The Half-life Of A Certain Radioactive Element Is 35 Hours. If There Are 904 Grams Of This Element, How

The Half-life Of A Certain Radioactive Element Is 35 Hours. If There Are 904 Grams Of This Element, How is a question that delves into the fascinating world of nuclear physics and radioactive decay. Understanding how radioactive elements diminish over time is crucial in various scientific and practical applications, including radiometric dating, nuclear medicine, and nuclear power management. This article explores the concept of half-life in detail, providing insights into how to calculate remaining quantities of radioactive substances, the significance of half-life in real-world scenarios, and the mathematical principles underpinning radioactive decay processes.

Understanding Radioactive Decay and Half-life

What Is Radioactive Decay?

Radioactive decay is a natural process where unstable atomic nuclei lose energy by emitting radiation in the form of particles or electromagnetic waves. This process transforms the original unstable isotope into a more stable form, often leading to a different element altogether. The decay occurs spontaneously and is random at the level of individual atoms but predictable statistically across large populations.

Defining Half-life

The half-life of a radioactive isotope is the time required for half of the radioactive atoms in a sample to decay. It is an intrinsic property of each isotope and remains constant regardless of the sample size. For example, if a substance has a half-life of 35 hours, then after 35 hours, only half of the original atoms remain; after another 35 hours (70 hours total), only a quarter remain, and so on.

The Importance of Half-life in Science and Industry

Applications of Half-life

Radioactive half-life plays a critical role in numerous fields:

- **Radiometric Dating:** Determining the age of fossils and archaeological artifacts by measuring the remaining radioactive isotopes.
- **Nuclear Medicine:** Using radioactive isotopes with specific half-lives for diagnosis and treatment.
- **Nuclear Power:** Managing reactor fuel and waste by understanding decay rates.
- **Environmental Monitoring:** Tracking radioactive contaminants over time.

Why Is Knowing the Remaining Quantity Important?

Understanding how much of a radioactive element remains after a certain period allows scientists and engineers to:

- Predict the longevity and safety of radioactive materials.
- Calculate the dosage of radioactive isotopes in medical applications.
- Determine the age of archaeological samples with radiocarbon dating.
- Manage nuclear waste disposal effectively.

Calculating Remaining Radioactive Material: The Mathematical Approach

The Decay Formula

The amount of radioactive material remaining after a certain period can be calculated using the exponential decay formula:

$$ N(t) = N_0 \times \left(\frac{1}{2} \right)^{\frac{t}{T_{1/2}}} $$

Where:
- $N(t)$ = amount remaining after time t
- N_0 = initial amount
- $T_{1/2}$ = half-life of the isotope
- t = elapsed time

This formula reflects that after each half-life period, the quantity halves.

Applying the Formula to Our Problem

Given:
- $N_0 = 904$ grams
- $T_{1/2} = 35$ hours
- t = unknown (to find remaining quantity after a specific period)

Suppose we want to find the remaining amount after a certain number of hours or determine how long it takes for the sample to decay to a specific mass.

Worked Example: How Much Radioactive Material Remains?

Scenario 1: Remaining Quantity After 70 Hours

Since the half-life is 35 hours, 70 hours equates to two half-lives:

$$ t = 70 \text{ hours} $$
$$ n = \frac{t}{T_{1/2}} = \frac{70}{35} = 2 $$

Applying the decay formula:

$$ N(70) = 904 \times \left(\frac{1}{2} \right)^2 = 904 \times \frac{1}{4} = 226 \text{ grams} $$

So, after 70 hours, approximately 226 grams of the original radioactive element remains.

Scenario 2: Time Needed to Reduce to 100 Grams

Suppose we want to find out how long it takes for the initial 904 grams to decay to 100 grams.

Rearranged decay formula:

$$N(t) = N_0 \times \left(\frac{1}{2} \right)^{\frac{t}{T_{1/2}}}$$

Dividing both sides by N_0:

$$\frac{N(t)}{N_0} = \left(\frac{1}{2} \right)^{\frac{t}{T_{1/2}}}$$

Plugging in the values:

$$\frac{100}{904} \approx 0.1106$$

Taking natural logarithms:

$$\ln(0.1106) = \frac{t}{T_{1/2}} \times \ln(0.5)$$

Calculating:

$$\ln(0.1106) \approx -2.201$$
$$\ln(0.5) \approx -0.693$$

Solving for t:

$$t = T_{1/2} \times \frac{\ln(0.1106)}{\ln(0.5)} = 35 \times \frac{-2.201}{-0.693} \approx 35 \times 3.177 \approx 111.2 \text{ hours}$$

Therefore, it takes approximately 111.2 hours for the sample to decay from 904 grams to 100 grams.

Understanding Decay Rate and Half-life Through Examples

Key Points to Remember

- The decay process is exponential; the amount halves every half-life period.
- The number of half-lives passed determines the remaining quantity.
- The formula $N(t) = N_0 \times (1/2)^{t/T_{1/2}}$ is fundamental for calculations.

