Suppose The First Derivative Of F(x) Is F'(x) = X3 6x + 11x 6. - Where Is F(x) Increasing? Use Interval

Suppose The First Derivative Of F(x) Is F'(x) = X3 6x + 11x 6. - Where Is F(x) Increasing? Use Interval

Understanding where a function is increasing or decreasing is a fundamental aspect of calculus, providing insights into the function's behavior and its graph's shape. In this article, we will analyze the derivative F'(x) = x^3 - 6x + 11x^6 to determine the intervals where the original function F(x) is increasing. This involves examining the sign of the derivative, solving inequalities, and interpreting the results in terms of intervals on the real number line.

Understanding the Derivative and Its Significance

The Role of the First Derivative

The first derivative of a function, F'(x), measures the rate at which F(x) changes with respect to x. Its sign indicates the nature of the function's slope:

- If F'(x) > 0, then F(x) is increasing on that interval.
- If F'(x) < 0, then F(x) is decreasing on that interval.
- If F'(x) = 0, then F(x) may have a local maximum, minimum, or a point of inflection at that point.

Given Derivative Function

In our case, F'(x) = x^3 - 6x + 11x^6. To analyze where F(x) is increasing, we must determine where F'(x) > 0.

Analyzing the Derivative F'(x) = x^3 - 6x + 11x^6

Rewriting the Derivative

Let’s write the derivative in a standard polynomial form:
\[
F'(x) = 11x^6 + x^3 - 6x
\]
This polynomial combines terms of different degrees, with the highest degree being 6.

Factoring the Derivative

Factoring helps identify critical points and analyze the sign of F'(x):
1. Factor out common factors:
$$
F'(x) = x (11x^5 + x^2 - 6)
$$
2. Now, analyze the quadratic and higher-degree polynomial:
$$
11x^5 + x^2 - 6
$$
While not all of these terms are straightforward to factor, we can attempt to factor F'(x) completely or find roots by other means.

Finding Critical Points (Roots of F'(x))

Critical points occur where F'(x) = 0:

$$
x (11x^5 + x^2 - 6) = 0
$$

This gives us:
$$
x = 0
$$
or
$$
11x^5 + x^2 - 6 = 0
$$

The first root is straightforward: x=0.

The second equation, $11x^5 + x^2 - 6 = 0$, is of higher degree. To find solutions:
- Use Rational Root Theorem to test potential rational roots.
- Rational roots are factors of 6 divided by factors of 11: $\pm1, \pm2, \pm3, \pm6, \pm \frac{1}{11}, \pm \frac{2}{11}, \pm \frac{3}{11}, \pm \frac{6}{11}$.

Test x=1:
$$
11(1)^5 + (1)^2 - 6 = 11 + 1 - 6 = 6 \neq 0
$$

Test x=-1:
$$
11(-1)^5 + (-1)^2 - 6 = -11 + 1 - 6 = -16 \neq 0
$$

Test x=2:
$$

$$
11(32) + 4 - 6 = 352 + 4 - 6 = 350 \neq 0
$$

Test x=-2:

$$
11(-32) + 4 - 6 = -352 + 4 - 6 = -354 \neq 0
$$

Test x=3:

$$
11(243) + 9 - 6 = 2673 + 9 - 6 = 2676 \neq 0
$$

Test x=-3:

$$
11(-243) + 9 - 6 = -2673 + 9 - 6 = -2670 \neq 0
$$

Test x=6:

$$
11(6)^5 + (6)^2 - 6 = 11(7776) + 36 - 6 = 85536 + 36 - 6 = 85566 \neq 0
$$

Test x=-6:

$$
11(-6)^5 + 36 - 6 = -11(7776) + 36 - 6 = -85536 + 36 - 6 = -85506 \neq 0
$$

Testing fractional roots yields unlikely solutions; thus, the roots of $11x^5 + x^2 - 6 = 0$ may not be rational or easily expressible. Numerical methods or graphing tools are appropriate here.

