The Point (-4,-6,10) Lies On The Surface Of A Sphere With Centre (2,3,-1). Hence Find The Volume Of The

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Understanding the geometric relationship between points and spheres is fundamental in mathematics, especially in the fields of coordinate geometry and spatial analysis. In this article, we will explore how to determine the volume of a sphere when given a specific point lying on its surface and the sphere's center. Specifically, we analyze the point (-4, -6, 10) and the sphere with center at (2, 3, -1), guiding you step-by-step through the process of finding the sphere's radius and subsequently its volume.

Introduction to Coordinate Geometry and Sphere Equations

Coordinate geometry provides a framework for analyzing geometric figures in three-dimensional space using Cartesian coordinates. A sphere in 3D space is defined as the set of all points equidistant from a fixed point called the center. The general equation of a sphere with center (h, k, l) and radius r is:

$$
(x - h)^2 + (y - k)^2 + (z - l)^2 = r^2
$$

where:
- (x, y, z) are the coordinates of any point on the sphere,
- (h, k, l) is the center of the sphere,
- r is the radius.

Given the center and a point on the surface, the task reduces to calculating the radius and then the volume.

Step 1: Understanding the Given Data

The problem provides:
- A point $P(-4, -6, 10)$ lying on the sphere's surface.

- The center $C(2, 3, -1)$ of the sphere.

This information allows us to determine the radius by calculating the distance between the center and the point P.

Step 2: Calculating the Radius of the Sphere

The radius r of the sphere is the distance from the center C to any point P on its surface. Using the distance formula in three-dimensional space:

$$
r = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}
$$

where:
- $(x_1, y_1, z_1) = (2, 3, -1)$, the center,
- $(x_2, y_2, z_2) = (-4, -6, 10)$, the point on the surface.

Calculating step-by-step:

1. Difference in x-coordinates:

$$
-4 - 2 = -6
$$

2. Difference in y-coordinates:

$$
-6 - 3 = -9
$$

3. Difference in z-coordinates:

$$
10 - (-1) = 10 + 1 = 11
$$

4. Calculate the squares:

$$
(-6)^2 = 36
$$

$$
(-9)^2 = 81
$$

$$
11^2 = 121
$$

5. Sum of squares:

$$
36 + 81 + 121 = 238
$$

6. Compute the radius:

$$
r = \sqrt{238} \approx 15.43
$$

Thus, the radius of the sphere is approximately 15.43 units.

Step 3: Calculating the Volume of the Sphere

The volume V of a sphere is given by the formula:

$$
V = \frac{4}{3} \pi r^3
$$

Using the radius calculated:

$$
V = \frac{4}{3} \pi (15.43)^3
$$

Calculating step-by-step:

1. Cube the radius:

$$
15.43^3 \approx 15.43 \times 15.43 \times 15.43
$$

First, $15.43^2 \approx 238.00$ (since $15.43^2 = 238$ from previous calculation), then multiply by 15.43:

$$
238 \times 15.43 \approx 3669.34
$$

2. Calculate the volume:

$$
V \approx \frac{4}{3} \times \pi \times 3669.34
$$

Using $\pi \approx 3.1416$:

$$
V \approx \frac{4}{3} \times 3.1416 \times 3669.34
$$

Multiply:

$$
3.1416 \times 3669.34 \approx 11523.3
$$

Then multiply by $\frac{4}{3}$:

$$
V \approx \frac{4}{3} \times 11523.3 \approx 1.3333 \times 11523.3 \approx 15364.4
$$

Final answer:

$$
\boxed{
\text{The volume of the sphere} \approx 15,364.4 \text{ cubic units}
}
$$

Additional Considerations and Applications

Understanding how to find the volume of a sphere given a point on its surface and the center has numerous applications across various fields:

- Physics: Calculating the volume of celestial bodies like planets or stars.
- Engineering: Designing spherical tanks, domes, or other structures.
- Computer Graphics: Rendering spherical objects accurately in 3D space.
- Mathematics Education: Teaching concepts of distance, volume, and coordinate systems.

Summary of the Calculation Process

To summarize, the steps involved in solving such problems are:

1. Identify the given data:
- Center coordinates (h, k, l)
- Surface point coordinates (x, y, z)

2. Calculate the radius:
- Use the 3D distance formula.

3. Apply the sphere volume formula:
- Use $V = \frac{4}{3} \pi r^3$

4. Compute the numerical value:
- Simplify step-by-step for accuracy.

Conclusion

Determining the volume of a sphere from a given point on its surface and its center is a fundamental problem in three-dimensional coordinate geometry. By calculating the distance between the center and the point, we find the radius, which then directly feeds into the volume formula. For the sphere with center at (2, 3, -1) and passing through (-4, -6, 10), the radius is approximately 15.43 units, leading to a volume of about 15,364.4 cubic units. This process exemplifies the practical application of distance formulas and volume calculations in spatial mathematics, essential for various scientific and engineering disciplines.

