The Fifth Term Of An Arithmetic Sequence Is 23 And The 12th Term Is 72? Determine The First Three Terms

The Fifth Term Of An Arithmetic Sequence Is 23 And The 12th Term Is 72? Determine The First Three Terms

Introduction to Arithmetic Sequences

Understanding arithmetic sequences is fundamental in algebra and many real-world applications. An arithmetic sequence is a list of numbers in which each term after the first is obtained by adding a constant difference to the previous term. This constant difference is called the common difference, often denoted by d.

Mathematically, an arithmetic sequence can be expressed as:
$$ a_n = a_1 + (n - 1)d $$
where:
- a_n is the n^th term,
- a_1 is the first term,
- d is the common difference,
- n is the position of the term in the sequence.

In this article, we are given specific information about the fifth and twelfth terms and need to determine the first three terms of the sequence.

Understanding the Given Data

Given Information

- The fifth term, a_5, is 23.
- The twelfth term, a_{12}, is 72.

From these, the goal is to find:
- a_1 (the first term),
- a_2,
- a_3.

Knowing a_5 and a_{12} allows us to set up equations involving a_1 and d.

Formulating the Equations

Using the general term formula

Applying the formula $a_n = a_1 + (n - 1)d$ to the known terms:

- For the 5th term:
$$a_5 = a_1 + 4d = 23$$

- For the 12th term:
$$a_{12} = a_1 + 11d = 72$$

Deriving the equations

This leads to a system of two equations:

1. $a_1 + 4d = 23$
2. $a_1 + 11d = 72$

Our goal is to solve these equations simultaneously to find a_1 and d.

Solving for the Common Difference and First Term

Subtracting the equations

Subtract equation 1 from equation 2:
$$(a_1 + 11d) - (a_1 + 4d) = 72 - 23$$
$$a_1 + 11d - a_1 - 4d = 49$$
$$7d = 49$$

Calculating the common difference d

Dividing both sides by 7:
$$d = \frac{49}{7} = 7$$

Finding the first term a_1

Substitute $d = 7$ into equation 1:
$$a_1 + 4(7) = 23$$
$$a_1 + 28 = 23$$
$$a_1 = 23 - 28 = -5$$

Determining the First Three Terms

Now that $a_1 = -5$ and $d = 7$, the first three terms are:

1. $a_1 = -5$
2. $a_2 = a_1 + d = -5 + 7 = 2$
3. $a_3 = a_1 + 2d = -5 + 14 = 9$

Thus, the first three terms of the sequence are -5, 2, and 9.

Verification of the Results

To ensure correctness, verify the fifth and twelfth terms:

- Fifth term:
$$a_5 = a_1 + 4d = -5 + 4 \times 7 = -5 + 28 = 23$$
Correct, matches given data.

- Twelfth term:
$$a_{12} = a_1 + 11d = -5 + 11 \times 7 = -5 + 77 = 72$$
Correct, matches given data.

Both checks confirm the accuracy of our calculations.

Additional Insights and Applications

Understanding the Role of the Common Difference

The common difference d determines whether the sequence is increasing or decreasing:
- If $d > 0$, the sequence increases.
- If $d < 0$, the sequence decreases.
- If $d = 0$, all terms are equal.

In this case, $d = 7$, which signifies an increasing sequence.

Applications of Arithmetic Sequences

Arithmetic sequences appear in various real-life scenarios such as:
- Salary increments over time.
- Planning savings with fixed monthly contributions.
- Arrangements in linear patterns.

Understanding how to determine initial terms from given data is essential for modeling and analyzing such situations.

Summary

- Given the fifth term $a_5 = 23$ and twelfth term $a_{12} = 72$, we formulated two equations based on the general term formula.
- Solving these equations yielded the common difference $d = 7$ and the first term $a_1 = -5$.
- The first three terms of the sequence are -5, 2, and 9.

This problem exemplifies the importance of algebraic techniques in solving for unknowns in sequences and highlights the practical value of understanding arithmetic progressions.

Conclusion

Determining the initial terms of an arithmetic sequence from partial information involves setting up and solving equations derived from the general term formula. By carefully manipulating the equations, we can uncover the sequence's structure and predict subsequent terms. This process demonstrates the power of algebra in solving real-world mathematical problems and reinforces the foundational concepts of sequences and series.

Frequently Asked Questions

What is the first step to find the first term of an arithmetic sequence given the 5th and 12th terms?

