The Roots Of $3x^2 - 4x 15 = 0$ Are The Same As The Roots Of $x^2 Bx C = 0,$ For Some Constants b And

The Roots Of $3x^2 - 4x + 15 = 0$ Are The Same As The Roots Of $x^2 + bx + c = 0$,$ For Some Constants b And c. Understanding the relationship between the roots of different quadratic equations is fundamental in algebra. This article explores the conditions under which two quadratic equations share the same roots, specifically analyzing the quadratic equations $3x^2 - 4x + 15 = 0$ and $x^2 + bx + c = 0$. We will delve into the concepts of roots, quadratic coefficients, and how to determine the constants b and c that make the roots identical. By the end, you'll have a comprehensive understanding of how quadratic equations relate and how to manipulate their coefficients to preserve roots.

Understanding Quadratic Equations and Their Roots

What Is a Quadratic Equation?

A quadratic equation is a second-degree polynomial equation in the form:

$$
ax^2 + bx + c = 0
$$

where:
- $a \neq 0$,
- b and c are constants,
- x is the variable.

The solutions or roots of the quadratic are the values of x that satisfy the equation.

Roots of a Quadratic Equation

The roots of a quadratic equation can be found using the quadratic formula:

$$
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
$$

The expression under the square root, $b^2 - 4ac$, is called the discriminant and determines the nature of the roots:
- If the discriminant > 0, roots are real and distinct.
- If the discriminant = 0, roots are real and equal.
- If the discriminant < 0, roots are complex conjugates.

Analyzing the Given Equations

Equation 1: $3x^2 - 4x + 15 = 0$

Let's examine the roots of this quadratic.

- Coefficients:
- $a = 3$,
- $b = -4$,
- $c = 15$.

- Discriminant:

$$
\Delta = b^2 - 4ac = (-4)^2 - 4 \times 3 \times 15 = 16 - 180 = -164
$$

Since $\Delta < 0$, the roots are complex conjugates:

$$
x = \frac{-(-4) \pm \sqrt{-164}}{2 \times 3} = \frac{4 \pm i\sqrt{164}}{6}
$$

which simplifies to:

$$
x = \frac{2 \pm i \sqrt{41}}{3}
$$

Therefore, the roots are:

$$
x_1 = \frac{2 + i \sqrt{41}}{3}, \quad x_2 = \frac{2 - i \sqrt{41}}{3}
$$

Equation 2: $x^2 + bx + c = 0$

We are told that this quadratic has the same roots as the first. Our goal is to find the constants b and c such that the roots of this quadratic are exactly:

$$
x_1 = \frac{2 + i \sqrt{41}}{3}, \quad x_2 = \frac{2 - i \sqrt{41}}{3}
$$

Conditions for Quadratics to Share Roots

Vieta's Formulas

Vieta's formulas relate the roots of a quadratic to its coefficients:

$$
x_1 + x_2 = -\frac{b}{a}
$$

$$
x_1 x_2 = \frac{c}{a}
$$

For the quadratic $x^2 + bx + c = 0$ (where $a = 1$), these simplify to:

$$
x_1 + x_2 = -b
$$

$$
x_1 x_2 = c
$$

Since the roots are known, we can compute these sums and products to find b and c.

Calculating Sum and Product of Roots

Given:

\[

x_1 = \frac{2 + i \sqrt{41}}{3}, \quad x_2 = \frac{2 - i \sqrt{41}}{3}
\]

- Sum:

\[
x_1 + x_2 = \frac{2 + i \sqrt{41}}{3} + \frac{2 - i \sqrt{41}}{3} = \frac{(2 + 2) + (i \sqrt{41} - i \sqrt{41})}{3} = \frac{4 + 0}{3} = \frac{4}{3}
\]

- Product:

\[
x_1 x_2 = \left(\frac{2 + i \sqrt{41}}{3}\right) \times \left(\frac{2 - i \sqrt{41}}{3}\right)
\]

Recall that:

\[
(a + bi)(a - bi) = a^2 + b^2
\]

