The Point P = (x, 1/2) Lies On The Unit Curcle Shown Below. What Is The Value Of X In The Simplest Form

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Understanding how to find the value of x when a point lies on a specific circle is a fundamental concept in coordinate geometry. In this article, we will explore the problem: Given the point P = (x, 1/2) lies on a circle, specifically the unit circle, what is the value of x in its simplest form? We will analyze the problem step-by-step, review the relevant equations, and provide clear explanations to help you grasp the underlying concepts.

Understanding the Unit Circle

What Is the Unit Circle?

The unit circle is a circle with a radius of 1 centered at the origin (0, 0) in the coordinate plane. Its standard equation is:

$$ x^2 + y^2 = 1 $$

This equation states that any point (x, y) lying on the circle must satisfy the relationship where the sum of the squares of x and y equals 1.

Importance of the Unit Circle in Geometry and Trigonometry

The unit circle serves as a fundamental tool for understanding trigonometric functions, angles, and their relationships. It helps in visualizing sine, cosine, tangent, and other functions, especially when dealing with angles in radians or degrees.

Given Point and the Problem Statement

Point P = (x, 1/2)

The problem states that the point P with coordinates (x, 1/2) lies on the circle. Our goal is to find the value of x.

Why Is This Important?

Knowing that a point lies on a circle allows us to substitute its coordinates into the circle's equation to find unknown variables—in this case, x.

Setting Up the Equation

Using the Equation of the Unit Circle

Since P lies on the circle with the equation $x^2 + y^2 = 1$, and the y-coordinate is given as 1/2, we can substitute y = 1/2 into the equation:

$$x^2 + \left(\frac{1}{2}\right)^2 = 1$$

Solving for x

Our goal is to isolate x and find its value:

$$x^2 + \frac{1}{4} = 1$$

Subtract $\frac{1}{4}$ from both sides:

$$x^2 = 1 - \frac{1}{4}$$

Simplify the right side:

$$x^2 = \frac{4}{4} - \frac{1}{4} = \frac{3}{4}$$

Now, take the square root of both sides to solve for x:

$$x = \pm \sqrt{\frac{3}{4}}$$

Simplify the square root:

$$x = \pm \frac{\sqrt{3}}{\sqrt{4}}$$

$$x = \pm \frac{\sqrt{3}}{2}$$

Final Answer: The Value of x

In Simplest Form

The value of x in simplest radical form is:

$$ x = \pm \frac{\sqrt{3}}{2} $$

This indicates that there are two possible points on the circle with y-coordinate 1/2, corresponding to x being positive or negative.

Visualizing the Solution

Graphical Interpretation

- When y = 1/2, the circle intersects the horizontal line y = 1/2 at two points.
- The x-coordinates of these intersection points are $\pm \frac{\sqrt{3}}{2}$.
- These points are symmetric about the y-axis because the circle is centered at the origin.

Sample Coordinates of the Points

- The two points are:

$$ P_1 = \left(\frac{\sqrt{3}}{2}, \frac{1}{2} \right) $$
$$ P_2 = \left(-\frac{\sqrt{3}}{2}, \frac{1}{2} \right) $$

Additional Examples and Applications

Example 1: Points on the Unit Circle with Different y-values

Suppose you are asked to find x when y = 0. Then:

$$ x^2 + 0^2 = 1 $$
$$ x^2 = 1 $$
$$ x = \pm 1 $$

Hence, the points are (1, 0) and (-1, 0).

Example 2: Using the Point to Find the Angle

Knowing x and y coordinates on the unit circle can help determine the angle θ in radians:

$$ \cos \theta = x $$
$$ \sin \theta = y $$

For our point:

$$\cos \theta = \pm \frac{\sqrt{3}}{2}$$
$$\sin \theta = \frac{1}{2}$$

The positive x corresponds to $\theta = 30^\circ$ or $\pi/6$ radians, and the negative x corresponds to 150° or $5\pi/6$ radians.

Summary and Key Takeaways

- The point $P = (x, 1/2)$ lies on the unit circle $x^2 + y^2 = 1$.
- Substituting y = 1/2 into the circle’s equation yields:

$$x^2 = \frac{3}{4}$$

- Taking the square root gives:

$$x = \pm \frac{\sqrt{3}}{2}$$

- The solutions are symmetric about the y-axis and represent the x-coordinates of the points where the circle intersects the line y = 1/2.

