

The Area Of A Rectangular Room Is 750 Square Feet. The Width Of The Room Is 5 Feet Less Than The Length

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Understanding the dimensions of a room is essential for various purposes, including interior design, renovation planning, furniture placement, and even real estate assessments. In this article, we will explore a specific problem involving a rectangular room with an area of 750 square feet, where the width of the room is 5 feet less than its length. We will analyze how to determine the dimensions of such a room using algebraic methods, interpret the results, and discuss practical applications.

Understanding the Problem

Before diving into calculations, it's important to clearly understand what is being asked:

- The area of the rectangular room is 750 square feet.
- The width of the room is 5 feet less than its length.

Our goal is to find the length and width of the room based on this information.

Defining Variables and Setting Up Equations

To solve this problem, we need to introduce variables:

- Let L represent the length of the room (in feet).
- Since the width is 5 feet less than the length, define W as:

- $W = L - 5$

Given that the area of a rectangle is calculated as:

$$\text{Area} = \text{Length} \times \text{Width}$$

We can express the area as:

$$750 = L \times (L - 5)$$

This is a quadratic equation in terms of L.

Formulating and Solving the Quadratic Equation

Expressed explicitly, the equation becomes:

$$L \times (L - 5) = 750$$

Expanding the equation:

$$L^2 - 5L = 750$$

Rearranged into standard quadratic form:

$$L^2 - 5L - 750 = 0$$

This quadratic equation can be solved using various methods: factoring (if possible), completing the square, or the quadratic formula.

Applying the Quadratic Formula

The quadratic formula is:

$$L = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For our equation:

$$- a = 1$$

$$- b = -5$$

$$- c = -750$$

Calculating the discriminant:

$$\Delta = b^2 - 4ac = (-5)^2 - 4(1)(-750) = 25 + 3000 = 3025$$

Finding the square root:

$$\sqrt{\Delta} = \sqrt{3025} = 55$$

Now, compute the two possible solutions:

$$1. L = [5 + 55] / 2 = 60 / 2 = 30$$

$$2. L = [5 - 55] / 2 = -50 / 2 = -25$$

Since a length cannot be negative, we discard $L = -25$.

Therefore, the length of the room is 30 feet.

Using the relationship $W = L - 5$:

$$W = 30 - 5 = 25 \text{ feet}$$

Final Dimensions of the Room

Based on our calculations:

- Length (L): 30 feet
- Width (W): 25 feet

These dimensions satisfy the original conditions:

- Area: $30 \times 25 = 750$ sq. ft.
- Width is 5 feet less than the length: $25 = 30 - 5$

Practical Applications of the Calculated Dimensions

Knowing the dimensions of a room has numerous practical applications:

1. **Interior Design and Furniture Placement:** Accurate measurements help in selecting furniture that fits comfortably within the space.
2. **Renovation Planning:** When installing new flooring, wallpaper, or ceiling fixtures, precise dimensions ensure proper material estimates.
3. **Heating, Ventilation, and Air Conditioning (HVAC):** Proper sizing of units depends on room dimensions.
4. **Real Estate Assessments:** Accurate room measurements impact property valuations and listings.

Additional Considerations When Measuring Rooms

While algebra provides an effective method for solving such problems, real-world measurements can involve additional complexities:

Measuring Accuracy

- Use of proper tools such as tape measures or laser distance meters.
- Ensuring measurements are taken at multiple points to account for irregularities.

Room Shape and Features

- Not all rooms are perfect rectangles; some may have alcoves, nooks, or irregular shapes.
- Adjust calculations accordingly by breaking the space into manageable sections.

Conversion of Units

- Sometimes, measurements may be in inches or meters, requiring conversions to feet for consistency.

