Subject To The Conditions 0x10,0y5 The Minimum Value Of The Function 4x5y+10 Is (1) 10 (2) 0 (3) 25 (4)

Subject To The Conditions 0x10,0y5 The Minimum Value Of The Function 4x5y+10 Is (1) 10 (2) 0 (3) 25 (4)

In the realm of calculus and mathematical optimization, understanding how to determine the minimum or maximum values of a function under certain constraints is fundamental. This article explores a specific constrained optimization problem involving the function $4x^5 y + 10$, with the constraints $0 \leq x \leq 10$ and $0 \leq y \leq 5$. We aim to identify the minimum value of this function within the specified domain and analyze the conditions under which this minimum occurs. This problem exemplifies key concepts in constrained optimization, Lagrange multipliers, and the importance of boundary analysis in calculus.

Understanding the Problem: Function and Constraints

Definition of the Function

The function we are analyzing is:

$$f(x, y) = 4x^5 y + 10$$

This is a multivariable function involving both x and y. The function's structure suggests that it is increasing with respect to both variables under typical conditions, but the presence of the constraints significantly influences the minimum value.

Constraints on Variables

The constraints define the domain in which we are searching for the minimum:

- $0 \leq x \leq 10$
- $0 \leq y \leq 5$

These bounds create a rectangular region in the xy-plane:

- The corners are at $(0, 0)$, $(10, 0)$, $(0, 5)$, and $(10, 5)$.
- The interior of the domain is where both variables are within these bounds.

Approach to Solving the Optimization Problem

Step 1: Find Critical Points Inside the Domain

To locate potential minima, we first check for critical points within the interior of the domain where the gradient of $f(x, y)$ is zero:

$$
\nabla f(x, y) = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right)
$$

Calculate the partial derivatives:

$$
\frac{\partial f}{\partial x} = 20x^4 y
$$
$$
\frac{\partial f}{\partial y} = 4x^5
$$

Set these equal to zero:

$$
20x^4 y = 0
$$
$$
4x^5 = 0
$$

From the second equation:

$$
4x^5 = 0 \Rightarrow x = 0
$$

Substituting into the first:

$$
20 \times 0^4 \times y = 0
$$

which is always true for any y.

Thus, the critical point inside the domain occurs at:

$$
x = 0, \quad y \in [0, 5]
$$

Note: Since the gradient is zero at any point where $x=0$, the entire line segment $x=0, y \in [0,5]$ consists of critical points.

Step 2: Evaluate the Function at Critical Points

At $x=0$:

$$
f(0, y) = 4 \times 0^5 \times y + 10 = 10
$$

This means the function value along the entire line $x=0$ is constant at 10, regardless of y.

Step 3: Evaluate the Boundary Points

Since critical points in the interior don't provide a minimum less than or equal to the boundary, examine the boundaries:

- At $x=0$, as above, $f(0, y) = 10$
- At $x=10$:

$$
f(10, y) = 4 \times (10)^5 \times y + 10 = 4 \times 100000 \times y + 10 = 400000 y + 10
$$

Varying y:

- For $y=0$:

$$
f(10, 0) = 10
$$

- For $y=5$:

$$
f(10, 5) = 400000 \times 5 + 10 = 2,000,010
$$

The function increases with y at $x=10$.

- At $y=0$:

$$
f(x, 0) = 10

$$
$$

- At $y=5$:

$$
f(x, 5) = 4x^5 \times 5 + 10 = 20 x^5 + 10
$$

Varying x:

- At $x=0$, $f=10$
- At $x=10$:

$$
f(10, 5) = 20 \times 100000 + 10 = 2,000,010
$$

Similarly, the minimum at these boundaries is at $x=0, y=0$, with $f=10$.

Summary of Findings: Minimum Value of the Function

From the above analysis:

- The critical points along $x=0$ yield $f=10$.
- Boundary points at $x=0, y=0$ and $x=0, y=5$ give $f=10$.
- Other boundary points, such as $x=10, y=0$ or $x=10, y=5$, produce values larger than 10.
- The interior critical points do not exist besides $x=0$.

Thus, the minimum value of $f(x,y) = 4x^5 y + 10$ within the given constraints is 10.

Conclusion: The Minimum Value and Its Conditions

The minimum value of the function $4x^5 y + 10$ under the constraints $0 \leq x \leq 10$ and $0 \leq y \leq 5$ is 10. This minimum occurs along the entire line segment where $x=0$, regardless of the value of y within $[0, 5]$. Specifically, at the point $(0,0)$, the function attains this minimum value.

Implications for Optimization and Real-World Applications

Key Points to Remember

- Critical points are found by setting the gradient to zero; here, they exist along $x=0$.
- Boundary analysis is essential in constrained optimization problems, especially when the interior critical points do not provide minima lower than boundary points.
- The minimum value within the constraints can often be found at boundary points or critical points, making boundary analysis a crucial step.

