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THE LENGTH OF A RECTANGLE IS 7 CM LONGER THAN ITS WIDTH. THE AREA OF THE RECTANGLE IS 120 CM. WHAT IS THE

UNDERSTANDING THE DIMENSIONS OF RECTANGLES IS FUNDAMENTAL IN GEOMETRY, ESPECIALLY WHEN SOLVING REAL-WORLD PROBLEMS INVOLVING AREA, LENGTH, AND WIDTH. IN THIS ARTICLE, WE EXPLORE A SPECIFIC PROBLEM: GIVEN A RECTANGLE WHERE THE LENGTH IS 7 CM LONGER THAN ITS WIDTH AND THE AREA IS 120 CM², HOW DO WE DETERMINE THE DIMENSIONS OF THE RECTANGLE? WE WILL COVER THE PROBLEM-SOLVING PROCESS COMPREHENSIVELY, INCLUDING STEP-BY-STEP ALGEBRAIC METHODS, FORMULAS, AND PRACTICAL APPLICATIONS. BY THE END OF THIS GUIDE, YOU'LL BE EQUIPPED WITH THE KNOWLEDGE TO SOLVE SIMILAR PROBLEMS CONFIDENTLY.

UNDERSTANDING THE PROBLEM

BEFORE DELVING INTO CALCULATIONS, IT'S CRUCIAL TO UNDERSTAND THE PROBLEM'S COMPONENTS:

- THE LENGTH OF THE RECTANGLE IS 7 CM LONGER THAN ITS WIDTH.
- THE AREA OF THE RECTANGLE IS 120 CM².
- THE GOAL IS TO FIND THE LENGTH AND WIDTH OF THE RECTANGLE.

THIS PROBLEM INVOLVES BASIC ALGEBRA AND UNDERSTANDING OF RECTANGLE PROPERTIES, SPECIFICALLY THE RELATIONSHIP BETWEEN LENGTH, WIDTH, AND AREA.

KEY CONCEPTS AND FORMULAS

TO SOLVE THIS PROBLEM, FAMILIARIZE YOURSELF WITH THE FOLLOWING KEY CONCEPTS AND FORMULAS:

1. RECTANGLE DIMENSIONS

- WIDTH (W): THE SHORTER SIDE OF THE RECTANGLE.
- LENGTH (L): THE LONGER SIDE OF THE RECTANGLE.

2. RELATIONSHIP BETWEEN LENGTH AND WIDTH

- GIVEN: $L = W + 7$

3. AREA FORMULA OF A RECTANGLE

- AREA (A) = LENGTH $\times$ WIDTH
- IN SYMBOLS: $A = L \times W$

4. SOLVING FOR DIMENSIONS

- USE ALGEBRA TO SUBSTITUTE THE RELATIONSHIP BETWEEN LENGTH AND WIDTH INTO THE AREA FORMULA TO FIND THE UNKNOWN SIDES.

STEP-BY-STEP SOLUTION APPROACH

LET'S WALK THROUGH THE PROCESS TO FIND THE DIMENSIONS OF THE RECTANGLE.

STEP 1: DEFINE THE VARIABLES

- LET w BE THE WIDTH OF THE RECTANGLE (IN CM).
- SINCE THE LENGTH IS 7 CM LONGER THAN THE WIDTH, $L = w + 7$.

STEP 2: SET UP THE EQUATION USING AREA

- GIVEN THE AREA IS 120 cm^2 :

$$\begin{aligned} & \backslash[\\ & A = L \times w = 120 \\ & \backslash] \end{aligned}$$

SUBSTITUTING $L = w + 7$:

$$\begin{aligned} & \backslash[\\ & (w + 7) \times w = 120 \\ & \backslash] \end{aligned}$$

SIMPLIFY:

$$\begin{aligned} & \backslash[\\ & w^2 + 7w = 120 \\ & \backslash] \end{aligned}$$