The first step is to identify the known terms ($a_5 = 23$ and $a_{12} = 72$) and write their formulas in terms of the first term (a_1) and common difference (d), then set up equations to solve for these variables.

How do you express the 5th and 12th terms of an arithmetic sequence mathematically?

The nth term of an arithmetic sequence is given by $a_n = a_1 + (n - 1)d$. So, $a_5 = a_1 + 4d$ and

$a_{12} = a_1 + 11d$.

What equations can be formed using the given terms to find the first term and common difference?

Using the formulas: $a_5 = a_1 + 4d = 23$ and $a_{12} = a_1 + 11d = 72$. These two equations can be solved simultaneously to find a_1 and d.

How do you solve for the common difference d using the given information?

Subtract the equation for a_5 from that for a_{12}: $(a_1 + 11d) - (a_1 + 4d) = 72 - 23$, which simplifies to $7d = 49$, so $d = 7$.

Once the common difference is known, how do you find the first term a_1?

Substitute $d = 7$ into the equation $a_5 = a_1 + 4d = 23$, resulting in $a_1 + 4(7) = 23$, so $a_1 + 28 = 23$, leading to $a_1 = -5$.

What are the first three terms of the arithmetic sequence based on the calculations?

The first term is $a_1 = -5$, the second term is $a_2 = a_1 + d = -5 + 7 = 2$, and the third term is $a_3 = a_2 + d = 2 + 7 = 9$.

Can you verify the first three terms satisfy the given 5th and 12th terms?

Yes. Calculating the 5th term: $a_5 = a_1 + 4d = -5 + 28 = 23$; and the 12th term: $a_{12} = a_1 + 11d = -5 + 77 = 72$. Both match the given information, confirming the accuracy.

How can this problem be generalized to find the first n terms of any arithmetic sequence given two terms?

Identify the known terms and their positions, set up equations based on the formula $a_n = a_1 + (n - 1)d$, and solve for a_1 and d. Then, use these to find any number of terms in the sequence.

Additional Resources

Understanding the Fifth and Twelfth Terms of an Arithmetic Sequence: A Deep Dive into the First Three Terms

In the realm of mathematics, sequences serve as fundamental building blocks that help us comprehend patterns, predict future values, and analyze relationships between numbers. Among the various types of sequences, arithmetic sequences hold a special place due to their simplicity and wide applicability. When given specific terms within an arithmetic sequence, such as the fifth and twelfth, mathematicians and students alike are often tasked with uncovering the initial terms, particularly the first three, which lay the foundation for understanding the entire sequence. This article explores the intriguing problem: The fifth term of an arithmetic sequence is 23 and the 12th term is 72. How do we determine the first three terms? Through detailed explanations, step-by-step calculations, and analytical insights, we aim to demonstrate the methods and reasoning processes involved in solving such problems.

Introduction to Arithmetic Sequences

Before delving into the specific problem, it is essential to establish a clear understanding of arithmetic sequences.

What Is an Arithmetic Sequence?

An arithmetic sequence is a sequence of numbers where each term after the first is obtained by adding a fixed number, called the common difference, to the preceding term. Formally, an arithmetic sequence can be expressed as:

$$ a_n = a_1 + (n - 1)d $$

where:
- a_n is the n^th term,
- a_1 is the first term,
- d is the common difference,
- n is the position of the term in the sequence.

Characteristics of Arithmetic Sequences

- Linear pattern: The sequence increases or decreases uniformly.
- Predictability: Knowing any two terms allows us to find the common difference and initial term.
- Applications: Used in finance (installments), physics (uniform motion), and computer science (algorithm analysis).

Problem Restatement and Initial Analysis

The core problem is:

Given an arithmetic sequence where the fifth term is 23 and the twelfth term is 72, determine the first three terms.

Restating the Known Information

- $a_5 = 23$
- $a_{12} = 72$

Objective

- Find a_1, a_2, and a_3.

Approach Overview

To solve this, the strategy involves:

1. Expressing the given terms using the general formula for an arithmetic sequence.
2. Setting up equations based on the known terms.
3. Solving these equations to find the common difference d and the first term a_1.
4. Calculating the first three terms.

