

Suppose $F(x)=13/x$.(a) The Rectangles In The Graph On The Left Illustrate A Leftendpoint Riemann Sum For

Suppose $F(x)=13/x$.(a) The Rectangles In The Graph On The Left Illustrate A Leftendpoint Riemann Sum For

Understanding the concept of Riemann sums is fundamental in calculus, especially when approximating the area under a curve. In this article, we delve into the specifics of how the function $(F(x) = \frac{13}{x})$ can be approximated using a leftendpoint Riemann sum, illustrated by a series of rectangles over a given interval. We'll explore the theoretical background, interpret the graphical representation, derive the sum mathematically, and discuss its significance in integral calculus.

Introduction to Riemann Sums and Their Significance

A Riemann sum provides a method to approximate the total area under a curve between two points. This approximation is especially useful when the exact integral is difficult to compute directly. The fundamental idea involves dividing the interval into subintervals, calculating the area of rectangles over these subintervals, and summing these areas.

Key Concepts:

- Partition of the interval: Dividing the domain into smaller subintervals.
- Sample points: Selecting a point within each subinterval (left, right, or midpoints).
- Rectangle height: Determined by the function value at the sample point.
- Approximate area: Sum of the areas of these rectangles.

Understanding the Function $(F(x) = \frac{13}{x})$

Before analyzing the Riemann sum, it's critical to understand the behavior of the function:

- Domain: $(x > 0)$ (since division by zero is undefined).
- Range: $(F(x) > 0)$ for $(x > 0)$.
- Graph characteristics:
 - Decreases monotonically as (x) increases.
 - Hyperbolic shape, approaching zero as $(x \rightarrow \infty)$.
 - Vertical asymptote at $(x = 0)$.

Understanding the graph's shape helps in selecting appropriate subintervals and sample points for the Riemann sum.

Graphical Illustration of the Leftendpoint Riemann Sum

The key feature of a leftendpoint Riemann sum is that the height of each rectangle is determined by the function value at the left endpoint of each subinterval.

Visual description:

- The interval $[a, b]$ is divided into n equal subintervals.
- For each subinterval $[x_{i-1}, x_i]$, the rectangle height is $f(x_{i-1})$.
- The width of each rectangle is $\Delta x = \frac{b - a}{n}$.
- The sum of the areas of these rectangles approximates the area under $f(x)$ from a to b .

Why use a left endpoint?

- It provides an underestimate or overestimate depending on the function's monotonicity.
- For decreasing functions like $(13/x)$, the left endpoint method tends to overestimate the area.

Mathematical Derivation of the Leftendpoint Riemann Sum

Suppose we are evaluating the Riemann sum over an interval $[a, b]$. The general process involves:

1. Partition the interval:

$$\text{Divide } [a, b] \text{ into } n \text{ equal subintervals}$$

$$\Delta x = \frac{b - a}{n}$$

$$x_i = a + i \Delta x, \quad i = 0, 1, 2, \dots, n$$

2. Determine the left endpoints:

$$x_{i-1} = a + (i - 1) \Delta x$$

for $(i = 1, 2, \dots, n)$.

3. Construct the sum:

$$L_n = \sum_{i=1}^n F(x_{i-1}) \Delta x$$

Applying this to $(F(x) = \frac{13}{x})$:

$$L_n = \sum_{i=1}^n \frac{13}{x_{i-1}} \Delta x$$

where

$$x_{i-1} = a + (i - 1) \Delta x$$

and

$$\Delta x = \frac{b - a}{n}$$

Choosing the Interval and Number of Subintervals

The specific interval $([a, b])$ over which the Riemann sum is computed significantly affects the approximation. For the purpose of illustration, assume:

- Interval: $([1, 4])$
- Number of subintervals: $(n = 4)$

This choice simplifies calculations and provides a clear visual depiction.

