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Understanding quadratic functions is fundamental in algebra and calculus, especially when analyzing their graphs, properties, and applications. The quadratic function given by the equation [tex]y=-x^2 - 8x + 1[/tex] is a classic example that demonstrates key features such as the axis of symmetry, vertex, and parabola shape. This article explores the detailed aspects of this quadratic function, focusing on how the axis of symmetry at [tex]x = -4[/tex] informs us about the parabola's shape and properties, and answering the question: which of the features or points lie on or relate to this axis?

Understanding the Quadratic Function and Its Graph

Quadratic functions are polynomial functions of degree 2, typically written in the form:

- [tex]y = ax^2 + bx + c[/tex]

where [tex]a \neq 0[/tex]. The graph of a quadratic function is a parabola that opens upward if [tex]a > 0[/tex], and downward if [tex]a < 0[/tex].

In our case, the function is:

- [tex]y = -x^2 - 8x + 1[/tex]

which opens downward because [tex]a = -1[/tex].

Calculating the Axis of Symmetry

The axis of symmetry of a parabola is a vertical line that passes through its vertex, dividing the parabola into two mirror-image halves.

Standard Formula

For a quadratic in the form [tex]y = ax^2 + bx + c[/tex], the axis of symmetry is given by:

- [tex]x = -\frac{b}{2a}[/tex]

Applying this to our function:

- [tex]a = -1[/tex]
- [tex]b = -8[/tex]

Calculating:

- [tex]x = -\frac{-8}{2 \times -1} = -\frac{-8}{-2} = -\frac{8}{-2} = 4[/tex]

However, note that the question states the axis of symmetry is at [tex]x = -4[/tex], which suggests a different interpretation or a potential typo. Let's verify this:

Verifying the Given Axis of Symmetry

Given the function:

- [tex]y = -x^2 - 8x + 1[/tex]

Calculate the axis of symmetry:

- [tex]x = -\frac{b}{2a} = -\frac{-8}{2 \times -1} = -\frac{-8}{-2} = -4[/tex]

Note: Here, the calculation yields [tex]x = -4[/tex], confirming the initial statement.

Correction: Previously, I made an error in the calculation; the correct calculation is:

- [tex]x = -\frac{b}{2a} = -\frac{-8}{2 \times -1} = -\frac{-8}{-2} = -4[/tex]

which aligns with the given axis at [tex]x = -4[/tex].

Thus, the axis of symmetry is correctly at [tex]x = -4[/tex].

Finding the Vertex of the Parabola

The vertex is the point where the parabola reaches its maximum or minimum. Since our parabola opens downward, the vertex is the maximum point.

Vertex Coordinates

Using the axis of symmetry:

- [tex]x_{vertex} = -4[/tex]

To find the corresponding y-coordinate, substitute [tex]x = -4[/tex] into the original function:

- [tex]y = -(-4)^2 - 8(-4) + 1[/tex]
- [tex]y = -16 + 32 + 1 = 17[/tex]

So, the vertex is at:

- [tex](-4, 17)[/tex]

Properties of the Quadratic Function

Understanding the properties of this quadratic function helps interpret its graph and its relationship with the axis of symmetry.

1. Parabola Opening Direction

Since [tex]a = -1 < 0[/tex], the parabola opens downward.

2. Vertex

Located at [tex](-4, 17)[/tex], it's the highest point on the graph.

3. Axis of Symmetry

A vertical line at [tex]x = -4[/tex], passing through the vertex.

4. Y-Intercept

Find the y-intercept by setting [tex]x=0[/tex]:

- [tex]y = -0^2 - 8(0) + 1 = 1[/tex]

So, the y-intercept is at [tex](0, 1)[/tex].

5. X-Intercepts (Roots)

Solve [tex]-x^2 - 8x + 1 = 0[/tex]:

Multiply both sides by -1:

- [tex]x^2 + 8x - 1 = 0[/tex]

Apply quadratic formula:

- [tex]x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-8 \pm \sqrt{64 - 4 \times 1 \times (-1)}}{2} = \frac{-8 \pm \sqrt{64 + 4}}{2} = \frac{-8 \pm \sqrt{68}}{2}[/tex]

Simplify:

- [tex]x = \frac{-8 \pm 2\sqrt{17}}{2} = -4 \pm \sqrt{17}[/tex]

Thus, the x-intercepts are at:

- [tex]x = -4 + \sqrt{17} \approx -4 + 4.123 = 0.123[/tex]
- [tex]x = -4 - \sqrt{17} \approx -4 - 4.123 = -8.123[/tex]

Which Points Lie On the Axis of Symmetry?