Deriving the Equations from Known Terms

Using the General Term Formula

Recall that:

$$ a_n = a_1 + (n - 1)d $$

Applying this to the known terms:

- For the 5th term:

$$ a_5 = a_1 + (5 - 1)d = a_1 + 4d = 23 $$

- For the 12th term:

$$ a_{12} = a_1 + (12 - 1)d = a_1 + 11d = 72 $$

Establishing the Equations

From the above, we have two equations:

1. $a_1 + 4d = 23$ — (Equation 1)
2. $a_1 + 11d = 72$ — (Equation 2)

Our goal is to solve these simultaneous equations to find a_1 and d.

Solving for the Common Difference and First Term

Step 1: Subtract Equation 1 from Equation 2

Subtracting Equation 1 from Equation 2 to eliminate a_1:

$$
(a_1 + 11d) - (a_1 + 4d) = 72 - 23
$$

Simplifies to:

$$
(11d - 4d) = 49
$$
$$
7d = 49
$$

Step 2: Find d

Dividing both sides by 7:

$$
d = \frac{49}{7} = 7
$$

Step 3: Find a_1

Using Equation 1:

$$
a_1 + 4d = 23
$$
$$
a_1 + 4 \times 7 = 23
$$
$$
a_1 + 28 = 23
$$
$$
a_1 = 23 - 28 = -5
$$

Calculating the First Three Terms

First Term a_1:

$$
a_1 = -5
$$

Second Term a_2:

Using the formula:

$$
a_2 = a_1 + d = -5 + 7 = 2
$$

Third Term a_3:

Similarly,

$$
a_3 = a_1 + 2d = -5 + 2 \times 7 = -5 + 14 = 9
$$

Summary of the First Three Terms:

- $a_1 = -5$
- $a_2 = 2$
- $a_3 = 9$

Verification and Consistency Checks

Verifying the earlier findings ensures the correctness of our calculations.

Verify Fifth Term:

$$
a_5 = a_1 + 4d = -5 + 4 \times 7 = -5 + 28 = 23
$$

Matches the given value.

Verify Twelfth Term:

$$
a_{12} = a_1 + 11d = -5 + 11 \times 7 = -5 + 77 = 72
$$

\]

Matches the given value.

Conclusion:

Our solution aligns perfectly with the provided data, confirming the accuracy of the first three terms.

Extended Insights and Analytical Perspectives

Significance of the Solution

Determining the first three terms offers a window into the entire sequence's behavior. It reveals that the sequence starts at \(-5\) and increases by 7 each time, forming an increasing arithmetic sequence with a linear growth pattern.

Broader Implications

Understanding how to derive initial terms from specific data points is crucial in various fields:

- Finance: Calculating initial investments when future values are known.
- Physics: Reconstructing initial velocities or positions based on observed data.
- Computer Science: Debugging algorithms that generate sequences with known outputs at particular steps.

Potential Variations and Challenges

While the problem presented was straightforward, several challenges can arise:

- Incomplete Data: When only one term is known, additional constraints are necessary.
- Non-Uniform Differences: For sequences where differences vary, methods become more complex, involving quadratic or higher-order formulas.
- Sequence with Unknown Starting Point: When the initial term isn't given, but some terms are, the approach remains similar but requires solving multiple equations.

Conclusion and Final Remarks

The task of determining the first three terms of an arithmetic sequence when given specific terms exemplifies the elegance and utility of algebraic methods. By translating the problem into equations, solving for the common difference and initial term, and verifying the results, we gain a comprehensive understanding of the sequence's structure. This analytical process not only reinforces core mathematical concepts but also demonstrates their practical applications across various disciplines.

In this particular case, the sequence begins at \(-5\) and increases by 7 each step, with the initial three terms being \(-5\), 2, and 9. These insights enable us to reconstruct the sequence fully and appreciate the interconnectedness of mathematical principles that govern such patterns.

Whether in academic settings or real-world scenarios, mastering these problem-solving techniques equips individuals to analyze, interpret, and leverage sequences effectively, fostering a deeper appreciation of the mathematical patterns that underpin many aspects of our universe.

The Fifth Term Of An Arithmetic Sequence Is 23 And The 12th Term Is 72 Determine The First Three Terms

The Fifth Term Of An Arithmetic Sequence Is 23 And The 12th Term Is 72 Determine The First Three Terms

Related Articles

- The Total Cost Of Producing X Food Processors Is C(x) = 2,000 + 50x 0.5x^2 A Find The Actual Additional
- Show All Your Work. Indicate Clearly The Methods You Use, Because You Will Be Scored On The Correctness
- Try To Answer The Questions Below About Deductive Reasoning And Rules.Using DeductionWhat Rule Does The

Back to Home