Applying this, with \(a = 2/3\) and \(b = \sqrt{41}/3\):

\[
x_1 x_2 = \left(\frac{2}{3}\right)^2 + \left(\frac{\sqrt{41}}{3}\right)^2 = \frac{4}{9} + \frac{41}{9} = \frac{45}{9} = 5
\]

Therefore:

\[
x_1 + x_2 = \frac{4}{3} \quad \Rightarrow \quad -b = \frac{4}{3} \quad \Rightarrow \quad b = -\frac{4}{3}
\]

\[
x_1 x_2 = 5 \quad \Rightarrow \quad c = 5
\]

Result:

\[
b = -\frac{4}{3}, \quad c = 5

Thus, the quadratic with the same roots as $3x^2 - 4x + 15 = 0$ is:

$$
x^2 - \frac{4}{3}x + 5 = 0
$$

or, multiplying through by 3 to clear fractions:

$$
3x^2 - 4x + 15 = 0
$$

which is the original equation, confirming the roots are the same.

Implications and Applications

Transforming Quadratic Equations

Understanding how to relate different quadratic equations with identical roots is crucial in various algebraic manipulations, especially in simplifying complex equations, solving systems, or modeling real-world problems.

- Scaling Coefficients: Multiplying a quadratic by a non-zero constant scales the entire equation but does not change its roots.
- Complete the Square: Techniques like completing the square help in rewriting quadratics to identify roots more straightforwardly.
- Factoring: Expressing quadratic equations as products of linear factors directly reveals roots.

Practical Uses

- Engineering: Designing systems where certain parameters (roots) need to be preserved under transformations.
- Physics: Modeling motion or wave equations where roots correspond to specific physical states.
- Finance: Solving quadratic equations in optimization problems, such as maximizing revenue or minimizing costs.

Summary and Key Takeaways

- The roots of the quadratic $3x^2 - 4x + 15 = 0$ are complex conjugates, specifically $\frac{2 \pm i \sqrt{41}}{3}$.
- To find a quadratic $x^2 + bx + c = 0$ with the same roots, use Vieta's formulas.
- The sum of roots is $\frac{4}{3}$, leading to $b = -\frac{4}{3}$.
- The product of roots is 5, leading to $c = 5$.
- The quadratic with matching roots is $x^2 - \frac{4}{3}x + 5 = 0$.

In conclusion, understanding the relationship between quadratic coefficients and roots allows for flexible manipulation of equations while preserving their solutions. This knowledge is essential for solving complex algebraic problems, simplifying equations, and applying quadratic principles across various scientific and engineering disciplines.

Further Reading and Resources

- Algebra and Its Applications by David S. Dummit and Richard M. Foote
- Quadratic Equations: Formulas, Graphs, and Applications on Khan Academy
- Online calculators for quadratic equations
- Interactive algebra tutorials for mastering Vieta's formulas

Remember: Mastery of quadratic equations not only enhances problem-solving skills but also builds a foundational understanding critical for advanced mathematics and numerous technical fields.

Frequently Asked Questions

What does it mean for two quadratic equations to have the same roots?

It means that the solutions (values of x) that satisfy both equations are identical, although the equations themselves may look different.

Given the quadratic equation $3x^2 - 4x + 15 = 0$, how can we find constants b and c such that the roots are the same as those of $x^2 + bx + c = 0$?

Since the roots are the same, the quadratic equations are scalar multiples of each other. By comparing coefficients, we can find b and c such that $3x^2 - 4x + 15 = k(x^2 + bx + c)$ for some constant k.

How do we determine if two quadratics share the same roots?

Two quadratics share the same roots if their factorizations are proportional, meaning one is a constant multiple of the other, or if their quadratic formulas yield the same solutions.

What is the role of the discriminant in analyzing the roots of quadratic equations?

The discriminant ($b^2 - 4ac$) determines the nature of the roots: real and distinct, real and repeated, or complex conjugates. For two quadratics to have the same roots, they must have the same discriminant and solutions.