Applications in Engineering and Economics

- Resource allocation: Determining minimal costs or resource usage within given constraints.
- Design optimization: Ensuring system parameters stay within safe or optimal limits.
- Mathematical modeling: Finding extremal points helps in understanding the behavior of complex systems.

Answer to the Multiple Choice Question

Based on the detailed analysis:

The minimum value of the function $4x^5 y + 10$ subject to the given constraints is 10.

Thus, the correct choice is:

1. 10

Final Remarks

Constrained optimization problems, like the one analyzed here, are fundamental in various scientific and engineering disciplines. Understanding how to systematically evaluate critical points and boundary conditions ensures accurate solutions. Recognizing that the minimum of the function occurs at the boundary $x=0$, and specifically at $(0,0)$, highlights the importance of boundary analysis in the optimization process.

Whether dealing with simple functions or complex systems, mastering these principles provides a solid foundation for tackling real-world problems involving optimization under constraints.

Frequently Asked Questions

What is the main goal when solving the problem 'Subject To The Conditions 0x10, 0y5 The Minimum Value Of The Function 4x5y + 10'?

The main goal is to find the minimum value of the function 4x5y + 10 subject to the given constraints 0x10 and 0y5.

Are the constraints '0x10' and '0y5' properly formatted, and what do they imply?

The constraints seem to be improperly formatted; they likely mean $0 \leq x \leq 10$ and $0 \leq y \leq 5$, which are bounds for the variables x and y.

How do you approach solving for the minimum of the function 4x5y + 10 under these constraints?

You substitute the boundary values of x and y within their constraints into the function and evaluate to find the minimum value, or use calculus and optimization methods considering the constraints.

What is the value of the function when x = 0 and y = 0?

When x = 0 and y = 0, the function is 4050 + 10 = 10.

What is the value of the function when x = 10 and y = 5?

When x = 10 and y = 5, the function is 41055 + 10 = 41025 + 10 = 4250 + 10 = 1000 + 10 = 1010.

Based on the boundary evaluations, what is the minimum value of the function within the given constraints?

Evaluating at the corners, the minimum value is at x=0, y=0 with a value of 10, which is less than other boundary points.

What is the correct answer choice for the minimum value of the function?

The minimum value is 10, which corresponds to answer choice (1).

Why is the minimum value of the function 10 in this problem?

Because at the lowest boundary point (x=0, y=0), the function evaluates to 10, which is less than at other boundary points within the constraints.

Additional Resources

Subject To The Conditions 0x10, 0y5: An In-Depth Examination of the Function’s Minimum Value

---

Introduction

In the realm of calculus and optimization, understanding how to determine the minimum or maximum of a function under certain constraints is fundamental. The problem posed—“Subject To The Conditions 0x10, 0y5 The Minimum Value Of The Function 4x5y + 10 Is (1) 10 (2) 0 (3) 25 (4)”—serves as a classic example illustrating the application of constrained optimization techniques. This article aims to dissect this problem thoroughly, providing a comprehensive overview of the methods involved, the reasoning behind each step, and ultimately, the conclusion.

Clarifying the Problem Statement

Before delving into the solution, it is crucial to interpret the problem accurately. The statement appears to have some typographical ambiguities, but based on standard conventions in optimization problems, it likely reads:

> Find the minimum value of the function $f(x, y) = 4x^5 y + 10$, subject to the constraints:
> - $0 \leq x \leq 10$
> - $0 \leq y \leq 5$

The phrase "Subject To The Conditions 0x10, 0y5" suggests boundary conditions or constraints on x and y, possibly indicating the feasible region: the rectangle defined by x in $[0, 10]$ and y in $[0, 5]$.

Understanding the Function

The core function under investigation is:

$$
f(x, y) = 4x^5 y + 10
$$

This is a multivariable function involving a high-degree polynomial in x and a linear term in y.

Key observations:

- The function increases rapidly with x, especially due to the x^5 term.
- The coefficient of y is positive, indicating that as y increases, $f(x, y)$ increases proportionally, scaled by $4x^5$.
- The constant term $+10$ shifts the entire function upward, ensuring the minimum value is at least 10.

Goal: Find the Minimum Value

Given the constraints, the goal is to identify the point(s) within the feasible region where $f(x, y)$ attains its least value.

Approach Overview

To systematically find the minimum, we employ the methods of constrained optimization:

1. Identify the feasible region — a rectangle defined by the bounds on x and y.

2. Analyze the function behavior within the region — determine whether the minimum occurs at interior points or on the boundaries.

3. Evaluate the function at boundary points — because polynomial functions often reach extrema at boundary points when constrained.

4. Use calculus techniques — such as setting partial derivatives to zero to find critical points in the interior, if any.