Solving Similar Problems: Practice Examples

To reinforce understanding, here are some practice scenarios similar to the original problem:

1. **Problem:** A rectangular garden has an area of 1200 square feet. Its width is 3 feet less than its length. Find the dimensions of the garden.
2. **Solution Approach:** Set variables, formulate a quadratic equation, and solve using the quadratic formula.
3. **Problem:** A classroom has an area of 900 square feet. The length is 10 feet more than the width. Determine the room's dimensions.
4. **Solution Approach:** Define variables, set up the equation, and solve accordingly.

Conclusion

Determining the dimensions of a rectangular room using algebraic methods allows for precise planning and efficient space utilization. In the case where the area is 750 square feet and the width is 5 feet less than the length, the dimensions are 30 feet for the length and 25 feet for the width. Understanding how to set up and solve such problems is valuable not only for students and professionals but also for homeowners and interior designers. By mastering these techniques, you can confidently approach similar real-world challenges involving measurements and spatial analysis.

Summary of Key Points

- Define variables based on the problem conditions.
- Set up the algebraic equation representing the area.
- Solve the quadratic equation using the quadratic formula.
- Interpret the solution within the context of the problem.
- Apply these methods to various practical scenarios involving room dimensions.

Frequently Asked Questions

How do you find the length and width of a rectangular room with an area of 750 square feet if the width is 5 feet less than the length?

Let the length be L and the width be W . Since $W = L - 5$, and $\text{area} = L \times W = 750$, substitute to get $L \times (L - 5) = 750$. Solve the quadratic equation $L^2 - 5L - 750 = 0$ to find L , then $W = L - 5$.

What is the length of the room if the width is 5 feet less and the area is 750 sq ft?

Solving $L^2 - 5L - 750 = 0$, the positive solution for L is approximately 30 feet. Therefore, the width $W = 30 - 5 = 25$ feet.

How can you verify the dimensions of the room once you find the length and width?

Multiply the length and width: $30 \text{ ft} \times 25 \text{ ft} = 750 \text{ sq ft}$, confirming the area matches the given information.

What are the steps to set up the equation for this problem?

Define the length as L , express width as $L - 5$, then set up the equation $L \times (L - 5) = 750$ and solve for L .

Can this problem be solved using the quadratic formula?

Yes, rewriting the equation as $L^2 - 5L - 750 = 0$ allows using the quadratic formula to find the value of L .

What are the approximate dimensions of the room based on the calculations?

The room measures approximately 30 feet in length and 25 feet in width.

Why is it important to check your solutions in problems like this?

To ensure the calculated dimensions satisfy both the area and the relationship between length and width, confirming the solution's accuracy.

Additional Resources

The area of a rectangular room is 750 square feet. The width of the room is 5 feet less than the length.

Introduction

Understanding the dimensions of a space is fundamental in various contexts—whether for interior design, construction, or renovation planning. When given the area of a rectangular room and a relationship between its length and width, one can determine the specific measurements involved. In this article, we will explore this problem comprehensively, analyzing how to derive the room's dimensions based on the given data. We will walk through the mathematical concepts involved, examine the implications of the measurements, and consider practical applications of such calculations.

The Mathematical Foundations

The Basic Rectangular Area Formula

At the core of this problem lies the fundamental geometric formula for the area of a rectangle:

$$A = L \times W$$

Given that the area of the room is 750 square feet, we can express this as:

$$L \times W = 750$$

This simple formula forms the basis for deriving the specific measurements of the room once the relationship between length and width is incorporated.

Relationship Between Length and Width

The problem states: "The width of the room is 5 feet less than the length." Mathematically, this can be expressed as:

$$W = L - 5$$

This relationship is critical because it introduces a variable dependency, enabling us to formulate an equation with a single unknown.

Developing the Mathematical Equation

Substituting the Relationship

Using the expression for width, the area formula becomes:

$$L \times (L - 5) = 750$$

Expanding this gives:

$$L^2 - 5L = 750$$

Rearranged into standard quadratic form:

$$L^2 - 5L - 750 = 0$$

This quadratic equation is the key to determining the length of the room.

Quadratic Equation in Standard Form

The quadratic equation is:

$$x^2 - 5x - 750 = 0$$

where x represents the length of the room.