Using Numerical and Graphical Methods to Find Critical Points

Graphical Approach

Plotting F'(x) helps visually identify where it crosses zero:
- The graph of $F'(x) = 11x^6 + x^3 - 6x$ shows where the derivative is positive or negative.
- From the graph, observe the points where the curve crosses the x-axis—these are the critical points.

Numerical Approximation of Roots

Utilize tools such as a graphing calculator or software (Desmos, WolframAlpha, GeoGebra) to approximate the roots:
- Approximate critical points around x ≈ -0.6, x ≈ 0, and x ≈ 0.6.

Suppose these approximate roots are:

$$
x \approx -0.65,\quad x=0,\quad x \approx 0.65
$$

These roots partition the real line into intervals:

$$
(-\infty, -0.65),\quad (-0.65, 0),\quad (0, 0.65),\quad (0.65, \infty)
$$

Sign Analysis of F'(x) on Intervals

Determining the sign of F'(x) in each interval involves selecting test points and evaluating the derivative.

Test Point in $(-\infty, -0.65)$

Choose x = -1:

$$
F'(-1) = 11(-1)^6 + (-1)^3 - 6(-1) = 11(1) - 1 + 6 = 11 - 1 + 6 = 16 > 0
$$

Hence, F'(x) > 0 in this interval, so F(x) is increasing here.

Test Point in $(-0.65, 0)$

Choose x = -0.5:

$$
F'(-0.5) = 11(0.5)^6 + (-0.5)^3 - 6(-0.5)
$$

Calculate:

$$
(0.5)^6 = 0.015625
$$

$$
11 \times 0.015625 = 0.171875
$$

$$
(-0.5)^3 = -0.125
$$

$$
-6 \times -0.5 = +3
$$

Sum:

$$
0.171875 - 0.125 + 3 = 3.046875 > 0
$$

Thus, F'(x) > 0 in $(-0.65, 0)$.

Test Point in $(0, 0.65)$

Choose x=0.5:

$$
F'(0.5) = 11(0.5)^6 + (0.5)^3 - 6(0.5)
$$

Calculate:

$$
(0.5)^6 = 0.015625
$$

$$
11 \times 0.015625 = 0.171875
$$

$$
(0.5)^3 = 0.125
$$

$$
-6 \times 0.5 = -3
$$

Sum:

$$
0.171875 + 0.125 - 3 = -2.703125 < 0
$$

Therefore, $F'(x) < 0$ in $(0, 0.65)$.

Test Point in $(0.65, \infty)$

Choose x=1:

$$
F'(1) = 11(1)^6 + 1^3 - 6(1) = 11 + 1 - 6 = 6 > 0
$$

Thus, $F'(x) > 0$ in $(0.65, \infty)$.

Summary of Sign Analysis

Based on the test points:
- F'(x)

Frequently Asked Questions

Given the derivative F'(x) = x^3 - 6x + 11x^6, where is the function F(x) increasing?

F(x) is increasing where its derivative F'(x) > 0. To determine this, analyze the sign of F'(x) considering the polynomial x^3 - 6x + 11x^6 and find the intervals where it is positive.

How do you find the intervals where F(x) is increasing based on F'(x)?

First, find the critical points by solving F'(x) = 0, then test the sign of F'(x) in the intervals around these points. F(x) is increasing where F'(x) > 0.

What are the steps to determine where F'(x) = x^3 - 6x + 11x^6 is positive?

Step 1: Rewrite the derivative. Step 2: Factor or analyze the polynomial to find its roots. Step 3: Use test points in each interval between roots to determine the sign of F'(x).

Can F'(x) be factored to find where F(x) is increasing?

Yes, factoring F'(x) helps identify critical points. For the polynomial x^3 - 6x + 11x^6, factor out common terms or use substitution to find roots, then analyze the sign of F'(x).

What is the significance of the critical points of F'(x) in determining where F(x) increases?

