# Suppose X Is A Uniform Random Variable Over The Interval [20, 90]. Find The Probability That A Randomly

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## Introduction to Uniform Random Variables

A uniform random variable is one of the simplest types of probability distributions, characterized by the property that every outcome within a specific interval has an equal likelihood of occurring. When dealing with continuous uniform distributions, the probability density function (PDF) remains constant over the interval, signifying that the chance of the variable falling within any subinterval depends solely on the length of that subinterval.

In this article, we explore the properties of a uniform random variable X, which is uniformly distributed over the interval [20, 90]. Our goal is to determine the probability that a randomly selected value of X falls within a particular subinterval or meets certain criteria. This involves understanding the fundamental concepts of the uniform distribution, calculating probabilities, and applying these principles to various scenarios.

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## Understanding the Uniform Distribution over [20, 90]

### Definition and Properties

A uniform distribution over the interval [a, b], denoted as X ~ U(a, b), has the following characteristics:

- Equal Likelihood: Every value within [a, b] is equally likely.
- Constant PDF: The probability density function is constant across the interval.
- Support: The distribution is supported on [a, b], meaning X cannot take values outside this interval.

For our specific case, X ~ U(20, 90):

- Interval: [20, 90]
- Length of the interval: $b - a = 90 - 20 = 70$

The constant probability density function (PDF) for X is:

$$
f_X(x) = \frac{1}{b - a} = \frac{1}{70}
$$

for all $x \in [20, 90]$, and $f_X(x) = 0$ otherwise.

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# Calculating Probabilities for Uniform Distributions

## Basic Probability Calculation

Since the distribution is uniform, the probability that X falls within a subinterval [c, d] within [20, 90] can be calculated using:

$$
P(c \leq X \leq d) = \frac{\text{Length of } [c, d] \cap [20, 90]}{\text{Length of } [20, 90]} = \frac{\text{Adjusted interval length}}{70}
$$

where the intersection ensures that the subinterval is within the support of the distribution.

Key points for calculating probabilities:

- If the subinterval [c, d] is entirely within [20, 90], then

$$
P(c \leq X \leq d) = \frac{d - c}{70}
$$

- If [c, d] extends beyond [20, 90], adjust to the intersection:

$$
P(c \leq X \leq d) = \frac{\max(0, \min(d, 90) - \max(c, 20))}{70}
$$

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## Examples of Probability Calculations

1. Probability that X is less than 30:

$$
P(X < 30) = P(20 \leq X < 30) = \frac{30 - 20}{70} = \frac{10}{70} = \frac{1}{7}
$$

2. Probability that X is greater than 80:

$$

$$
P(X > 80) = P(80 < X \leq 90) = \frac{90 - 80}{70} = \frac{10}{70} = \frac{1}{7}
$$

3. Probability that X falls between 40 and 60:

$$
P(40 \leq X \leq 60) = \frac{60 - 40}{70} = \frac{20}{70} = \frac{2}{7}
$$

4. Probability that X is between 15 and 25:

Since 15 is outside the support on the lower end, adjust to [20, 25]:

$$
P(15 \leq X \leq 25) = \frac{25 - 20}{70} = \frac{5}{70} = \frac{1}{14}
$$

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# General Approach to Finding Probabilities

## Step-by-Step Methodology

To solve probability questions involving a uniform distribution over [20, 90], follow these steps:

1. Identify the subinterval of interest, say [c, d].
2. Determine the intersection of [c, d] with the support [20, 90], i.e., $[\max(c, 20), \min(d, 90)]$.
3. Calculate the length of this intersection: $\min(d, 90) - \max(c, 20)$.
4. Divide this length by the total interval length, which is 70, to find the probability.

Mathematically:

$$
P(c \leq X \leq d) = \frac{\max(0, \min(d, 90) - \max(c, 20))}{70}
$$

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# Applications of the Uniform Distribution

# Probability

## Scenario 1: Probability that X is within a specific subinterval

Suppose you are asked: What is the probability that a randomly chosen value of X falls between 25 and 75?

- Step 1: Check if the subinterval is within [20, 90]. Yes, [25, 75] is within.

- Step 2: Calculate the length:

$$
75 - 25 = 50
$$

- Step 3: Divide by total length:

$$
P(25 \leq X \leq 75) = \frac{50}{70} = \frac{5}{7}
$$

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## Scenario 2: Probability that X exceeds a certain threshold

What is the probability that X > 85?

- Step 1: The intersection is [85, 90].

- Step 2: Length:

$$
90 - 85 = 5
$$

- Step 3: Probability:

$$
P(X > 85) = \frac{5}{70} = \frac{1}{14}
$$

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# Expected Value and Variance of X

Beyond probabilities of specific intervals, it is often useful to understand the expected value (mean) and variance of a uniform random variable.

### Expected Value (Mean)

For a uniform distribution over [a, b], the expected value:

$$
E[X] = \frac{a + b}{2}
$$

In our case:

$$
E[X] = \frac{20 + 90}{2} = \frac{110}{2} = 55
$$

This indicates that, on average, a randomly selected value from the distribution will be around 55.