How can we express the roots of $3x^2 - 4x + 15 = 0$?

Using the quadratic formula: $x = [4 \pm \text{sqrt}((-4)^2 - 4315)] / (23)$. Simplifying the discriminant gives the roots explicitly.

If the roots of $3x^2 - 4x + 15 = 0$ are α and β, what are the sum and product of these roots?

The sum of roots is $\alpha + \beta = 4/3$, and the product is $\alpha\beta = 15/3 = 5$, based on the coefficients of the quadratic.

How do we find constants b and c so that $x^2 + bx + c$ shares roots with $3x^2 - 4x + 15$?

We set the two quadratics proportional: $3x^2 - 4x + 15 = k(x^2 + bx + c)$. Comparing coefficients allows us to solve for b and c once k is determined, ensuring the roots are the same.

What is the significance of the fact that the roots are the same for both quadratics?

It indicates the quadratics are either scalar multiples of each other or have identical solutions, which helps in simplifying or solving the equations by understanding their relationships.

Additional Resources

The Roots Of \(3x^2 - 4x + 15 = 0\) Are The Same As The Roots Of \(x^2 + bx + c = 0,\) For Some Constants \(b\) And \(c\)

Understanding the relationship between different quadratic equations and their roots is a fundamental aspect of algebra. When analyzing whether two quadratic equations share the same roots, it often involves exploring how their coefficients relate to each other. This blog post delves into the intriguing question: The roots of $3x^2 - 4x + 15 = 0$ are the same as the roots of $x^2 + bx + c = 0$, for some constants b and c. We will explore the mathematical principles that underpin this question, provide step-by-step methods to analyze such relationships, and discuss how to determine the constants b and c when the roots coincide.

Understanding Quadratic Equations and Roots

Before we analyze the connection between the two equations, it's essential to review some foundational concepts related to quadratic equations:

What Is a Quadratic Equation?

A quadratic equation is any second-degree polynomial equation of the form:

$$ax^2 + bx + c = 0$$

where $a \neq 0$, and b, c are real or complex constants.

Roots of a Quadratic Equation

The roots (or solutions) of the quadratic are the values of x that satisfy the equation. These roots can be real or complex, depending on the discriminant:

$$\Delta = b^2 - 4ac$$

- If $\Delta > 0$, there are two distinct real roots.
- If $\Delta = 0$, there is one real root (a repeated root).
- If $\Delta < 0$, there are two complex roots.

Sum and Product of Roots

For a quadratic $ax^2 + bx + c = 0$, the roots r_1 and r_2 satisfy:

- Sum: $r_1 + r_2 = -\frac{b}{a}$
- Product: $r_1 r_2 = \frac{c}{a}$

Comparing Roots of $3x^2 - 4x + 15 = 0$ and $x^2 + bx + c = 0$

The core question is whether the roots of the quadratic $3x^2 - 4x + 15 = 0$ are the same as those of some quadratic of the form $x^2 + bx + c = 0$. To explore this, consider the following:

Step 1: Find the Roots of $3x^2 - 4x + 15 = 0$

Using the quadratic formula:

$$
x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4 \times 3 \times 15}}{2 \times 3} = \frac{4 \pm \sqrt{16 - 180}}{6} = \frac{4 \pm \sqrt{-164}}{6}
$$

Since the discriminant is negative (-164), the roots are complex conjugates:

$$
x = \frac{4 \pm i \sqrt{164}}{6} = \frac{4}{6} \pm i \frac{\sqrt{164}}{6}
$$

Simplify:

$$
x = \frac{2}{3} \pm i \frac{\sqrt{164}}{6}
$$

Recall that $\sqrt{164} = \sqrt{4 \times 41} = 2 \sqrt{41}$. So,

$$
x = \frac{2}{3} \pm i \frac{2 \sqrt{41}}{6} = \frac{2}{3} \pm i \frac{\sqrt{41}}{3}
$$