The axis of symmetry at [tex]x=-4[/tex] passes through the vertex and is a line of symmetry for the parabola. Points that lie on this line are symmetric counterparts on either side of the parabola.

Points on the Axis of Symmetry

- Vertex: [tex](-4, 17)[/tex], lies on the axis of symmetry.
- Any point with [tex]x = -4[/tex]: The only such point is the vertex itself, since the parabola is symmetric about this line.

Symmetric Points about the Axis

For any [tex]x \neq -4[/tex], the points [tex](x, y)[/tex] and [tex](x', y')[/tex] are symmetric if:

- [tex]x' = -8 - x[/tex]
- [tex]y' = y[/tex]

For example, the points at [tex]x = 0.123[/tex] and [tex]x = -8.123[/tex] are symmetric about [tex]x = -4[/tex].

Applications and Importance of the Axis of Symmetry

Understanding the axis of symmetry is vital in various mathematical and real-world problems:

- **Graphing:** Helps sketch the parabola accurately.
- **Vertex determination:** The axis passes through the vertex, giving its x-coordinate directly.
- **Optimization problems:** The vertex represents maximum or minimum values of quadratic functions.
- **Symmetry analysis:** Simplifies solving equations and analyzing the graph's properties.

Conclusion

The quadratic function [tex]y=-x^2 -8x + 1[/tex] possesses an axis of symmetry at [tex]x = -4[/tex], which passes through its vertex at [tex](-4, 17)[/tex]. This symmetry plays a crucial role in understanding the parabola's shape, roots, and maximum point. The points on the parabola that

Frequently Asked Questions

What is the axis of symmetry for the quadratic function y = -x^2 - 8x + 1?

The axis of symmetry is x = -4.

How do you find the axis of symmetry for a quadratic function in the form y = ax^2 + bx + c?

The axis of symmetry is given by x = -b / (2a).

Given the quadratic function y = -x^2 - 8x + 1, what are the coordinates of its vertex?

The vertex is at (-4, 17).

Is the parabola y = -x^2 - 8x + 1 opening upwards or downwards, and why?

It opens downward because the coefficient of x^2, which is -1, is negative.

How does the axis of symmetry relate to the graph of the quadratic function?

The axis of symmetry divides the parabola into two mirror-image halves and passes through the vertex.

Additional Resources

Quadratic Function Analysis: Unlocking the Secrets of $y = -x^2 - 8x + 1$

When delving into the fascinating world of quadratic functions, their graphs, and their properties, the quadratic function $y = -x^2 - 8x + 1$ stands out as a compelling example. This particular parabola offers rich insights into symmetry, vertex location, and key features that define its behavior. In this comprehensive review, we will explore this quadratic function in detail, emphasizing its axis of symmetry $x = -4$, and uncover how this property shapes the graph and its characteristics.

Understanding the Quadratic Function Fundamentals

Before diving into the specifics of our quadratic $y = -x^2 - 8x + 1$, it's essential to revisit the foundational concepts that underpin all quadratic functions.

What Is a Quadratic Function?

A quadratic function is a polynomial of degree 2, generally expressed in the form:

$$
y = ax^2 + bx + c
$$

where $a \neq 0$, and b and c are real numbers. The shape of its graph is a parabola, which can open upward or downward depending on the sign of a.

In our case:

$$
a = -1, \quad b = -8, \quad c = 1

\]

Since \(a \) is negative, the parabola opens downward.

Key Features of Quadratic Graphs

Quadratic graphs are symmetric with respect to a vertical line called the axis of symmetry. They have distinctive features:

- Vertex: The highest or lowest point of the parabola, depending on the direction it opens.
- Axis of Symmetry: The vertical line passing through the vertex, dividing the parabola into mirror images.
- Y-intercept: The point where the parabola crosses the y-axis.
- X-intercepts (Roots): The points where the parabola crosses the x-axis, if any.

Analyzing \(y = -x^2 - 8x + 1 \)

The focus of this review is to analyze the specific quadratic \(y = -x^2 - 8x + 1 \), with special attention to its axis of symmetry at \(x = -4 \).

Standard Form and Vertex Form

The quadratic is initially presented in standard form:

\[
y = -x^2 - 8x + 1
\]

To analyze its properties effectively, especially the vertex and symmetry, converting it to vertex form is advantageous. The vertex form is:

\[
y = a(x - h)^2 + k
\]

where \((h, k) \) is the vertex of the parabola.

Completing the Square

Let's transform the standard form into vertex form to identify the vertex explicitly.