Calculations:

$$\Delta x = \frac{4 - 1}{4} = \frac{3}{4} = 0.75$$

Left endpoints:

$$x_0 = 1, \text{quad } x_1 = 1.75, \text{quad } x_2 = 2.5, \text{quad } x_3 = 3.25$$

Rectangles' heights:

$$F(x_0) = \frac{13}{1} = 13$$

$$F(x_1) = \frac{13}{1.75} \approx 7.43$$

$$F(x_2) = \frac{13}{2.5} = 5.2$$

$$F(x_3) = \frac{13}{3.25} \approx 4.00$$

Approximate area:

$$L_4 = \left(13 + 7.43 + 5.2 + 4.00\right) \times 0.75$$

$$L_4 \approx (29.63) \times 0.75 \approx 22.22$$

This value approximates the area under $(F(x) = 13/x)$ over $([1, 4])$ using four rectangles with left endpoints.

Analyzing the Accuracy and Limit Behavior

The accuracy of the Riemann sum improves as the number of subintervals (n) increases. In the limit, as $(n \rightarrow \infty)$, the Riemann sum approaches the exact value of the definite integral:

$$\lim_{n \rightarrow \infty} L_n = \int_a^b \frac{13}{x} dx$$

Calculating this integral:

$$\int_a^b \frac{13}{x} dx = 13 \int_a^b \frac{1}{x} dx = 13 [\ln|x|]_a^b = 13 (\ln b - \ln a)$$

Applying the specific interval:

$$\int_1^4 \frac{13}{x} dx = 13 (\ln 4 - \ln 1) = 13 \ln 4$$

Since $(\ln 1 = 0)$, the exact area is:

$$13 \ln 4 \approx 13 \times 1.386 \approx 18.02$$

Notice that the left endpoint Riemann sum with $(n=4)$ overestimates this value (~ 22.22 vs. 18.02). Increasing the number of subintervals would bring the approximation closer to the actual integral.

Significance of the Leftendpoint Riemann Sum in Calculus

The leftendpoint Riemann sum is a fundamental concept in understanding the approximation of integrals, especially in numerical methods. It provides insight into the behavior of functions and the effects of choosing different sample points.

Applications:

- Numerical integration: Estimating integrals when an analytical solution is complex or impossible.
- Understanding function behavior: Recognizing if the sum overestimates or underestimates the true area based on the function's monotonicity.
- Educational purposes: Illustrating the concept of limits and convergence in calculus.

Comparison with Other Riemann Sums:

- Right endpoint sum: Uses the right endpoints for height; may underestimate or overestimate depending on the function.
- Midpoint sum: Uses midpoints; often provides a better approximation.
- Trapezoidal rule: Averages the left and right sums; improves accuracy.

Practical Implications and Further Explorations

Understanding the leftendpoint Riemann sum for $(F(x) = 13/x)$ is just a stepping stone toward mastering integral calculus. Students and practitioners can extend this knowledge by:

- Increasing the number of subintervals to improve approximation accuracy.

- Comparing left, right, and midpoint sums to understand their relative effectiveness.
- Applying similar techniques to other functions with different behaviors (increasing, decreasing, oscillatory).
- Using computational tools and software for large n to perform precise numerical integrations.

Conclusion

The leftendpoint Riemann sum for the function $F(x) = 13/x$ offers a tangible way to approximate the area under the curve over a specified

Frequently Asked Questions

What does the function $F(x) = 13/x$ represent in the context of Riemann sums?

$F(x) = 13/x$ is the function being integrated, and the Riemann sum approximates the area under its curve over a specified interval.

How do the rectangles in the leftendpoint Riemann sum relate to the function $F(x)$?

Each rectangle's height is determined by the value of $F(x)$ at the left endpoint of each subinterval.

What is the significance of using left endpoints in the Riemann sum for this function?

Using left endpoints provides an approximation of the area that tends to underestimate or overestimate the true value depending on whether the function is decreasing or increasing over the interval.

How does increasing the number of rectangles affect the accuracy of the Riemann sum for $F(x) = 13/x$?

Increasing the number of rectangles results in a finer partition, which generally increases the accuracy of the Riemann sum as it better approximates the integral.

What is the geometric interpretation of the leftendpoint Riemann sum for this function?

It represents the total area of rectangles under the curve of $F(x)$, with heights determined by the function's value at the left endpoints of each subinterval.

Can the leftendpoint Riemann sum be used to estimate the definite integral of $F(x)$ from a to b ?

Yes, it provides an approximation of the definite integral of $F(x)$ over the interval $[a, b]$.

What challenges might arise when using a leftendpoint Riemann sum on $F(x) = 13/x$ near $x=0$?