5. Compare boundary and interior values — to identify the global minimum within the feasible region.

Step-by-Step Solution

1. Feasible Region and Boundaries

The constraints:

$$
0 \leq x \leq 10,\quad 0 \leq y \leq 5
$$

define a rectangular feasible region.

---

2. Behavior of $f(x, y)$ in the Region

Since $f(x, y) = 4x^5 y + 10$:

- For fixed x, f increases with y because the coefficient $4x^5 \geq 0$.
- For fixed y, f increases with x because $x^5 \geq 0$ within the bounds, and the coefficient is positive.

Therefore, the minimum should occur at the lowest possible values of x and y, i.e., at $x=0$

and $y=0$, unless the function behavior indicates otherwise.

---

3. Evaluating at Corner Points

Corner points are often where extrema occur in constrained problems:

Point	x	y	$f(x,y)$
A	0	0	$4 \times 0^5 \times 0 + 10 = 10$
B	10	0	$4 \times 10^5 \times 0 + 10 = 10$
C	0	5	$4 \times 0^5 \times 5 + 10 = 10$
D	10	5	$4 \times 10^5 \times 5 + 10$

Calculating point D:

$$
f(10, 5) = 4 \times 10^5 \times 5 + 10 = 4 \times 100000 \times 5 + 10 = 4 \times 500000 + 10 = 2,000,000 + 10 = 2,000,010
$$

This is vastly larger than the other corner points.

---

4. Interior Critical Points

To find potential interior minima, set partial derivatives to zero.

Calculate:

$$
\frac{\partial f}{\partial x} = 4 \times 5 x^4 y = 20 x^4 y
$$

$$
\frac{\partial f}{\partial y} = 4 x^5
$$

Set derivatives to zero:

$$
\frac{\partial f}{\partial x} = 0 \Rightarrow 20 x^4 y = 0
$$

$$
\frac{\partial f}{\partial y} = 0 \Rightarrow 4 x^5 = 0
$$

From $\frac{\partial f}{\partial y} = 0$:

$$
4 x^5 = 0 \Rightarrow x=0
$$

From $\frac{\partial f}{\partial x} = 0$:

$$
20 x^4 y= 0
$$

If $x=0$, then this derivative is zero regardless of y.

Result: The only critical point in the interior is at $x=0$, for any y in the feasible region.

---

5. Evaluating $f(0, y)$

Since $f(0, y) = 4 \times 0^5 \times y + 10 = 10$, for all y in $[0, 5]$.

- At $y=0$, $f=10$.
- At $y=5$, $f=10$.

Thus, along the line $x=0$, the function is constantly 10.

---

6. Summary of Findings

- At points $(0, y)$, $f=10$.
- At other boundary points, f is larger:
- At $(10, 0)$, $f=10$.
- At $(10, 5)$, $f=2,000,010$.

- Interior critical points at $x=0$ give $f=10$, which is less than the values at other corners.

- The minimal value of the function within the feasible region is therefore 10.

Conclusion: The Minimum Value

Given the analysis, the minimum value of the function $f(x, y) = 4x^5 y + 10$ over the specified constraints is 10.

This minimum occurs along the entire segment where $x=0$, and $0 \leq y \leq 5$.

Final Answer:

Option (2) 0 is not correct—since f does not reach zero, it attains a minimum of 10 within the feasible region.

Option (1) 10 is the correct choice, as the minimal value of the function is 10.

Additional Insights on the Function and Constraints

- The function's growth is dominated by the x^5 term, which becomes large for x approaching 10.
- The constant addition of 10 ensures the function's minimum is never below 10.
- The problem exemplifies how boundary analysis and partial derivatives help identify extrema in constrained optimization.

Implications for Optimization Practice

This analysis underscores a common theme in constrained calculus problems: the extremum often occurs at boundary points or critical points within the interior. Recognizing the behavior of the function—such as monotonicity and polynomial growth—can significantly simplify the process.

Furthermore, understanding the feasible region's geometry guides efficient evaluation, avoiding unnecessary calculations in interior regions when the minimum or maximum is likely on the boundary.

Summary Table

Aspect	Explanation
Function	$f(x, y) = 4x^5 y + 10$
Constraints	$0 \leq x \leq 10$, $0 \leq y \leq 5$
Critical points	Along \(x=0

Subject To The Conditions 0x10 0y5 The Minimum Value Of The Function 4x5y 10 Is 1 10 2 0 3 25 4

Subject To The Conditions 0x10 0y5 The Minimum Value Of The Function 4x5y 10 Is 1 10 2 0 3 25 4

Related Articles

- The Degree To Which Members Of A Work Organization Are Able To Satisfy Important Personal Needs Through
- There Are 35 Rabbits At The Fair The Rabbits Are Kept In Blank Hutches With Blank Rabbits In Each Hutch

- The P-value For A Hypothesis Test Turns Out To Be 0.05038 . At A 2 % Level Of Significance, What Is The

Back to Home