STEP 3: REARRANGE TO STANDARD QUADRATIC FORM

- BRING ALL TERMS TO ONE SIDE:

$$\begin{aligned} & \backslash[\\ & w^2 + 7w - 120 = 0 \\ & \backslash] \end{aligned}$$

STEP 4: SOLVE THE QUADRATIC EQUATION

- THE QUADRATIC EQUATION:

$$\begin{aligned} & \backslash[\\ & w^2 + 7w - 120 = 0 \\ & \backslash] \end{aligned}$$

- USE THE QUADRATIC FORMULA:

$$w = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

WHERE:

- ($a = 1$),
- ($b = 7$),
- ($c = -120$).

CALCULATING DISCRIMINANT:

$$\Delta = b^2 - 4ac = 7^2 - 4 \times 1 \times (-120) = 49 + 480 = 529$$

SQUARE ROOT OF DISCRIMINANT:

$$\sqrt{529} = 23$$

CALCULATING ROOTS:

$$w = \frac{-7 \pm 23}{2}$$

- FIRST ROOT:

$$w = \frac{-7 + 23}{2} = \frac{16}{2} = 8$$

- SECOND ROOT:

$$w = \frac{-7 - 23}{2} = \frac{-30}{2} = -15$$

SINCE WIDTH CANNOT BE NEGATIVE, $w = 8$ CM.

STEP 5: FIND THE LENGTH

- RECALL $L = w + 7$:

$$L = 8 + 7 = 15, \text{ CM}$$

FINAL DIMENSIONS OF THE RECTANGLE

BASED ON THE CALCULATIONS:

- WIDTH (w): 8 cm
- LENGTH (L): 15 cm

THESE DIMENSIONS SATISFY THE ORIGINAL CONDITIONS: THE LENGTH IS 7 cm LONGER THAN THE WIDTH, AND THE AREA IS 120 cm².

VERIFICATION OF THE SOLUTION

ALWAYS VERIFY YOUR SOLUTION TO ENSURE ACCURACY.

- CALCULATE THE AREA:

$$\begin{aligned} & \text{Area} = L \times w = 15 \times 8 = 120 \\ & \text{cm}^2 \end{aligned}$$

- CHECK THE LENGTH DIFFERENCE:

$$15 - 8 = 7 \text{ cm}$$

BOTH CONDITIONS ARE SATISFIED; THUS, THE SOLUTION IS CORRECT.

PRACTICAL APPLICATIONS OF THE PROBLEM

UNDERSTANDING HOW TO FIND DIMENSIONS BASED ON AREA AND RELATIONSHIPS BETWEEN SIDES HAS MULTIPLE REAL-WORLD APPLICATIONS, SUCH AS:

- DESIGNING RECTANGULAR PLOTS OF LAND WITH SPECIFIC AREA REQUIREMENTS.
- MANUFACTURING ITEMS WHERE DIMENSIONS ARE CONSTRAINED BY AREA AND PROPORTIONAL RELATIONSHIPS.
- PLANNING ROOM LAYOUTS OR FURNITURE DIMENSIONS TO FIT SPECIFIC SPACE AND AESTHETIC CRITERIA.

ADDITIONAL TIPS FOR SOLVING SIMILAR PROBLEMS

- ALWAYS DEFINE VARIABLES CLEARLY.
- SUBSTITUTE KNOWN RELATIONSHIPS INTO FORMULAS BEFORE SOLVING.
- USE THE QUADRATIC FORMULA WHEN THE EQUATION IS QUADRATIC.
- CHECK FOR EXTRANEOUS SOLUTIONS, ESPECIALLY WHEN NEGATIVE ROOTS ARE OBTAINED.
- VERIFY YOUR ANSWER BY PLUGGING BACK INTO ORIGINAL CONDITIONS.

FREQUENTLY ASKED QUESTIONS (FAQs)

Q1: CAN THE WIDTH BE NEGATIVE?

- No, because width cannot be negative in real-world measurements.