Since $13/x$ is undefined at $x=0$ and tends to infinity as x approaches zero, the sum must be computed over an interval away from zero to avoid divergence.

How does the shape of $F(x) = 13/x$ influence the choice of partition points for the Riemann sum?

Because $F(x)$ decreases as x increases, choosing equal subintervals and left endpoints can give a consistent approximation, but more refined, non-uniform partitions may improve accuracy.

What is the limit of the Riemann sum as the number of rectangles approaches infinity?

It approaches the exact value of the definite integral of $F(x) = 13/x$ over the specified interval, representing the true area under the curve.

How can understanding the leftendpoint Riemann sum help in understanding the definite integral of $F(x)$?

It provides a concrete approximation method that illustrates how the area under the curve accumulates, offering insight into the concept of integration as a limit of sums.

Additional Resources

In-Depth Analysis of the Left-Endpoint Riemann Sum for $(F(x) = \frac{13}{x})$

Introduction to Riemann Sums and Their Significance

Understanding the concept of Riemann sums is fundamental to grasping the principles of integral calculus. Riemann sums serve as a bridge between the intuitive notion of summing areas of rectangles and the formal definition of the definite integral. They provide an approximation of the area under a curve over a specified interval by partitioning the region into smaller, manageable rectangles.

In particular, the left-endpoint Riemann sum involves constructing rectangles whose heights are

determined by the function value at the left endpoint of each subinterval. As the number of rectangles increases (and the width of each decreases), the Riemann sum approaches the exact value of the definite integral, reflecting the fundamental theorems of calculus.

Understanding the Function $(F(x) = \frac{13}{x})$

Before delving into the Riemann sum specifics, it is crucial to understand the behavior and properties of the function $(F(x) = \frac{13}{x})$:

- Domain: The function is defined for all $(x \neq 0)$. For the purpose of integration on a finite interval, typically $(x > 0)$ or $(x < 0)$.
- Range: The output values are all real numbers except zero (since (x) cannot be zero). As $(x \rightarrow 0^+)$, $(F(x) \rightarrow +\infty)$; as $(x \rightarrow +\infty)$, $(F(x) \rightarrow 0^+)$.
- Shape and Behavior: The graph of $(F(x))$ is a hyperbola with a vertical asymptote at $(x = 0)$. It is decreasing on its domain, and symmetric with respect to the axes if extended to negative (x) .
- Continuity: The function is continuous on any interval that does not include zero.

This behavior impacts how Riemann sums are calculated because the function's values change significantly over the interval, especially near the asymptote at zero.

Setting Up the Riemann Sum

Suppose we are asked to approximate the definite integral of $(F(x) = \frac{13}{x})$ over a specific interval, say $([a, b])$. To analyze the left-endpoint Riemann sum:

1. Partition the Interval

- Divide $([a, b])$ into (n) subintervals of equal width (Δx) .

- Calculate (Δx) :

$$\Delta x = \frac{b - a}{n}$$

- Determine Subintervals:

$$x_0 = a, \quad x_1 = a + \Delta x, \quad x_2 = a + 2\Delta x, \quad \dots, \quad x_n = b$$

2. Identify Left Endpoints

- For each subinterval $[x_k, x_{k+1}]$, the left endpoint is x_k .
- The height of each rectangle is determined by $F(x_k) = \frac{13}{x_k}$.

3. Construct the Riemann Sum

- The sum approximates the area under the curve:

$$L_n = \sum_{k=0}^{n-1} F(x_k) \Delta x$$

- Interpretation: This sum adds up the areas of rectangles with bases Δx and heights $F(x_k)$.

Visual Representation and the Significance of the Graph

The graph on the left, illustrating the rectangles, plays a vital role in understanding the approximation process:

- Rectangles: Each rectangle is positioned directly above its left endpoint, capturing the function's height at that point.
- Left-Endpoint Illustration: This method tends to underestimate or overestimate the true area depending on whether the function is decreasing or increasing over the interval.
- Behavior for $F(x) = \frac{13}{x}$:
 - Since $F(x)$ is decreasing for $x > 0$, the left-endpoint Riemann sum overestimates the actual area when integrating over an interval in the positive domain.
 - For example, over the interval $[a, b]$ with $0 < a < b$, the left endpoints are higher than the midpoints or right endpoints, leading to an overestimation.
- Importance of the Partition Size:
 - As $n \rightarrow \infty$ (meaning $\Delta x \rightarrow 0$), the sum approaches the exact integral.
 - The rectangles become narrower, capturing the curve more accurately, and the approximation improves.