### Variance

The variance for a uniform distribution:

$$
Var[X] = \frac{(b - a)^2}{12}
$$

Calculating for our interval:

$$
Var[X] = \frac{70^2}{12} = \frac{4900}{12} \approx 408.33
$$

The standard deviation (spread) of X is:

$$
\sigma = \sqrt{408.33} \approx 20.2
$$

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## Visualizing the Distribution

A helpful way to understand uniform distributions is through their probability density function graph. Since the distribution is uniform over [20, 90], the PDF is constant at $\frac{1}{70}$ between these points, forming a rectangle with height $\frac{1}{70}$.

This visualization emphasizes that:

- The probability of X falling into any subinterval is proportional to the width.
- The total area under the PDF over the support is 1.

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# Practical Applications and Implications

Understanding the properties of uniform random variables has numerous applications:

- Simulations and Random Sampling: Uniform distributions are fundamental in generating random samples for simulations.
- Modeling Equal Likelihood Events: When outcomes are equally likely within an interval, uniform distributions are appropriate.
- Estimating Probabilities in Real-Life Scenarios: For example, estimating the probability that a randomly selected product falls within a certain quality range.

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# Summary and Key Takeaways

- A uniform random variable over [20, 90] assigns equal probability density to all values within the interval.
- The probability of falling within any subinterval can be calculated via the ratio of the subinterval length to the total interval length.
- Key formulas:

$$
f_X(x) = \frac{1}{70} \quad \text{for } x \in [20, 90]
$$

$$
P(c \leq X \leq d) = \frac{\max(0, \min(d, 90) - \max(c, 20))}{70}
$$

- The expected value of X is 55

# Frequently Asked Questions

## Suppose X is a uniform random variable over the interval [20, 90]. What is the probability that X is less than 50?

Since X is uniform over [20, 90], P(X < 50) = (50 - 20) / (90 - 20) = 30 / 70 = 3/7.

## Suppose X is uniformly distributed over [20, 90]. What is the probability that X is greater than 75?

P(X > 75) = (90 - 75) / (90 - 20) = 15 / 70 = 3/14.

## If X is uniform over [20, 90], what is the

## probability that X falls between 40 and 60?

P(40 ≤ X ≤ 60) = (60 - 40) / (90 - 20) = 20 / 70 = 2/7.

## Suppose X is a uniform random variable over [20, 90]. What is the probability that X equals exactly 50?

For a continuous uniform distribution, the probability of X taking any exact value (like 50) is zero.

## Given X ~ Uniform[20, 90], what is the probability that X is between 30 and 80?

P(30 ≤ X ≤ 80) = (80 - 30) / (90 - 20) = 50 / 70 = 5/7.

# Additional Resources

Suppose X Is A Uniform Random Variable Over The Interval [20, 90]. Find The Probability That A Randomly

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Introduction

In the realm of probability and statistics, the uniform distribution is one of the most fundamental and intuitive models. It describes a scenario where every outcome within a specified interval is equally likely to occur. This property makes it a cornerstone for understanding more complex distributions and serves as an essential building block for statistical inference, simulation, and decision-making processes.

This article delves into a specific case: suppose $X$ is a uniform random variable over the interval $[20, 90]$. We will explore how to determine the probability of various events involving $X$, including the likelihood that $X$ falls within a particular subinterval, exceeds a certain value, or lies outside a specified range. Through rigorous analysis, we aim to illuminate the practical applications of uniform distributions and demonstrate the step-by-step procedures to compute these probabilities.

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Understanding the Uniform Distribution

Definition and Properties

A continuous uniform random variable $X$ over the interval $[a, b]$, denoted as $X \sim U(a, b)$, is characterized by the following:

- Equal Likelihood: Every outcome within $[a, b]$ has an equal probability density.
- Probability Density Function (PDF):

$$
f_X(x) = \frac{1}{b - a}, \quad \text{for } x \in [a, b]
$$
and $f_X(x) = 0$ elsewhere.

- Cumulative Distribution Function (CDF):

$$
F_X(x) = P(X \leq x) =
\begin{cases}
0, & x < a \\
\frac{x - a}{b - a}, & a \leq x \leq b \\
1, & x > b
\end{cases}
$$

- Expected Value (Mean):

$$
E[X] = \frac{a + b}{2}
$$

- Variance:

$$
\operatorname{Var}(X) = \frac{(b - a)^2}{12}
$$

Given these properties, the uniform distribution is mathematically straightforward, yet powerful for modeling scenarios with equally likely outcomes.

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Applying the Distribution to the Interval [20, 90]

In our specific case, $X \sim U(20, 90)$. Here, the parameters are:

- $a = 20$
- $b = 90$

The total length of the interval is:

$$
b - a = 90 - 20 = 70
$$

Thus, the probability density function simplifies to:

$$
f_X(x) = \frac{1}{70}, \quad \text{for } x \in [20, 90]
$$

and zero elsewhere.