Q2: WHAT IF THE AREA WAS DIFFERENT?

- The same method applies; set up the equation with the given area and solve for the dimensions.

Q3: HOW DO I HANDLE MORE COMPLEX RELATIONSHIPS?

- Break down the problem into smaller parts, define variables, and use algebraic methods like substitution and quadratic formulas.

CONCLUSION

Determining the dimensions of a rectangle with known relationships between length, width, and area involves understanding formulas, setting up equations, and solving quadratic equations. In this example, with the length being 7 cm longer than the width and the area being 120 cm^2 , the dimensions are 8 cm for the width and 15 cm for the length. Mastering these techniques empowers you to tackle a wide variety of geometric problems efficiently and accurately.

RELATED TOPICS FOR FURTHER STUDY

- Properties of rectangles and other quadrilaterals
- Solving quadratic equations
- Applications of algebra in geometry
- Real-world problem-solving involving area and perimeter
- Using formulas for compound shapes

If you want to improve your understanding of geometry and algebra, practicing problems like these will strengthen your problem-solving skills and prepare you for more advanced mathematical challenges.

FREQUENTLY ASKED QUESTIONS

WHAT IS THE WIDTH OF THE RECTANGLE IF ITS LENGTH IS 7 CM LONGER AND THE AREA IS 120 cm^2 ?

Let the width be x cm. Then, the length is $x + 7$ cm. Area = length $\times$ width, so $(x + 7) \times x = 120$. Solving for x : $x^2 + 7x = 120$ $\Rightarrow$ $x^2 + 7x - 120 = 0$. Factoring or using quadratic formula: $x = 5$ or $x = -24$. Since width can't be negative, the width is 5 cm.

HOW DO YOU FIND THE LENGTH OF THE RECTANGLE GIVEN ITS WIDTH AND AREA?

First, identify the width (x). The length is $x + 7$ cm. Use the formula for area: length $\times$ width = 120 cm^2 . Substitute length: $(x + 7) \times x = 120$. Solve for x to find the width, then add 7 to find the length.

WHAT ARE THE DIMENSIONS OF THE RECTANGLE IF ITS AREA IS 120 cm^2 AND THE LENGTH IS 7 CM LONGER THAN THE WIDTH?

THE WIDTH IS 5 CM, AND THE LENGTH IS 12 CM (SINCE $5 + 7 = 12$). THESE DIMENSIONS SATISFY THE AREA CONDITION: $5 \times 12 = 60$, BUT SINCE THE AREA IS 120 cm^2 , THE CORRECT DIMENSIONS ARE WIDTH = 8 CM AND LENGTH = 15 CM, BECAUSE $8 \times 15 = 120$. (RECALCULATION SHOWS THE QUADRATIC SOLUTION YIELDS WIDTH = 8 CM, LENGTH = 15 CM).

CAN THE WIDTH OF THE RECTANGLE BE NEGATIVE BASED ON THE GIVEN AREA AND LENGTH DIFFERENCE?

NO, THE WIDTH CANNOT BE NEGATIVE BECAUSE PHYSICAL DIMENSIONS LIKE WIDTH MUST BE POSITIVE. THE QUADRATIC SOLUTION YIELDS A NEGATIVE VALUE ($x = -24$), WHICH IS INVALID IN THIS CONTEXT, SO THE ONLY VALID WIDTH IS 8 CM.

WHAT IS THE STEP-BY-STEP APPROACH TO FIND THE DIMENSIONS OF A RECTANGLE WITH GIVEN AREA AND LENGTH DIFFERENCE?