- Understanding how to derive these values is essential for solving many problems in coordinate geometry, trigonometry, and related fields.

Conclusion

Finding the value of x when a point lies on a circle involves substituting the point's coordinates into the circle's equation and solving for the unknowns. For the specific case where the point is $(x, 1/2)$ on the unit circle, the simplest radical form of x is:

$$x = \pm \frac{\sqrt{3}}{2}$$

This process underscores the importance of algebraic manipulation and understanding the properties of the circle in coordinate geometry. Whether you're studying for exams or exploring the fundamentals of geometry, mastering these techniques is crucial for success in mathematics.

Frequently Asked Questions

What is the equation of the unit circle on which

point P = (x, 1/2) lies?

The equation of the unit circle is $x^2 + y^2 = 1$. Since P is on the circle, its coordinates satisfy this equation.

How do you find the value of x for the point P = (x, 1/2) on the unit circle?

Substitute $y = 1/2$ into the circle's equation: $x^2 + (1/2)^2 = 1$, then solve for x.

What is the simplified form of the value of x when solving for P = (x, 1/2) on the circle?

Solving $x^2 + 1/4 = 1$ gives $x^2 = 3/4$, so $x = \pm\sqrt{3/4} = \pm(\sqrt{3})/2$.

What are the possible values of x for point P = (x, 1/2) on the unit circle?

The possible values are $x = (\sqrt{3})/2$ and $x = -(\sqrt{3})/2$.

Why are there two values of x for the point on the circle at y = 1/2?

Because the circle is symmetric about the x-axis, points at $y=1/2$ can have positive or negative x-values, corresponding to both sides of the circle.

In what quadrant(s) can the point P = (x, 1/2) lie on the unit circle?

P can lie in the first quadrant ($x = (\sqrt{3})/2$) or the second quadrant ($x = -(\sqrt{3})/2$), since $y = 1/2$ is positive.

What is the significance of finding x in simplest form for the point on the circle?

Expressing x in simplest radical form ($\pm(\sqrt{3})/2$) provides the exact value, avoiding decimal approximation and maintaining mathematical accuracy.

Additional Resources

Understanding the Geometric Significance of the Point P = (x, 1/2) on the Unit Circle

When exploring the fascinating world of circles and their algebraic representations, one of the fundamental concepts is the relationship of points lying on a circle. Specifically, the unit circle, defined as the set of all points in the coordinate plane at a distance of 1 from the origin, serves as a cornerstone in trigonometry, geometry, and algebra. The problem at hand involves determining the value of x for a point $P = (x, \frac{1}{2})$ that resides on this circle. This question not only tests our understanding of circle equations but also offers insights into the interplay between algebraic and geometric representations.

Introduction to the Unit Circle

The unit circle is a fundamental concept in mathematics, especially in trigonometry and coordinate geometry. Its defining characteristic is that every point (x, y) on the circle satisfies the equation:

$$
x^2 + y^2 = 1
$$

This equation is derived from the Pythagorean theorem, considering the origin $(0,0)$ as the center and the radius $r = 1$. The significance of the unit circle extends to:

- Angles and Radians: It provides a geometric representation of trigonometric functions.
- Coordinate Relationships: It links algebraic expressions to geometric points.
- Symmetry and Periodicity: It exemplifies the periodic nature of sine and cosine functions.

Understanding the properties of the unit circle is crucial for solving the problem involving the point $P = (x, \frac{1}{2})$.

Formulating the Problem

The problem states:
> "The point $P = (x, \frac{1}{2})$ lies on the unit circle. Find the value of x in the simplest form."

This involves:

- Recognizing that P satisfies the circle's equation.

- Substituting $y = \frac{1}{2}$ into the equation.
- Solving for x.

This straightforward substitution leads to an algebraic problem, but the depth of the problem can be appreciated by exploring the geometric implications and the nature of solutions.

Step-by-Step Solution

1. Write down the equation of the circle

$$
x^2 + y^2 = 1
$$

2. Substitute $y = \frac{1}{2}$ into the equation

$$
x^2 + \left(\frac{1}{2}\right)^2 = 1
$$

3. Simplify the equation

$$
x^2 + \frac{1}{4} = 1
$$

4. Isolate x^2

$$
x^2 = 1 - \frac{1}{4}
$$
$$
x^2 = \frac{4}{4} - \frac{1}{4} = \frac{3}{4}
$$

5. Solve for x

$$
x = \pm \sqrt{\frac{3}{4}}
$$

Since the square root of a fraction can be written as the fraction of the square roots:

$$
x = \pm \frac{\sqrt{3}}{\sqrt{4}} = \pm \frac{\sqrt{3}}{2}
$$

Result:

$$
x = \pm \frac{\sqrt{3}}{2}
$$

Interpreting the Solutions and Their Geometric Significance

The solutions $x = \pm \frac{\sqrt{3}}{2}$ have rich geometric interpretations, especially when viewed on the unit circle.

