The Quadrilateral DEFG,show. On The Coordinate Plane Below Is Dilated From The Origin By A Space Factor

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Understanding how geometric figures transform on the coordinate plane is fundamental in both mathematics education and practical problem-solving. One particularly interesting transformation is dilation, which involves resizing a figure relative to a specific point—most commonly the origin—by a certain scale factor. In this article, we will explore the transformation of a quadrilateral, specifically quadrilateral DEFG, as it undergoes dilation from the origin by a given space factor. We will dissect the concept of dilation, analyze the effects on the quadrilateral's coordinates, and discuss the implications of different scale factors on its size and position.

What Is Dilation in Coordinate Geometry?

Dilation, also known as resizing or scaling, is a transformation that produces a similar figure to the original but with a different size. It preserves the shape of the figure while changing its size, either enlarging or reducing it. The key components of dilation include:

- Center of Dilation: The fixed point about which the figure expands or contracts. Often, the origin (0,0) is used for simplicity.
- Scale Factor (Space Factor): A number that determines how much the figure is enlarged or reduced. A scale factor greater than 1 enlarges the figure, while a scale factor between 0 and 1 shrinks it.

Mathematically, when a figure is dilated from the origin with a scale factor k, each point (x, y) on the figure transforms to a new point (x', y'), calculated as:

$$
x' = k \times x
$$
$$
y' = k \times y
$$

This straightforward rule applies to all points of the figure, making dilation a linear transformation.

Coordinates of Quadrilateral DEFG Before Dilation

Suppose the vertices of quadrilateral DEFG are given by the coordinates:

- D: (x_1, y_1)
- E: (x_2, y_2)
- F: (x_3, y_3)
- G: (x_4, y_4)

For example, assume the quadrilateral's vertices are:

- D (2, 3)
- E (5, 7)
- F (8, 4)

- G (3, 1)

Plotting these points on the coordinate plane, you can visualize the original shape as a quadrilateral with these vertices.

The Effect of Dilation on Quadrilateral DEFG

When applying dilation from the origin with a space factor k, each vertex of quadrilateral DEFG undergoes a proportional transformation. Specifically:

$$
\text{New point } (x', y') = (k \times x, k \times y)
$$

For the given vertices, the dilated points will be:

- D′: $(2k, 3k)$
- E′: $(5k, 7k)$
- F′: $(8k, 4k)$
- G′: $(3k, 1k)$

Impact of the Scale Factor

- When $k > 1$: The quadrilateral enlarges, maintaining its shape but increasing in size. The vertices move farther from the origin, stretching the figure outward.
- When $0 < k < 1$: The quadrilateral shrinks, with vertices moving closer to the origin.
- When $k = 1$: The figure remains unchanged.
- When $k < 0$: The figure is reflected across the origin while scaling, resulting in a size change and a rotation of the figure across the coordinate axes.

Graphical Representation

Visualizing the dilation helps in understanding the transformation:

- The original quadrilateral is plotted with its original vertices.
- The dilated quadrilateral is plotted based on the scaled vertices.
- Comparing both shapes illustrates how the figure expands or contracts relative to the origin.

Mathematical Examples of Dilation

Let's consider specific examples to clarify the effect of different space factors:

Example 1: Dilation with $k = 2$

Vertices after dilation:

- D′: $(2 \times 2, 3 \times 2) = (4, 6)$
- E′: $(5 \times 2, 7 \times 2) = (10, 14)$
- F′: $(8 \times 2, 4 \times 2) = (16, 8)$
- G′: $(3 \times 2, 1 \times 2) = (6, 2)$

The quadrilateral doubles in size, with each vertex located twice as far from the origin compared to the original.

Example 2: Dilation with $k = 0.5$

Vertices after dilation:

- D′: $(2 \times 0.5, 3 \times 0.5) = (1, 1.5)$
- E′: $(5 \times 0.5, 7 \times 0.5) = (2.5, 3.5)$
- F′: $(8 \times 0.5, 4 \times 0.5) = (4, 2)$
- G′: $(3 \times 0.5, 1 \times 0.5) = (1.5, 0.5)$

This results in a smaller, similar quadrilateral closer to the origin.