1. DEFINE THE WIDTH AS x CM. 2. WRITE THE LENGTH AS $x + 7$ CM. 3. SET UP THE AREA EQUATION: $x(x + 7) = 120$. 4. EXPAND: $x^2 + 7x = 120$. 5. REARRANGE TO QUADRATIC FORM: $x^2 + 7x - 120 = 0$. 6. SOLVE USING QUADRATIC FORMULA: $x = \frac{-7 \pm \sqrt{7^2 - 4 \times 1 \times (-120)}}{2 \times 1}$. 7. CALCULATE DISCRIMINANT: $49 + 480 = 529$. 8. FIND ROOTS: $x = \frac{-7 \pm \sqrt{529}}{2} = \frac{-7 \pm 23}{2}$. 9. VALID SOLUTION: $x = (16)/2 = 8$ CM. 10. FIND LENGTH: $8 + 7 = 15$ CM. 11. VERIFY: $8 \times 15 = 120 \text{ cm}^2$.

ADDITIONAL RESOURCES

THE LENGTH OF A RECTANGLE IS 7 CM LONGER THAN ITS WIDTH. THE AREA OF THE RECTANGLE IS 120 cm^2 . WHAT IS THE

UNDERSTANDING THE FUNDAMENTAL PROPERTIES OF RECTANGLES IS ESSENTIAL NOT ONLY IN MATHEMATICS BUT ALSO IN VARIOUS REAL-WORLD APPLICATIONS, FROM ARCHITECTURE TO ENGINEERING. IN THIS ARTICLE, WE DELVE INTO A SPECIFIC PROBLEM INVOLVING A RECTANGLE'S DIMENSIONS, EXPLORING HOW TO DETERMINE ITS LENGTH AND WIDTH WHEN GIVEN CERTAIN PARAMETERS. THE PROBLEM STATES: THE LENGTH OF A RECTANGLE IS 7 CENTIMETERS LONGER THAN ITS WIDTH, AND ITS TOTAL AREA IS $120 \text{ SQUARE CENTIMETERS}$. OUR GOAL IS TO FIND THE PRECISE MEASUREMENTS OF BOTH THE LENGTH AND THE WIDTH, UNRAVELING THE STEPS INVOLVED IN SOLVING SUCH A PROBLEM WITH CLARITY AND PRECISION.

BREAKING DOWN THE PROBLEM: WHAT ARE WE ASKED TO FIND?

AT ITS CORE, THE PROBLEM PRESENTS A SCENARIO INVOLVING TWO KEY MEASUREMENTS: LENGTH AND WIDTH. IT STATES:

- THE LENGTH OF THE RECTANGLE IS 7 CM LONGER THAN ITS WIDTH.
- THE AREA OF THE RECTANGLE IS 120 cm^2 .

GIVEN THESE CONDITIONS, THE QUESTION ASKS US TO DETERMINE:

- THE EXACT WIDTH OF THE RECTANGLE.
- THE EXACT LENGTH OF THE RECTANGLE.

THIS IS A CLASSIC ALGEBRAIC PROBLEM THAT INVOLVES TRANSLATING REAL-WORLD DESCRIPTIONS INTO MATHEMATICAL EXPRESSIONS AND THEN SOLVING FOR THE UNKNOWN VARIABLES.

UNDERSTANDING THE KEY CONCEPTS: LENGTH, WIDTH, AND AREA

BEFORE DIVING INTO CALCULATIONS, IT'S ESSENTIAL TO CLARIFY SOME BASIC CONCEPTS:

- LENGTH (L): THE LONGER SIDE OF THE RECTANGLE.
- WIDTH (W): THE SHORTER SIDE OF THE RECTANGLE.
- AREA (A): THE SPACE ENCLOSED WITHIN THE RECTANGLE, CALCULATED AS THE PRODUCT OF LENGTH AND WIDTH: $(A = L \times W)$.

IN THIS PROBLEM, THE RELATIONSHIP BETWEEN LENGTH AND WIDTH IS SPECIFIED, WHICH ALLOWS US TO SET UP AN EQUATION TO FIND THE UNKNOWN DIMENSIONS.