Mathematical Derivation of the Left-Endpoint Riemann Sum

Let's formalize the process with specific steps:

1. Choose the Interval

Suppose the interval is $[a, b]$. For example, let's consider $[1, 4]$:

- $a = 1$, $b = 4$.

2. Determine n and Δx

- For illustrative purposes, choose $n = 3$:

$$\Delta x = \frac{4 - 1}{3} = 1$$

- The subintervals are:

$$[1, 2], \quad [2, 3], \quad [3, 4]$$

- The left endpoints are:

$$x_0 = 1, \quad x_1 = 2, \quad x_2 = 3$$

3. Calculate $F(x_k)$ at left endpoints

$$F(1) = \frac{13}{1} = 13$$

$$F(2) = \frac{13}{2} = 6.5$$

$$F(3) = \frac{13}{3} \approx 4.333$$

4. Compute the Riemann sum

$$L_3 = \sum_{k=0}^2 F(x_k) \times \Delta x = (13 + 6.5 + 4.333) \times 1 = 23.833$$

This value approximates the area under the curve $(F(x) = \frac{13}{x})$ from 1 to 4 using three rectangles.

Limit of the Riemann Sum and Connection to the Definite Integral

In the limit as $(n \rightarrow \infty)$ (and thus $(\Delta x \rightarrow 0)$), the sum:

$$\lim_{n \rightarrow \infty} L_n = \int_a^b \frac{13}{x} \, dx$$

Calculating this integral:

$$\int_a^b \frac{13}{x} \, dx = 13 \int_a^b \frac{1}{x} \, dx = 13 [\ln|x|]_a^b = 13 (\ln|b| - \ln|a|) = 13 \ln \left| \frac{b}{a} \right|$$

This demonstrates that the Riemann sum converges to the natural logarithm scaled by 13, which is consistent with the properties of the logarithmic integral.

Practical Considerations and Applications

Understanding the process of constructing and analyzing the left-endpoint Riemann sum for $(F(x) = \frac{13}{x})$ has numerous applications:

- **Approximate Area Under Curves:** In many real-world scenarios, exact integration may be complex or impossible analytically. Riemann sums provide a practical method for approximation.
- **Numerical Integration Techniques:** The left-endpoint method is one of several numerical approaches, alongside right-endpoint, midpoint, trapezoidal, and Simpson's rules.
- **Modeling in Physics and Economics:** For functions like $(1/x)$, which appear in decay processes, economic models involving inverse relationships, or rate functions, understanding the integral and its approximation is essential.
- **Evaluating the Effect of Partition Size:** By increasing (n) , one can observe how the approximation converges, providing insights into the accuracy of numerical methods.

Comparison Between Different Riemann Sum Methods

While the focus here is on the left-endpoint Riemann sum, it's instructive to contrast it with other methods:

- Right-Endpoint Riemann Sum: Uses the function value at the right endpoint of each subinterval. For decreasing functions like $\frac{1}{x}$, this tends to underestimate the area.
- Midpoint Riemann Sum: Uses the function value at the midpoint of each subinterval, often providing a better approximation.
- Trapezoidal Rule: Averages the left and right sums, often yielding higher accuracy.
- Simpson's Rule: Uses quadratic polynomials to approximate the curve, providing even more precise results for smooth functions

Suppose $f(x) = \frac{1}{x}$ The Rectangles In The Graph On The Left Illustrate A Leftendpoint Riemann Sum For

Suppose $f(x) = \frac{1}{x}$ The Rectangles In The Graph On The Left Illustrate A Leftendpoint Riemann Sum For

Related Articles

- [The Smaller Circles Are Congruent. Find The Area Of The Shaded Region. Round Your Answer To The Nearest](#)
- [The Difference Between The Marginal Social Cost Curve And The Marginal Cost Of Production Curve Is The:](#)
- [True Or False: Consider A Voice Recognition System That Labels A Given Sound Signal As Belonging To One](#)

[Back to Home](#)