Solving the Quadratic Equation

Applying the Quadratic Formula

The quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

applies to equations of the form $ax^2 + bx + c = 0$. For our case:

$$- (a = 1)$$

$$- (b = -5)$$

$$- (c = -750)$$

Substituting these into the formula:

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(-750)}}{2(1)}$$

Simplify numerator and inside the square root:

$$x = \frac{5 \pm \sqrt{25 + 3000}}{2}$$

$$x = \frac{5 \pm \sqrt{3025}}{2}$$

Calculate the square root:

$$\sqrt{3025} = 55$$

Thus, the solutions are:

$$x = \frac{5 \pm 55}{2}$$

Evaluating the Solutions

1. First solution:

$$x = \frac{5 + 55}{2} = \frac{60}{2} = 30$$

2. Second solution:

$$x = \frac{5 - 55}{2} = \frac{-50}{2} = -25$$

Since a length cannot be negative in this context, the negative solution is invalid. Therefore, the length of the room is 30 feet.

Determining the Width

Using the relationship:

$$\text{Width} = \text{Length} - 5$$

Substitute the found length:

$$\text{Width} = 30 - 5 = 25 \text{ feet}$$

Result: The room measures 30 feet in length and 25 feet in width.

Verification of the Calculated Dimensions

To ensure accuracy, verify that the product of the length and width matches the given area:

$$30 \times 25 = 750$$

Indeed, the calculation confirms the dimensions satisfy the original area constraint.

Practical Implications and Applications

Interior Design and Space Planning

Knowing the exact dimensions of a room is essential for furniture arrangement, flooring, and decoration. For instance, with a length of 30 feet and width of 25 feet:

- Furniture Placement: Larger furniture pieces can be arranged to maximize space efficiency.
- Flooring: Flooring materials can be ordered based on precise measurements, minimizing waste.
- Lighting and Ventilation: Proper planning ensures adequate lighting fixtures and ventilation pathways.

Construction and Renovation

Accurate room dimensions are critical during:

- Building permits and compliance: Ensuring the space meets building codes.
- Structural modifications: Planning for walls, windows, and door placements.
- Cost estimation: Calculating material needs and associated costs.

Real Estate and Property Management

Understanding room sizes contributes to:

- Property valuation: Larger rooms typically enhance property value.
- Tenant satisfaction: Accurate room dimensions help set expectations and avoid disputes.

Broader Mathematical Context and Variations

Other Relationships Between Dimensions

While this problem involved a fixed difference between length and width, other relationships can be explored:

- Proportional relationships: e.g., width is twice the length.
- Other fixed differences: e.g., width is 3 feet less than the length.
- Multiple dimensions: Extending the problem to include height for volume calculations.

Extending to Different Areas

Suppose the area were different, or the difference between length and width varied; similar algebraic methods could be applied. For example, if the area were 1000 sq ft and the width was 4 feet less than the length, one would set up a quadratic similarly and solve accordingly.

Limitations and Considerations

Assumptions in the Model

- The room is perfectly rectangular.
- Measurements are precise, with no rounding errors.
- The relationship between length and width is exact.

Practical Measurement Challenges

In real-world scenarios, measurement inaccuracies can occur due to:

- Irregularities in room shape.
- Construction tolerances.
- Measurement tools' precision.

It's essential to account for these factors when applying mathematical models to physical spaces.

Conclusion

Understanding how to derive the dimensions of a rectangular room based on its area and a relationship between length and width demonstrates the practical application of algebraic principles in real-world contexts. By translating words into mathematical expressions, solving quadratic equations, and verifying results, one can accurately determine room measurements essential for various fields like interior design, construction, and property management. The specific case of a 750-square-foot room with a width 5 feet less than its length reveals a length of 30 feet and a width of 25 feet, exemplifying the power of algebra in solving everyday spatial problems. This approach underscores the importance of mathematical literacy in making informed decisions about space utilization and resource planning.

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