Common Misconceptions

- Half-life is the time it takes for the entire sample to decay: Incorrect. It’s only the time for half of the atoms to decay.
- Radioactive decay is predictable at the atomic level: While individual atom decay is random, the overall decay rate for large samples is predictable and follows exponential laws.

Real-World Implications and Safety Considerations

Managing Radioactive Waste

Knowledge of half-life helps in planning the storage and disposal of radioactive waste. Longer half-lives mean longer-lasting hazards, requiring secure containment over extended periods.

Nuclear Medicine and Radiotherapy

Radioisotopes with specific half-lives are chosen to optimize diagnostic imaging or targeted therapy, ensuring that radiation exposure is minimized while effectiveness is maximized.

Radiocarbon Dating

Scientists estimate the age of archaeological samples by measuring remaining ^{14}C, which has a half-life of about 5,730 years, making it suitable for dating ancient artifacts.

Conclusion: The Significance of Half-life Calculations

Understanding the half-life of radioactive elements, such as the one with a 35-hour half-life, is crucial for numerous scientific, medical, and environmental applications. Calculating how much of a substance remains after a given period allows professionals to make informed decisions regarding safety, treatment, and research. The exponential decay law provides a powerful mathematical framework that simplifies these calculations, enabling accurate predictions and effective management of radioactive materials.

In the specific case of starting with 904 grams of an element with a 35-hour half-life, knowing how to determine remaining quantities after any period or the time required to reach a certain level is essential. Whether for scientific research, medical treatment, or safety protocols, mastering these concepts ensures responsible and effective handling of radioactive substances.

Keywords: half-life, radioactive decay, decay calculation, exponential decay, nuclear physics, radiometric dating, nuclear medicine, radioactive waste management, decay formula, remaining radioactive material

Frequently Asked Questions

What is the half-life of the radioactive element in question?

The half-life of the element is 35 hours.

If you start with 904 grams of the radioactive element, how much remains after 35 hours?

After 35 hours, half of the original amount remains, so 452 grams.

How much of the radioactive element remains after 70 hours?

After 70 hours, which is two half-lives, the remaining amount is 904 / 2 / 2 = 226 grams.

How many half-lives pass in 140 hours for this radioactive element?

Since each half-life is 35 hours, 140 / 35 = 4 half-lives pass in 140 hours.

What is the general formula to calculate remaining radioactive material after a certain time?

Remaining amount = initial amount × (1/2)^(time / half-life).

Using the formula, how much of the original 904 grams remains after 105 hours?

Number of half-lives = 105 / 35 = 3. Remaining amount = 904 × (1/2)^3 = 904 × 1/8 = 113 grams.

How long will it take for the radioactive element to decay to 100 grams?

Set remaining amount = 100 grams in the decay formula and solve for time: 100 = 904 × (1/2)^(t/35). Solving gives t ≈ 69.56 hours.

What is the decay constant (λ) for this element based on its half-life?

λ = ln(2) / half-life = 0.693 / 35 hours ≈ 0.0198 per hour.

If the decay follows exponential decay, what is the remaining amount after 50 hours?

Remaining amount = 904 × (1/2)^(50/35) ≈ 904 × (1/2)^(1.43) ≈ 904 × 0.356 ≈ 322 grams.

Why is understanding half-life important in radioactive decay?

Understanding half-life helps determine how quickly a radioactive substance decays, which is crucial for safety, medical applications, and dating techniques.

Additional Resources

The Half-life of a Certain Radioactive Element Is 35 Hours. If There Are 904 Grams of This Element, How — these words open the door to a fascinating exploration of radioactive decay principles and their practical applications. Understanding how to calculate remaining quantities of a radioactive substance over time is fundamental not only in scientific research but also in fields such as medicine, archaeology, environmental science, and nuclear energy. This guide aims to provide a comprehensive, step-by-step approach to solving problems involving half-lives, illustrating the core concepts with clear explanations and practical examples.

Introduction to Radioactive Decay and Half-life

Radioactive decay is a natural process in which unstable atomic nuclei lose energy by emitting radiation. This process is inherently random at the level of individual atoms, but when considering large numbers of atoms, it follows a predictable exponential decay pattern.

What is Half-life?

The half-life of a radioactive element is the time it takes for half of the atoms in a sample to decay. For the element in question, the half-life is given as 35 hours. This means:

- After 35 hours, only 50% of the original amount remains.
- After another 35 hours (70 hours total), only 25% remains.
- And so on, with the remaining amount halving every half-life period.

Understanding this concept allows scientists to estimate how much of a radioactive material remains after a given period, which is crucial for applications like radiometric dating, nuclear medicine, and waste management.