Frequently Asked Questions

How do you verify if the point (-4, -6, 10) lies on the surface of a sphere with center (2, 3, -1)?

To verify, calculate the distance between the point and the sphere's center using the distance formula. If this distance equals the sphere's radius, the point lies on its surface.

What is the formula to find the radius of a sphere given its center and a point on its surface?

The radius is the distance between the center (x_1, y_1, z_1) and the point (x, y, z), calculated as $\sqrt{[(x - x_1)^2 + (y - y_1)^2 + (z - z_1)^2]}$.

How do you compute the volume of a sphere once the radius is known?

The volume of a sphere is given by the formula $V = (4/3)\pi r^3$, where r is the radius of the sphere.

Can you determine the volume of the sphere if the point (-4, -6, 10) lies on its surface and the center is at (2, 3, -1)?

Yes. First, find the radius using the distance formula: $r = \sqrt{[(-4-2)^2 + (-6-3)^2 + (10+1)^2]}$, then substitute r into $V = (4/3)\pi r^3$ to find the volume.

What steps are involved in solving for the volume of the sphere given the point and center coordinates?

Step 1: Calculate the radius using the distance formula between the center and the point. Step 2: Use the radius in the volume formula $V = (4/3)\pi r^3$. Step 3: Simplify to find the volume.

Additional Resources

The Point (-4, -6, 10) Lies On The Surface Of A Sphere With Centre (2, 3, -1). Hence Find The Volume Of The

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Introduction

In the realm of three-dimensional geometry, the relationship between points and spherical surfaces is fundamental. Understanding whether a point lies on the surface of a sphere, and subsequently calculating the sphere’s volume, involves a clear grasp of geometric principles and algebraic computations. The problem at hand exemplifies these concepts: given a specific point in space and the sphere’s centre, determine if the point resides on the sphere’s surface, and if so, derive the sphere's volume.

This exploration provides a comprehensive analysis of the problem, breaking down each step with detailed explanations, mathematical formulas, and contextual insights. It aims to serve as a thorough guide for students, educators, or enthusiasts interested in geometric applications, emphasizing not just the solution but also the underlying principles that govern three-dimensional spatial relationships.

Understanding the Geometry of the Sphere and Point Coordinates

The Sphere in Three-Dimensional Space

A sphere is a perfectly symmetrical three-dimensional shape characterized by all points equidistant from a fixed point called the centre. The distance from the centre to any point on the surface is the radius of the sphere.

Mathematically, a sphere with centre \((x_0, y_0, z_0)\) and radius \(r\) can be represented as:

\[
(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 = r^2
\]

where \((x, y, z)\) are the coordinates of any point on the sphere's surface.

Coordinates of the Given Point and Sphere's Centre

In the problem, the given point is \((-4, -6, 10)\), and the sphere's centre is \((2, 3, -1)\). These coordinates are essential for calculating the distance between the point and the centre, which in turn determines whether the point lies on the sphere.

Step-by-Step Solution Approach

Step 1: Verify if the Point Lies on the Sphere Surface

The initial step involves calculating the distance between the point and the sphere's centre. If this distance equals the sphere's radius, the point lies on the surface; otherwise, it does not.

Distance Formula in 3D:

\[
d = \sqrt{(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2}
\]

Applying the formula:

\[
d = \sqrt{(-4 - 2)^2 + (-6 - 3)^2 + (10 - (-1))^2}
\]

Calculating each term:

- \((-4 - 2) = -6\)
- \((-6 - 3) = -9\)
- \((10 + 1) = 11\)

Squaring these:

- \((-6)^2 = 36\)
- \((-9)^2 = 81\)
- \(11^2 = 121\)

Sum:

\[
36 + 81 + 121 = 238

\]

Therefore, the distance:

\[
d = \sqrt{238} \approx 15.43
\]

Step 2: Determining the Radius of the Sphere

Since the point is on the surface of the sphere, the distance from the centre to this point is the radius \(r\):

\[
r = \sqrt{238} \approx 15.43
\]

This confirms that the point indeed lies on the sphere's surface as its distance from the centre matches the radius.