Critical points are where F'(x) = 0. The sign change of F'(x) around these points indicates intervals where F(x) is increasing (F'(x) > 0) or decreasing (F'(x) < 0).

Is the polynomial F'(x) = x^3 - 6x + 11x^6 increasing or decreasing at large |x|?

As |x| becomes large, the term 11x^6 dominates, making F'(x) positive for large positive x, indicating F(x) is increasing at large positive x. For large negative x, since x^6 is positive, the same applies, so F'(x) is positive at large |x|, and F(x) increases there.

How does the degree of the polynomial F'(x) = x^3 - 6x + 11x^6 affect the intervals of increase?

Since the highest degree term is 11x^6 (even degree, positive coefficient), F'(x) tends to infinity as |x| → ∞, and the polynomial's roots determine the change in signs, influencing the intervals where F(x) increases.

What tools can be used to analyze the sign of F'(x) = x^3 - 6x + 11x^6?

Tools include factoring the polynomial, using the Rational Root Theorem, synthetic division, or numerical methods like graphing to find roots and test intervals.

Why is understanding where F'(x) > 0 important in calculus?

Because it tells us where the original function F(x) is increasing, which is crucial for understanding the function's behavior, graphing, and optimization problems.

Can the intervals where F(x) is increasing be expressed explicitly for the given derivative?

Yes, once the roots of F'(x) are found, the intervals of increase are the regions where $F'(x) > 0$, which can be expressed explicitly based on those critical points.

Additional Resources

Understanding the Behavior of Functions Through Derivatives: An Expert Analysis of $F'(x) = x^3 - 6x + 11x^6$

Introduction

In the realm of calculus, the derivative of a function serves as a powerful tool to analyze the function’s behavior—particularly, where it is increasing or decreasing. When examining a function F(x), understanding the sign of its first derivative, F'(x), across different intervals provides critical insights into its overall shape and characteristics.

Today, we delve deep into a specific derivative—$F'(x) = x^3 - 6x + 11x^6$—and explore where this function is increasing by analyzing its interval behavior. This examination not only uncovers the nuanced behavior of the underlying function but also demonstrates how calculus concepts like critical points, sign analysis, and interval testing come together to paint a comprehensive picture.

The Significance of the First Derivative

Before diving into the specifics, it’s important to understand the foundational role of the first derivative:

- $F'(x) > 0$ indicates that the function F(x) is increasing on that interval.
- $F'(x) < 0$ indicates that F(x) is decreasing.
- $F'(x) = 0$ marks potential critical points, which can be local maxima, minima, or points of inflection depending on the behavior of F'(x) around these points.

Our task: Identify the intervals where $F'(x) > 0$, i.e., where the function is increasing.

Step 1: Analyzing the Derivative F'(x)

Given:
$$
F'(x) = x^3 - 6x + 11x^6
$$

This derivative combines polynomial terms of different degrees, with the dominant term being $11x^6$. As $x \to \pm \infty$, the $11x^6$ term dominates, indicating that the derivative tends toward positive infinity for large magnitudes of x.

Step 2: Factoring and Simplifying F'(x)

To analyze the sign of F'(x), it’s prudent to factor where possible:

$$
F'(x) = 11x^6 + x^3 - 6x
$$

Notice that the polynomial contains terms with powers 6, 3, and 1. Let's attempt to factor out common factors:

- The lowest degree term is $-6x$ and the highest is $11x^6$.
- The middle term x^3 can be factored or combined with others.

First, factor out an x:

$$
F'(x) = x(11x^5 + x^2 - 6)
$$

Now, the expression becomes:

$$
F'(x) = x \cdot (11x^5 + x^2 - 6)
$$

This factorization is crucial because it clearly shows that $x=0$ is a critical point, and the sign of $F'(x)$ depends on the signs of both factors.

Step 3: Find Critical Points

Critical points occur at values of x where $F'(x) = 0$:

$$
x(11x^5 + x^2 - 6) = 0
$$

This yields two cases:

1. When $x=0$: Directly, $F'(0) = 0$.

2. When $11x^5 + x^2 - 6=0$: This is a quintic in x, which is complex to solve algebraically. However, we can analyze its roots qualitatively or numerically.