The expected value is:

$$
E[X] = \frac{20 + 90}{2} = 55
$$

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Calculating Probabilities: Fundamental Techniques

Calculating the probability that $X$ falls within a certain subinterval $[c, d]$, where $20 \leq c < d \leq 90$, involves integrating the PDF over that interval. Since the PDF is constant over $[20, 90]$, the probability simplifies to:

$$
P(c \leq X \leq d) = \int_c^d f_X(x) \, dx = \frac{d - c}{70}
$$

Similarly, for any event involving $X$, we utilize the CDF:

$$
P(X \leq x) = F_X(x) = \frac{x - 20}{70} \quad \text{for } 20 \leq x \leq 90
$$

and for $x < 20$, $P(X \leq x) = 0$; for $x > 90$, $P(X \leq x) = 1$.

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Specific Probability Calculations

1. Probability that $X$ falls within a given subinterval $[c, d]$

Suppose we are asked: What is the probability that $X$ is between 30 and 60?

Applying the formula:

$$
P(30 \leq X \leq 60) = \frac{60 - 30}{70} = \frac{30}{70} = \frac{3}{7} \approx 0.429
$$

This indicates there's approximately a 42.9% chance that a randomly selected value of $X$ from $[20, 90]$ falls between 30 and 60.

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2. Probability that $X$ exceeds a specific value $x_0$

Suppose we want to find: What is the probability that $X$ exceeds 70?

Using the complement rule:

$$
P(X > 70) = 1 - P(X \leq 70)
$$

Compute $P(X \leq 70)$:

$$
P(X \leq 70) = F_X(70) = \frac{70 - 20}{70} = \frac{50}{70} = \frac{5}{7} \approx 0.714
$$

Therefore,

$$
P(X > 70) = 1 - \frac{5}{7} = \frac{2}{7} \approx 0.286
$$

$$
$$

So, there's about a 28.6% chance that $X$ exceeds 70.

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3. Probability that $X$ is outside a certain interval

Suppose we are asked: What is the probability that $X$ is less than 25 or greater than 85?

Since these are two disjoint events:

$$
P(X < 25 \text{ or } X > 85) = P(X < 25) + P(X > 85)
$$

Calculate each:

$$
P(X < 25) = F_X(25) = \frac{25 - 20}{70} = \frac{5}{70} = \frac{1}{14} \approx 0.0714
$$
$$
P(X > 85) = 1 - F_X(85) = 1 - \frac{85 - 20}{70} = 1 - \frac{65}{70} = 1 - \frac{13}{14} = \frac{1}{14} \approx 0.0714
$$

Adding these:

$$
P(X < 25 \text{ or } X > 85) = \frac{1}{14} + \frac{1}{14} = \frac{2}{14} = \frac{1}{7} \approx 0.1429
$$

Thus, approximately a 14.3% chance that $X$ falls outside the interval $[25, 85]$.

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Advanced Considerations: Expectation and Variance

Beyond simple probability calculations, understanding the distribution's mean and variance offers insight into the typical values and spread.

- Expected Value:

$$
E[X] = \frac{20 + 90}{2} = 55
$$

This indicates the "center" or average of the distribution.

- Variance:

$$
\operatorname{Var}(X) = \frac{(90 - 20)^2}{12} = \frac{70^2}{12} = \frac{4900}{12} \approx 408.33
$$

- Standard Deviation:

$$
\sigma = \sqrt{\operatorname{Var}(X)} \approx \sqrt{408.33} \approx 20.21
$$

These parameters are useful for understanding the spread and variability of outcomes.

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Real-World Applications and Implications

Uniform distributions over finite intervals are used in numerous practical contexts:

- Simulation and Modeling: When modeling scenarios where outcomes are equally likely (e.g., rolling a fair die scaled over a continuous range).
- Quality Control: Estimating the probability of a measurement falling within certain bounds when the measurement error is uniformly distributed.
- Decision Analysis: When lacking detailed information, assuming a uniform distribution provides a neutral basis for probabilistic inference.

In our case, knowing the probability that $X$ falls within certain ranges can inform decisions in fields such as manufacturing, risk assessment, and resource allocation.

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Summary and Conclusion

This comprehensive review has explored the properties of a uniform random variable $X \sim U(20, 90)$, providing clear methods to compute the probability of various events. The key takeaways include:

- The probability that $X$ falls within a subinterval $[c, d]$ is proportional to the length of that interval relative to the total length (70 units).
- The probability that $X$ exceeds or is less than a specific value can be directly obtained using the CDF.
- Probabilities involving outside events or unions of disjoint events can be effectively computed through the sum of individual probabilities, leveraging the uniform distribution's simplicity.

Understanding these principles enhances our ability to analyze and interpret uniform random variables across diverse applications. Whether assessing risks, conducting simulations, or making informed decisions, the uniform

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