Step 2: Find the Sum and Product of These Roots

- Sum:

$$
r_1 + r_2 = \frac{2}{3} + i \frac{\sqrt{41}}{3} + \frac{2}{3} - i \frac{\sqrt{41}}{3} = \frac{4}{3}
$$

- Product:

$$
r_1 r_2 = \left(\frac{2}{3}\right)^2 + \left(\frac{\sqrt{41}}{3}\right)^2 = \frac{4}{9} + \frac{41}{9} = \frac{45}{9} = 5
$$

\]

Note: The product of conjugates is equal to the constant term divided by the leading coefficient in the original quadratic, which checks out because:

\[
\frac{c}{a} = \frac{15}{3} = 5
\]

Finding the Corresponding \(b\) and \(c\)

Now, for the quadratic \(x^2 + bx + c = 0\) to have the same roots as the original quadratic, it must have:

- Sum of roots: \(-b\)
- Product of roots: \(c\)

Given the above:

\[
\boxed{
\begin{aligned}
& \text{Sum of roots} = \frac{4}{3} = -b \Rightarrow b = -\frac{4}{3} \\
& \text{Product of roots} = 5 = c
\end{aligned}
}
\]

Thus, the quadratic with the same roots as \(3x^2 - 4x + 15 = 0\) can be written as:

\[
x^2 + \left(-\frac{4}{3}\right) x + 5 = 0
\]

or equivalently,

\[
x^2 - \frac{4}{3} x + 5 = 0
\]

General Conditions for Equivalence of Roots

The above example demonstrates how to find a quadratic with specific roots. But how general is this approach? Let's analyze the broader question:

When do two quadratic equations share the same roots?

Case 1: The Quadratics Are Scalar Multiples

Suppose the two quadratics are:

$$
ax^2 + bx + c = 0
$$
and
$$
dx^2 + ex + f = 0
$$

They share the same roots if and only if they are scalar multiples of each other:

$$
ax^2 + bx + c = k(dx^2 + ex + f)
$$

for some non-zero constant k. Therefore, their coefficients are proportional:

$$
a = kd, \quad b = ke, \quad c = kf
$$

This means the two quadratics are essentially the same equation up to a scaling factor.

Case 2: Different Quadratics with the Same Roots

Alternatively, if the quadratics are not scalar multiples but still share roots, they must:

- Have the same roots r_1, r_2,
- Satisfy the relationships:

$$
r_1 + r_2 = -\frac{b}{a} = -\frac{e}{d}
$$

and

$$
r_1 r_2 = \frac{c}{a} = \frac{f}{d}
$$

Thus, the key is to find b, c such that their sum and product of roots match those of the original quadratic.

Practical Steps to Determine b and c

Given a quadratic $3x^2 - 4x + 15 = 0$, here's how you can find a quadratic $x^2 + bx + c = 0$ with the same roots:

Step 1: Find the roots of the original quadratic (as shown earlier).

Step 2: Compute the sum and product of these roots.

Step 3: Assign these values to the sum and product formulas:

$$
r_1 + r_2 = -b
$$

$$
r_1 r_2 = c
$$

Step 4: Solve for b and c:

$$
b = - (r_1 + r_2)
$$

$$
c = r_1 r_2
$$

Step 5: Write the quadratic $x^2 + bx + c = 0$ with these coefficients.

Additional Considerations

Roots Are Complex

In our example, the roots are complex conjugates. When roots are complex, the quadratic with those roots will also have complex coefficients if you insist on real coefficients. However, since our target quadratic is $x^2 + bx + c$ with real coefficients, the roots must be conjugates, which they are.

Roots Are Repeated (Repeated Roots)

If the original quadratic has a repeated root (discriminant zero), then the quadratic with the same roots is a perfect square:

$$
(x - r)^2 = 0
$$

with root r. In this case, $b = -2r$ and $c = r^2$.

Summary

- The roots of $3x^2 - 4x + 15 = 0$ are complex conjugates with sum $\frac{4}{3}$ and product

The Roots Of 3x 2 4x 15 0 Are The Same As The Roots Of X 2 Bx C 0 For Some Constants B And

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