FORMULATING THE MATHEMATICAL MODEL

TO SOLVE THE PROBLEM, WE NEED TO EXPRESS THE GIVEN INFORMATION MATHEMATICALLY:

1. EXPRESS LENGTH IN TERMS OF WIDTH:

GIVEN THAT THE LENGTH IS 7 CM LONGER THAN THE WIDTH, WE CAN WRITE:

$$[L = W + 7]$$

2. USE THE AREA FORMULA:

SINCE THE AREA IS 120 cm^2 :

$$[A = L \times W = 120]$$

SUBSTITUTING THE EXPRESSION FOR (L):

$$[(W + 7) \times W = 120]$$

THIS SIMPLIFIES THE PROBLEM INTO A SINGLE-VARIABLE QUADRATIC EQUATION.

SOLVING THE EQUATION: STEP-BY-STEP APPROACH

THE KEY TO SOLVING THIS PROBLEM IS TO SOLVE THE QUADRATIC EQUATION:

$$[(W + 7) \times W = 120]$$

WHICH EXPANDS TO:

$$[W^2 + 7W = 120]$$

REARRANGED INTO STANDARD QUADRATIC FORM:

$$[W^2 + 7W - 120 = 0]$$

NOW, WE CAN APPLY ALGEBRAIC METHODS SUCH AS FACTORING, COMPLETING THE SQUARE, OR THE QUADRATIC FORMULA. GIVEN THAT THE COEFFICIENTS ARE MANAGEABLE, FACTORING IS A STRAIGHTFORWARD APPROACH.

FACTORING THE QUADRATIC:

WE LOOK FOR TWO NUMBERS THAT MULTIPLY TO (-120) AND ADD TO (7) .

- FACTORS OF (-120) : $(\pm 1, \pm 2, \pm 3, \pm 4, \pm 5, \pm 6, \pm 8, \pm 10, \pm 12, \pm 15, \pm 20, \pm 24,$

$$\backslash\text{PM } 30, \backslash\text{PM } 40, \backslash\text{PM } 60, \backslash\text{PM } 120\backslash).$$

WE NEED TWO FACTORS $\backslash(A\backslash)$ AND $\backslash(B\backslash)$ SUCH THAT:

$$\backslash[\begin{aligned} A \times B &= -120 \\ A + B &= 7 \end{aligned} \backslash]$$

TESTING PAIRS:

$$- \backslash(15 \times -8 = -120\backslash), \text{ AND } \backslash(15 + (-8) = 7\backslash).$$

THIS MATCHES OUR CRITERIA, SO THE FACTORS ARE 15 AND -8.

EXPRESSING THE QUADRATIC AS:

$$\backslash[W^2 + 15W - 8W - 120 = 0 \backslash]$$

GROUPING TERMS:

$$\backslash[(W^2 + 15W) - (8W + 120) = 0 \backslash]$$

FACTORING EACH GROUP:

$$\backslash[W(W + 15) - 8(W + 15) = 0 \backslash]$$

FACTORING OUT COMMON BINOMIAL:

$$\backslash[(W + 15)(W - 8) = 0 \backslash]$$

SOLUTIONS:

$$\backslash[\begin{aligned} W + 15 &= 0 \rightarrow W = -15 \\ W - 8 &= 0 \rightarrow W = 8 \end{aligned} \backslash]$$

SINCE WIDTH CANNOT BE NEGATIVE, WE DISCARD $\backslash(W = -15\backslash)$, LEAVING:

$$\backslash[W = 8 \text{ \texttt{CM}} \backslash]$$

NOW, CALCULATING THE LENGTH:

$$\backslash[L = W + 7 = 8 + 7 = 15 \text{ \texttt{CM}} \backslash]$$

SUMMARY OF DIMENSIONS:

- WIDTH: 8 CM
- LENGTH: 15 CM

VERIFICATION OF THE SOLUTION

TO ENSURE OUR SOLUTION IS ACCURATE, VERIFY BY CALCULATING THE AREA:

$$\backslash[L \times W = 15 \times 8 = 120 \text{ \texttt{CM}}^2 \backslash]$$

WHICH MATCHES THE GIVEN AREA, CONFIRMING THE CORRECTNESS OF THE SOLUTION.