Properties of Dilation and Similarity

Dilation preserves the shape of the figure, meaning the dilated quadrilateral DE′F′G′ is similar to the original quadrilateral DEFG. The key properties include:

- Corresponding angles are equal: The internal angles of the original and dilated quadrilaterals are the same.
- Corresponding sides are proportional: The ratios of corresponding side lengths equal the scale factor k.

This property is crucial in similarity proofs and geometric problem-solving, especially when establishing relationships between figures after transformation.

Applications of Dilation in Real Life and Mathematics

Dilation has numerous practical applications beyond pure mathematics:

- Engineering and Design: Scaling blueprints or models to different sizes.
- Computer Graphics: Resizing images and objects while maintaining proportions.
- Physics: Understanding expansion or contraction phenomena.
- Geometry Education: Visualizing similarity, transformations, and scale models.

In educational settings, exercises involving dilation help students understand the concepts of similarity, scale factors, and coordinate transformations.

Conclusion: Mastering Dilation and Its Effects on Quadrilaterals

Dilation is a powerful and intuitive transformation in coordinate geometry that highlights the proportionality and similarity of figures. By understanding how the quadrilateral DEFG transforms when dilated from the origin by a space factor, learners can deepen their comprehension of geometric transformations. Remember that each point's coordinate multiplies by the scale factor, and the resulting figure remains similar to the original, just scaled accordingly. Whether enlarging or reducing the size, dilation provides a fundamental tool for exploring geometric relationships and modeling real-world scenarios.

By practicing dilation with various figures and scale factors, students can develop a stronger intuition about geometric transformations and their applications in mathematics and beyond.

Frequently Asked Questions

What does it mean for quadrilateral DEFG to be dilated from the origin by a scale factor?

It means that all the vertices of quadrilateral DEFG are scaled relative to the origin, so each coordinate is multiplied by the scale factor, enlarging or reducing the shape proportionally.

How do you find the new coordinates of quadrilateral DEFG after dilation from the origin?

Multiply each original coordinate (x, y) of the vertices by the scale factor to get the new coordinates (kx, ky), where k is the scale factor.

What is the significance of the origin in the dilation process for quadrilateral DEFG?

The origin serves as the fixed point from which the shape is dilated, meaning all points move directly away from or toward the origin based on the scale factor.

If the scale factor is greater than 1, what happens to quadrilateral DEFG during dilation?

The quadrilateral enlarges, becoming larger while maintaining its shape and proportions.

What happens if the scale factor is between 0 and 1 when dilating quadrilateral DEFG?

The quadrilateral reduces in size, becoming smaller but retaining its shape and proportions.

How can you verify that the dilation preserves the shape of quadrilateral DEFG?

Check that the ratios of corresponding sides remain consistent and that angles are unchanged, confirming the shape is similar to the original.

Why is understanding dilation from the origin important in coordinate geometry?

It helps in understanding similarity transformations, how shapes change size while maintaining proportions, and is fundamental in geometric scaling and modeling.

Additional Resources

The Quadrilateral DEFG, show. On The Coordinate Plane Below Is Dilated From The Origin By A Scale Factor is a fundamental concept in coordinate geometry that demonstrates how figures can be transformed through dilation. This process not only helps in understanding the properties of geometric shapes but also provides insights into how shapes change size while maintaining their fundamental structure. In this guide, we will explore the concept of dilation in the context of quadrilaterals, specifically focusing on quadrilateral DEFG, and examine how dilating a figure from the origin by a scale factor impacts its position and size on the coordinate plane.

Understanding Dilation on the Coordinate Plane

Dilation is a transformation that produces an image that is the same shape as the original but is scaled by a certain factor. When dilating a figure from the origin (0,0), each point of the original figure moves along a line radiating from the origin, either closer to or farther from the origin depending on the scale factor.

Key Concepts:
- Center of Dilation: The point from which the figure is dilated. In this case, it is the origin (0,0).
- Scale Factor (k): A number that determines how much the figure is enlarged or reduced.
- If $k > 1$, the image is an enlargement.
- If $0 < k < 1$, the image is a reduction.
- Coordinates Transformation: For each point (x, y), the dilated point (x', y') is given by:
- $x' = k\, x$
- $y' = k\, y$

Step-by-Step Breakdown of Dilation of Quadrilateral DEFG

Step 1: Identify Coordinates of Quadrilateral DEFG

Suppose the quadrilateral DEFG is plotted on the coordinate plane with the following vertices:
- $D(x_1, y_1)$
- $E(x_2, y_2)$
- $F(x_3, y_3)$
- $G(x_4, y_4)$

Note: For illustration, let's assume specific coordinates:
- D(2, 3)
- E(4, 5)
- F(6, 2)
- G(3, 1)

Step 2: Determine the Scale Factor

Let's assume the figure is dilated from the origin by a scale factor of $k = 2$. This means each coordinate of the quadrilateral will be multiplied by 2.