Positive and Negative Roots

- Positive root $x = \frac{\sqrt{3}}{2}$: Corresponds to a point on the circle in the first or second quadrant where x is positive.
- Negative root $x = -\frac{\sqrt{3}}{2}$: Corresponds to a point in the second or third quadrant where x is negative.

Connection to Trigonometry

- The x-coordinate on the unit circle corresponds to $\cos \theta$, where θ is the angle made with the positive x-axis.
- Since $y = \frac{1}{2}$, which equals $\sin \theta$, the point P corresponds to angles where:

$$
\sin \theta = \frac{1}{2}
$$

- Known from standard angles:

$$
\theta = \frac{\pi}{6} \quad \text{or} \quad \frac{5\pi}{6}
$$

- Correspondingly, the x-values (cosines) at these angles are:

$$
\cos \frac{\pi}{6} = \frac{\sqrt{3}}{2}
$$
$$
\cos \frac{5\pi}{6} = - \frac{\sqrt{3}}{2}
$$

This confirms that the solutions are consistent with well-known trigonometric values.

Alternative Approach: Using Trigonometry

Instead of solving purely algebraically, one could approach the problem from a trigonometric perspective:

- Recognize that the point (x, y) on the unit circle corresponds to $(\cos \theta, \sin \theta)$.
- Given $y = \frac{1}{2}$, find θ such that:

$$
\sin \theta = \frac{1}{2}
$$

- The solutions for θ are:

$$
\theta = \frac{\pi}{6} \quad \text{or} \quad \frac{5\pi}{6}
$$

- The corresponding x-values are:

$$
x = \cos \theta
$$
$$
x = \cos \frac{\pi}{6} = \frac{\sqrt{3}}{2}
$$
$$
x = \cos \frac{5\pi}{6} = -\frac{\sqrt{3}}{2}
$$

\]

This approach aligns with the algebraic solution, confirming its correctness and providing an intuitive understanding based on angles.

Understanding the Simplest Form of the Solution

The problem requests the value of x in the simplest form. The solutions:

$$
x = \pm \frac{\sqrt{3}}{2}
$$

are already in their simplest radical form because:

- The numerator $\sqrt{3}$ cannot be simplified further.
- The denominator 2 is prime and simple.
- There are no common factors to reduce.

Expressing solutions in radical form is standard practice in mathematics, especially for exact values related to special angles.

Further Insights and Related Concepts

Symmetry of the Circle

- The solutions reflect the symmetry about the y-axis.
- For a fixed $y = \frac{1}{2}$, the two x-values are symmetric with respect to the y-axis, which is a characteristic of circles.

Application in Trigonometry and Calculus

- These solutions are foundational for understanding sine and cosine functions.
- They are used in deriving identities, solving equations, and analyzing wave functions.

Other Points on the Circle with Similar Coordinates

- For different y-values, similar algebraic procedures can find corresponding x-values.
- Understanding the structure of these solutions enhances comprehension of the circle's geometry.

Summary and Final Remarks

The problem of finding the x-coordinate of the point $P = (x, \frac{1}{2})$ on the unit circle encapsulates core principles of algebra and trigonometry. The key steps involve:

1. Recognizing the circle's equation $x^2 + y^2 = 1$.
2. Substituting $y = \frac{1}{2}$, leading to a quadratic in x.
3. Simplifying to find:

$$
x^2 = \frac{3}{4}
$$

4. Taking the square root to obtain:

$$
x = \pm \frac{\sqrt{3}}{2}
$$

These solutions are expressed in their simplest radical form, aligning with well-known cosine values at specific angles. They demonstrate the deep connection between algebraic solutions and geometric/trigonometric interpretations.

In conclusion, the value of x in the simplest form is:

$$
\boxed{
x = \pm \frac{\sqrt{3}}{2}
}
$$

This result is not only a straightforward algebraic solution but also a window into the elegant symmetry and structure of the unit circle, reinforcing foundational concepts in mathematics and their applications across various fields.

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