Step 4: Analyzing the Roots of $11x^5 + x^2 - 6=0$

Given the complexity of solving this equation analytically, we turn to methods such as:

- Sign analysis of the polynomial for different x values.
- Numerical approximation or graphing to identify roots.

Let's evaluate the polynomial at strategic points:

x	$11x^5 + x^2 - 6$	Sign
-1	$11(-1)^5 + (-1)^2 - 6 = -11 + 1 - 6 = -16$	Negative
0	$0 + 0 - 6 = -6$	Negative
0.5	$11(0.5)^5 + (0.5)^2 - 6 \approx 11 \times 0.03125 + 0.25 - 6 \approx 0.344 + 0.25 - 6 = -5.406$	Negative
1	$11(1)^5 + 1 - 6 = 11 + 1 - 6 = 6$	Positive
1.2	$11(1.2)^5 + (1.2)^2 - 6$ — approximate: $11 \times 2.488 + 1.44 - 6 \approx 27.37 + 1.44 - 6 = 22.81$	Positive

From these values, the polynomial is negative at $x=0$ and positive at $x=1$. Since it changes from negative to positive between $x=0.5$ and $x=1$, there exists at least one root in this interval.

Similarly, at $x=-1$, the polynomial is negative, and at $x=0$, it’s negative, so no root in -1 to 0.

Conclusion:

- There is one real root of $11x^5 + x^2 - 6=0$ in $(0.5, 1)$.
- For $x<0$, the polynomial remains negative, indicating no additional roots there.

Step 5: Sign Analysis of $F'(x)$

Recall:

$$
F'(x) = x \cdot (11x^5 + x^2 - 6)
$$

- For $x<0$:

- $x<0$,
- $11x^5 + x^2 - 6$ is negative (from previous evaluations),
- Product of two negatives is positive.

Therefore, F'(x) > 0 for all $x<0$.

- At $x=0$:

- $F'(0) = 0$.

- For $x>0$:

- $x>0$,
- $11x^5 + x^2 - 6$:

- Negative for $0 < x < r$ (where r is the root between 0.5 and 1),
- Positive for $x > r$.

From the sign change:

- For $0 < x < r$: $11x^5 + x^2 - 6 < 0$, so $F'(x) = (+) \times (-) = -$, indicating F(x) decreasing.

- For $x > r$: $11x^5 + x^2 - 6 > 0$, so $F'(x) = (+) \times (+) = +$, indicating F(x) increasing.

Summary of critical points and sign:

Interval	Sign of $F'(x)$	Behavior of $F(x)$
$(-\infty, 0)$	positive	Increasing
0	zero	Critical point
$(0, r)$	negative	Decreasing
r	zero	Critical point
(r, ∞)	positive	Increasing

Final Conclusion: Where Is F(x) Increasing?

Based on the above detailed analysis, the function $F(x)$ is increasing on the intervals:

$$
\boxed{
(-\infty, 0) \quad \text{and} \quad (r, \infty)
}
$$

where r is the real root of $11x^5 + x^2 - 6=0$ lying between 0.5 and 1.

Implications and Insights

This comprehensive exploration emphasizes the importance of derivative analysis in understanding function behavior:

- Negative $F'(x)$ indicates the function is decreasing.
- Positive $F'(x)$ indicates the function is increasing.
- Critical points mark potential local maxima or minima, aiding in understanding the function's turning points.

By combining algebraic factorization, sign analysis, and numerical approximation, we can accurately determine the intervals of increase, even when dealing with complex polynomial derivatives.

In Summary

- The derivative $F'(x) = x^3 - 6x + 11x^6$ factors as $x(11x^5 + x^2 - 6)$.
- The function $F(x)$ is increasing where $F'(x) > 0$, which occurs:

- For all

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