Step 3: Calculate the Coordinates of the Dilated Quadrilateral

Applying the dilation formulas:

- $D'(x', y') = (k\, x, k\, y)$
- $E'(x', y') = (k\, x, k\, y)$
- $F'(x', y') = (k\, x, k\, y)$
- $G'(x', y') = (k\, x, k\, y)$

Calculations:

- D'(2, 3) ⍰ (2 2, 2 3) = (4, 6)
- E'(4, 5) ⍰ (2 4, 2 5) = (8, 10)
- F'(6, 2) ⍰ (2 6, 2 2) = (12, 4)
- G'(3, 1) ⍰ (2 3, 2 1) = (6, 2)

Step 4: Plot the Original and Dilated Quadrilaterals

- Original Quadrilateral DEFG:
- D(2, 3)
- E(4, 5)

- F(6, 2)
- G(3, 1)

- Dilated Quadrilateral D'E'F'G':
- D'(4, 6)
- E'(8, 10)
- F'(12, 4)
- G'(6, 2)

By plotting these points on the coordinate plane, you will observe that the dilated quadrilateral maintains the same shape and proportional angles, but its size is scaled by a factor of 2, and it is located farther from the origin.

Analyzing the Effects of Dilation

1. Size and Area Changes

- The lengths of sides are multiplied by the scale factor.
- The area of the quadrilateral is multiplied by the square of the scale factor:
- Area' = k^2 Area

This means that if the original quadrilateral had an area A, after dilation:
- The new area will be k^2 A.

2. Shape Preservation

- Dilation is an image similarity transformation.
- It preserves the shape and angles of the quadrilateral.
- The proportionality of the sides remains intact.

3. Position and Orientation

- Since the dilation is centered at the origin, the position of the figure changes based on the scale factor.
- The orientation of the quadrilateral remains unchanged.

Visualizing Dilations: Practical Tips

- Use graph paper or digital graphing tools to accurately plot points.
- Label all original points clearly, then plot the dilated points.
- Connect the vertices in order to visualize the shape before and after dilation.
- Observe how the shape scales outward from the origin.

Applications of Dilation in Geometry

Understanding dilation has broad applications, including:
- Scaling diagrams in engineering and architecture.
- Modeling real-world objects at different sizes.
- Understanding transformations in computer graphics.
- Solving similarity problems and proving geometric theorems.

Common Mistakes to Avoid

- Incorrect scale factor application: Remember to multiply both x and y coordinates by the scale factor.
- Center of dilation assumptions: If dilation is not centered at the origin, the calculation becomes more complex, involving additional translation steps.
- Ignoring the impact on area: While side lengths scale linearly, areas scale quadratically, which is crucial for accuracy in calculations.

Practice Problems

1. Suppose quadrilateral DEFG has vertices D(1, 2), E(3, 4), F(5, 3), G(2, 1). If it is dilated from the origin by a scale factor of 3, what are the coordinates of the dilated quadrilateral?

2. If the original quadrilateral has an area of 6 square units, what will be the area after dilation with scale factor 4?

3. How does the dilation change if the scale factor is less than 1? Illustrate with an example.

Conclusion

The Quadrilateral DEFG, show. On The Coordinate Plane Below Is Dilated From The Origin By A Space Factor exemplifies a fundamental transformation in coordinate geometry—dilation. By understanding how each point transforms with a given scale factor, students and practitioners can manipulate figures accurately and predict their new positions and sizes. Recognizing the properties of dilation, including the preservation of shape and angles, as well as the proportional increase in lengths and area, is essential for mastering geometric transformations and applying them in various mathematical and real-world contexts. Whether you're analyzing geometric figures, creating scaled models, or exploring the principles of similarity, grasping the concept of dilation from the origin forms a cornerstone of spatial reasoning and geometric understanding.

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