The Mathematical Foundation of Radioactive Decay

The decay of a radioactive substance can be described mathematically using the exponential decay formula:

$N(t) = N_0 (1/2)^{(t / T_1/_2)}$

Where:
- $N(t)$ is the amount of substance remaining after time t.
- N_0 is the initial amount of the substance.
- $T_1/_2$ is the half-life of the substance.
- t is the elapsed time.

Alternatively, the decay can be expressed using the exponential function:

$N(t) = N_0 e^{(-\lambda t)}$

Where:
- λ is the decay constant, related to the half-life by:

$\lambda = \ln(2) / T_1/_2$

In our case:
- $T_1/_2 = 35$ hours
- $N_0 = 904$ grams

Step-by-step Solution to the Problem

Suppose we are asked: "Given that there are 904 grams of this radioactive element, how long will it take for the amount to decay to a certain level?" or "How much remains after a certain period?". The exact question will determine the approach.

Common Questions and Their Solutions

1. How much of the element remains after a certain period?
2. How long does it take for the element to decay to a specified amount?
3. How many half-lives have passed for a given reduction?

In this guide, we'll cover both types of questions with detailed solutions.

Calculating Remaining Quantity After a Given Time

Example: How much of the element remains after 70 hours?

Given:
- Initial amount: $N_0 = 904$ grams
- Half-life: $T_1/_2 = 35$ hours
- Time elapsed: $t = 70$ hours

Step 1: Determine the number of half-lives elapsed

Number of half-lives, n, is:

$n = t / T_1/_2 = 70 / 35 = 2$

Step 2: Use the decay formula

Since the amount halves every half-life:

$N(t) = N_0 (1/2)^n$

Plugging in the values:

$N(70) = 904 (1/2)^2 = 904 (1/4) = 226$ grams

Answer: After 70 hours, approximately 226 grams of the element remains.

Calculating Time for a Specific Remaining Quantity

Suppose we want to find out how long it takes for the initial 904 grams to decay to 113 grams.

Step 1: Set up the decay formula

$N(t) = N_0 (1/2)^{(t / T_{1/2})}$

Plugging in the knowns:

$113 = 904 (1/2)^{(t / 35)}$

Step 2: Solve for t

Divide both sides by 904:

$(1/2)^{(t / 35)} = 113 / 904 \approx 0.1248$

Step 3: Take the natural logarithm of both sides

$\ln[(1/2)^{(t / 35)}] = \ln(0.1248)$

Using log rules:

$(t / 35) \ln(1/2) = \ln(0.1248)$

Recall that $\ln(1/2) = -0.6931$, so:

$(t / 35) (-0.6931) = -2.082$

Step 4: Solve for t

$t / 35 = -2.082 / -0.6931 \approx 3$

$t \approx 3 \; 35 = 105$ hours

Answer: It takes approximately 105 hours for the amount to decay from 904 grams to 113 grams.

Understanding the Decay Constant and Its Role

The decay constant λ is a key parameter that characterizes the rate at which a substance decays and is related to the half-life:

$\lambda = \ln(2) / T_1/_2$

For our element:

$\lambda = 0.6931 / 35 \approx 0.0198$ per hour

This means every hour, approximately 1.98% of the remaining substance decays.

Using the decay constant, we can also compute:

$N(t) = N_0$ e^(-λt)

which is useful for continuous decay calculations.

Practical Applications and Implications

Understanding the half-life and decay calculations has numerous real-world applications:

- Radiometric Dating: Estimating the age of archaeological samples.
- Medical Treatments: Determining dosage and timing for radioactive tracers.
- Nuclear Power: Managing radioactive waste decay over time.
- Environmental Monitoring: Tracking decay of radioactive contaminants.

Additional Tips for Solving Half-life Problems

- Always identify whether the question asks for remaining quantity, elapsed time, or number of half-lives.

- Convert all units consistently (e.g., hours, days).
- Remember that each half-life reduces the remaining amount by half.
- Use logarithmic functions for non-integer half-lives or when solving for time.

Summary of Key Formulas

Scenario	Formula	Notes
Remaining amount after t hours	$N(t) = N_0 (1/2)^{(t / T_{1/2})}$	Use for direct calculation
Time to decay to a certain amount	$t = T_{1/2} \log_2(N_0 / N(t))$	Logarithm base 2
Decay constant	$\lambda = \ln(2) / T_{1/2}$	Converts half-life to exponential decay rate

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Final Thoughts

The problem involving The half-life of a certain radioactive element is 35 hours and starting with 904 grams is representative of the fundamental principles of nuclear science. The exponential nature of decay means that quantities decrease rapidly at first, but never truly reach zero, reflecting the probabilistic nature of atomic decay processes. Mastery of these calculations enables scientists and engineers to accurately predict the behavior of radioactive materials over time, ensuring safety, efficacy, and scientific accuracy in various applications.

By understanding these principles and practicing with different scenarios, you can confidently analyze radioactive decay problems and appreciate the elegant simplicity of exponential decay laws governing the atomic world.

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