Step 3: Calculating the Volume of the Sphere

The volume \(V\) of a sphere with radius \(r\) is given by the well-known formula:

\[
V = \frac{4}{3} \pi r^3
\]

Substituting the value of \(r\):

\[
V = \frac{4}{3} \pi (\sqrt{238})^3
\]

Simplify the cube:

\[
V = \frac{4}{3} \pi \times (238)^{3/2}
\]

Recall that:

\[
(238)^{3/2} = (\sqrt{238})^3
\]

which we already have as \(r^3 \). Therefore:

\[
V = \frac{4}{3} \pi \times (\sqrt{238})^3
\]

Expressed explicitly:

$$
V = \frac{4}{3} \pi \times 238 \times \sqrt{238}
$$

since:

$$
(\sqrt{238})^3 = 238 \times \sqrt{238}
$$

Detailed Calculation and Approximation

Numerical Approximation of the Volume

Given:

$$
r = \sqrt{238} \approx 15.43
$$

Compute r^3:

$$
r^3 \approx (15.43)^3
$$

Calculating:

- $15.43^2 \approx 238$ (by definition)
- $15.43^3 = 15.43 \times 238 \approx 15.43 \times 238$

Calculate:

$$
15.43 \times 238 \approx 15.43 \times 238
$$

Breaking down:

- $15.43 \times 200 = 3086$
- $15.43 \times 38 = 586.34$

Adding:

$$
3086 + 586.34 \approx 3672.34
$$

Now, substitute back into the volume formula:

$$
V \approx \frac{4}{3} \pi \times 3672.34
$$

Using $\pi \approx 3.1416$:

$$
V \approx \frac{4}{3} \times 3.1416 \times 3672.34
$$

Calculating:

- $\frac{4}{3} \times 3.1416 \approx 4.1888$

Then:

$$
V \approx 4.1888 \times 3672.34 \approx 15359.48
$$

Final Approximate Volume:

$$
\boxed{
V \approx 15359.48 \text{ cubic units}
}
$$

Additional Considerations and Context

Significance of the Calculation

The calculation confirms that the point $(-4, -6, 10)$ indeed lies on the sphere’s surface, as verified through distance computation. With the radius determined, computing the volume becomes straightforward, grounded in fundamental geometric formulas.

Applicability and Real-World Relevance

Understanding such geometric principles is vital across various domains:

- Physics: Modeling celestial bodies and their gravitational spheres.
- Engineering: Designing spherical tanks or domes, where precise volume calculations are crucial.
- Computer Graphics: Rendering spherical objects and understanding spatial relationships in 3D modeling.
- Geography and Earth Sciences: Calculating the volume of spherical sections like planetary models.

Limitations and Assumptions

- The calculations assume a perfect sphere with no distortions.
- The point coordinates are precise, and the calculations are based on Euclidean geometry.
- Approximate numerical values are used; for exact results, symbolic computation involving $\sqrt{238}$ should be preferred.

Broader Mathematical Insights

The Role of Coordinates in Spatial Geometry

Coordinate systems enable precise calculations in three-dimensional space. The ability to analyze whether a point lies on a sphere’s surface hinges on calculating distances and comparing them to the sphere’s radius, highlighting the importance of vector and distance formulas.

The Relationship Between Distance and Volume

The radius is the critical parameter determining the sphere's volume. Slight variations in the radius exponentially affect the volume due to the cubic relationship, emphasizing the importance of accurate measurements in practical applications.

Extension to Related Problems

- Finding the Sphere’s Surface Area: Using $A = 4 \pi r^2$.
- Determining the Sphere’s Equation: With known centre and radius, the explicit equation can be written as:

$$
(x - 2)^2 + (y - 3)^2 + (z + 1)^2 = 238
$$

- Volume of Spherical Segments: For partial sections, calculus techniques are employed.

Conclusion

The problem of verifying a point’s position relative to a sphere and calculating its volume encapsulates core principles of three-dimensional geometry. The process involves calculating the distance between a point and the sphere’s centre, confirming if it matches the radius, and then applying the volume formula.

In this case, the point $(-4, -6, 10)$ lies precisely on the surface of the sphere with centre $(2, 3, -1)$, and the sphere’s radius is approximately 15.43 units. Consequently, the volume of this sphere is approximately 15,359.48 cubic units.

This exploration underscores the interconnectedness of spatial reasoning, algebra, and geometry—fundamental tools that find applications across science, engineering, and technology. Understanding these principles not only facilitates solving specific problems but also enhances one's capacity to analyze complex three-dimensional structures in the real world.

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