A vector perpendicular to $\vec{AC}$ in 2D can be $(-3, 4)$ or $(3, -4)$, since:

$$
(4)(-3) + (3)(4) = -12 + 12 = 0
$$

Step 2: Determine the length of $\vec{BD}$

The length of diagonal BD should be the same as AC, which is 5.

- The magnitude of the perpendicular vector $\vec{v} = (a, b)$ is:

$$
|\vec{v}| = \sqrt{a^2 + b^2}
$$

For $\vec{v} = (3, 4)$:

$$
|\vec{v}| = \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5
$$

Thus, the vector $\vec{v} = (3, 4)$ has the same length as the diagonal, which is perfect.

Step 3: Find the coordinates of B and D

Since the diagonals bisect at the same point $M(0, 2.5)$, then:

- The vertices B and D are located at:

$$
B = M + \frac{\vec{v}}{2} \quad \text{and} \quad D = M - \frac{\vec{v}}{2}
$$

Calculating:

$$
\frac{\vec{v}}{2} = \left(\frac{3}{2}, 2 \right)
$$

- Coordinates of B:

$$
B = (0 + 1.5, 2.5 + 2) = (1.5, 4.5)
$$

- Coordinates of D:

$$
D = (0 - 1.5, 2.5 - 2) = (-1.5, 0.5)
$$

Alternatively, using the other perpendicular vector $(-3, -4)$:

- $\frac{\vec{v}}{2} = (-1.5, -2)$

- Coordinates of B:

$$
B = (0 - 1.5, 2.5 - 2) = (-1.5, 0.5)
$$

- Coordinates of D:

$$
D = (0 + 1.5, 2.5 + 2) = (1.5, 4.5)
$$

This confirms that the two options are symmetric, and the orientation of the square can be determined based on the labeling of vertices.

Verifying the Solution

To ensure the correctness of the coordinates:

1. Distance from B to D:

Calculate $|BD|$:

$$
|BD| = \sqrt{(x_D - x_B)^2 + (y_D - y_B)^2}
$$

Using:

$$
B(1.5, 4.5), \quad D(-1.5, 0.5)
$$

\]

$$
|BD| = \sqrt{(-1.5 - 1.5)^2 + (0.5 - 4.5)^2} = \sqrt{(-3)^2 + (-4)^2} = \sqrt{9 + 16} = \sqrt{25} = 5
$$

This confirms that BD has length 5, matching the diagonal AC.

2. Check right angles:

- The vectors along sides can be checked for perpendicularity to confirm the square's shape.
- The vector from B to A:

$$
A(-2, 1), \quad B(1.5, 4.5)
$$
$$
\vec{BA} = (-2 - 1.5, 1 - 4.5) = (-3.5, -3.5)
$$

- The vector from B to C:

$$
C(2, 4), \quad B(1.5, 4.5)
$$
$$
\vec{BC} = (2 - 1.5, 4 - 4.5) = (0.5, -0.5)
$$

- Dot product:

$$
\vec{BA} \cdot \vec{BC} = (-3.5)(0.5) + (-3.5)(-0.5) = -1.75 + 1.75 = 0
$$

Since the dot product is zero, the sides are perpendicular, confirming the right angle at B.

Similarly, check at other vertices to confirm the orthogonality, which is consistent with a square.

Consolidated Coordinates of All Vertices

Based on the calculations and verifications:

```
- Vertex A: \((-2, 1)\)
- Vertex C: \((2, 4)\)
- Vertex B: \(\left(1.5, 4.5\right)\)
- Vertex D: \(\left(-1.5, 0.5\right)\)

These points form a square with the known diagonal AC, with the other
diagonal BD passing through the midpoint
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