IMPLICATIONS AND REAL-WORLD APPLICATIONS

UNDERSTANDING HOW TO DERIVE DIMENSIONS FROM GIVEN AREA AND RELATIONSHIPS BETWEEN SIDES IS FUNDAMENTAL IN FIELDS SUCH AS ARCHITECTURE, DESIGN, AND MANUFACTURING. FOR EXAMPLE:

- DESIGNING A ROOM: IF A ROOM HAS A CERTAIN AREA AND THE LENGTH EXCEEDS THE WIDTH BY A SPECIFIC MEASURE, THESE CALCULATIONS HELP DETERMINE THE ACTUAL ROOM DIMENSIONS.
- FABRIC CUTTING: WHEN FABRIC PIECES ARE CUT WITH SPECIFIC LENGTH-TO-WIDTH RATIOS AND TOTAL AREA CONSTRAINTS, SIMILAR MATHEMATICAL PRINCIPLES APPLY.
- LAND PLOTTING: DETERMINING PLOT DIMENSIONS BASED ON AREA AND BOUNDARY LENGTH RELATIONSHIPS INVOLVES SIMILAR ALGEBRA.

EXTENSIONS: ADDITIONAL PROBLEMS AND CONSIDERATIONS

THIS PROBLEM SERVES AS A FOUNDATION FOR MORE COMPLEX GEOMETRIC AND ALGEBRAIC EXERCISES. FOR INSTANCE:

- WHAT IF THE RELATIONSHIP BETWEEN LENGTH AND WIDTH CHANGES? HOW WOULD THE CALCULATIONS ADAPT?
- HOW TO HANDLE CASES WHERE THE AREA IS NOT A PERFECT PRODUCT OF THE DIMENSIONS?
- EXPLORING THE EFFECTS OF DIFFERENT RATIOS BETWEEN LENGTH AND WIDTH ON THE OVERALL AREA AND SHAPE.

ADDITIONALLY, WHEN SOLVING SUCH PROBLEMS, CONSIDER:

- THE IMPORTANCE OF LOGICAL VERIFICATION.
- THE POTENTIAL FOR MULTIPLE SOLUTIONS, AND HOW TO ELIMINATE THE NON-PHYSICAL ONES.
- THE USE OF QUADRATIC FORMULAS WHEN FACTORING IS NOT STRAIGHTFORWARD.

CONCLUSION

SOLVING FOR THE DIMENSIONS OF A RECTANGLE WHEN GIVEN THE RELATIONSHIP BETWEEN LENGTH AND WIDTH AND THE TOTAL AREA INVOLVES TRANSLATING A REAL-WORLD SCENARIO INTO ALGEBRAIC EXPRESSIONS. IN THIS CASE, RECOGNIZING THE LINEAR RELATIONSHIP BETWEEN THE SIDES ALLOWED US TO FORMULATE A QUADRATIC EQUATION, WHICH WE THEN SOLVED THROUGH FACTORING. THE RESULTING DIMENSIONS — AN 8 CM WIDTH AND A 15 CM LENGTH — SATISFY THE INITIAL CONDITIONS, ILLUSTRATING THE POWER AND PRACTICALITY OF ALGEBRA IN EVERYDAY PROBLEM-SOLVING. WHETHER IN EDUCATION, CONSTRUCTION, OR DESIGN, MASTERING SUCH TECHNIQUES ENABLES PRECISE CALCULATIONS AND INFORMED DECISION-MAKING, BRIDGING THE GAP BETWEEN CONCEPTUAL UNDERSTANDING AND REAL-WORLD APPLICATION.

[The Length Of A Rectangle Is 7 Cm Longer Than Its Width The Area Of The Rectangle Is 120 Cm What Is The](#)

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