Starting with:

\[
y = -x^2 - 8x + 1
\]

Factor out the coefficient of \(x^2 \):

\[
y = -(x^2 + 8x) + 1
\]

Complete the square inside the parentheses:

- Take half of the coefficient of \(x \), which is 8, divide by 2:

\[
\frac{8}{2} = 4
\]

- Square it:

\[
4^2 = 16
\]

Add and subtract this inside the parentheses:

\[
y = -\left(x^2 + 8x + 16 - 16 \right) + 1
\]

Rewrite:

\[
y = -\left((x + 4)^2 - 16 \right) + 1
\]

Distribute the negative:

\[
y = - (x + 4)^2 + 16 + 1
\]

Simplify:

\[
\boxed{
y = - (x + 4)^2 + 17

}
\]

Vertex form: \(y = - (x + 4)^2 + 17 \)

Vertex: \((-4, 17) \)

This confirms the vertex of the parabola is at \((-4, 17) \).

Axis of Symmetry: \(x = -4 \)

The axis of symmetry for any parabola in vertex form is the vertical line that passes through its vertex. Since our vertex is at \((-4, 17) \), the axis of symmetry is:

\[
\boxed{
x = -4
}
\]

This line divides the parabola into two mirror-image halves, providing a clear axis around which symmetry is established.

Why is the axis of symmetry at \(x = -4 \)?

In quadratic functions, the axis of symmetry is directly linked to the vertex's \(x \)-coordinate:

\[
x = h
\]

where \(h \) is the \(x \)-coordinate of the vertex. Our transformation explicitly shows that \(h = -4 \), confirming the symmetry line at \(x = -4 \).

Implications of the Axis of Symmetry in the Graph

Understanding the axis of symmetry allows us to analyze and predict the graph's behavior:

- Mirror Image: For any point \((x, y) \) on the parabola, there exists a corresponding point at \((2h -

x, y) $.
- Vertex as the Peak: Since $a = -1$, the parabola opens downward, making the vertex the maximum point at $(-4, 17)$.
- Symmetric X-Values: Points equidistant from $x = -4$ have the same y-value.

Calculating Key Features of the Parabola

With the vertex and axis of symmetry established, we can further analyze the parabola's features.

Y-Intercept

Set $x = 0$:

$$
y = - (0)^2 - 8(0) + 1 = 1
$$

Y-intercept is at $(0, 1)$.

X-Intercepts (Roots)

Set $y = 0$:

$$
0 = -x^2 - 8x + 1
$$

Multiply both sides by -1:

$$
0 = x^2 + 8x - 1
$$

Use quadratic formula:

$$
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
$$

where $a=1$, $b=8$, $c=-1$:

$$
x = \frac{-8 \pm \sqrt{8^2 - 4(1)(-1)}}{2}

\]

Calculate discriminant:

$$
64 - (-4) = 64 + 4 = 68
$$

Square root:

$$
\sqrt{68} = \sqrt{4 \times 17} = 2 \sqrt{17}
$$

Calculate roots:

$$
x = \frac{-8 \pm 2 \sqrt{17}}{2} = -4 \pm \sqrt{17}
$$

Thus, the roots are approximately:

$$
x \approx -4 + 4.1231 \approx 0.1231
$$

$$
x \approx -4 - 4.1231 \approx -8.1231
$$

X-intercepts: $\left(-8.1231, 0 \right)$ and $\left(0.1231, 0 \right)$

Graphical Interpretation and Visual Features

By analyzing the features above, the graph of $y = -x^2 - 8x + 1$ can be visualized as follows:

- Opens downward, with the maximum point at $(-4, 17)$.
- Symmetric about the line $x = -4$.
- Crosses the y-axis at $(0, 1)$.
- Crosses the x-axis approximately at $(-8.1231, 0)$ and $(0.1231, 0)$.

Real-World Applications and Insights

Quadratic functions such as \(y = -x^2 - 8x + 1 \) are prevalent in various fields:

- Physics: Describing projectile motion where the maximum height occurs at the vertex.
- Economics: Modeling profit or cost functions with diminishing returns.
- Engineering: Designing parabolic reflectors or antennas.
- Optimization Problems: Finding maximum or minimum values within constraints.

Understanding the axis of symmetry, vertex, and intercepts enables professionals and students to interpret data, optimize solutions, and make predictions effectively.

Conclusion: Unlocking the Power of Symmetry

The quadratic function \(y = -x^2 - 8x + 1 \) exemplifies the beauty and utility of quadratic analysis. Its axis of symmetry at \(x = -4 \) is not just a mathematical construct but a pivotal feature that simplifies understanding the parabola's shape, identifying key points, and predicting behavior

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