# The Surface Area Of A Cylinder Is 66 Cm. If Its Radius Is Increasing At The Rate Of 0.4 Cms-1, Find The

**The Surface Area Of A Cylinder Is 66 Cm. If Its Radius Is Increasing At The Rate Of 0.4 Cms-1, Find The** is a classic problem in calculus and geometry that involves understanding the relationship between the surface area of a cylinder, its radius, and height, especially when the radius changes over time. This problem not only tests your knowledge of surface area formulas but also your ability to apply related rates in calculus to find unknown rates or quantities. In this article, we will explore how to approach this problem step-by-step, understand the underlying concepts, and provide comprehensive solutions to help you master related problems involving cylinders.

## Understanding the Surface Area of a Cylinder

Before diving into the problem, it’s essential to understand the fundamental formula for the surface area of a cylinder and how it relates to its dimensions.

### Surface Area Formula of a Cylinder

The total surface area (SA) of a cylinder includes the areas of its two circular bases and the curved surface (lateral surface). The formula is:

- $SA = 2\pi r^2 + 2\pi rh$

where:
- r = radius of the cylinder
- h = height of the cylinder
- $\pi \approx 3.1416$

The first term, $2\pi r^2$, accounts for the area of the two circular bases, and the second term, $2\pi rh$, accounts for the lateral surface area.

## Given Data and What To Find

In the problem, we are told:
- The surface area (SA) of the cylinder is 66 cm.
- The radius (r) is increasing at a rate of 0.4 cm/sec, i.e., dr/dt = 0.4 cm/sec.
- The goal is to find the rate of change of the height (dh/dt), or other related quantities

depending on the specific question.

Understanding these parameters helps us set up the related rates problem properly.

# Setting Up the Problem

To solve the problem, we need to relate the known quantities and the rates of change. The key is to differentiate the surface area formula with respect to time (t), which involves the chain rule.

## Step 1: Express the Surface Area Equation

Given:
$SA = 2\pi r^2 + 2\pi rh = 66$ cm

Since the surface area is given as a constant (66 cm), the derivative of SA with respect to time will be zero:

$d(SA)/dt = 0$

## Step 2: Differentiate the Surface Area Equation

Differentiating both sides with respect to t:

$d/dt\ [2\pi r^2 + 2\pi rh] = 0$

Applying differentiation term by term:

1. For $2\pi r^2$: $d/dt\ [2\pi r^2] = 4\pi r\ (dr/dt)$
2. For $2\pi rh$: $d/dt\ [2\pi rh] = 2\pi\ (r\ dh/dt + h\ dr/dt)$ (product rule)

Putting it together:

$4\pi r\ (dr/dt) + 2\pi\ [r\ dh/dt + h\ dr/dt] = 0$

Simplify:

$4\pi r\ (dr/dt) + 2\pi r\ dh/dt + 2\pi h\ dr/dt = 0$

Collect like terms:

$(4\pi r \, dr/dt + 2\pi h \, dr/dt) + 2\pi r \, dh/dt = 0$

Factor where possible:

$2\pi \, dr/dt \, (2r + h) + 2\pi r \, dh/dt = 0$

This is the differential equation we will use to find the unknown rate, typically dh/dt, given the other quantities.

# Determining the Missing Variables and Values

To proceed, sometimes additional data such as the current radius (r), height (h), or the specific time at which the rates are measured are given or can be inferred.

## Common Assumptions or Additional Data

- If the problem states at a particular instant, the radius r is known, then it can be substituted into the differentiated equation.
- If the height h is not changing (i.e., the cylinder's height is constant), then dh/dt = 0, simplifying the calculations.
- If the height h is not specified, the problem might be asking for the rate of change of the height or radius at a specific moment.

In our case, since the problem states the surface area is 66 cm and the radius is increasing at 0.4 cm/sec, let’s proceed with an example assumption: suppose that at the instant of interest, the radius r is a specific value, say r = 3 cm, and the height h is such that the surface area condition holds.

# Calculating the Height of the Cylinder

Given the surface area formula:

$SA = 2\pi r^2 + 2\pi rh = 66$

Plugging in r = 3 cm:

$2\pi(3)^2 + 2\pi(3)h = 66$

Calculate:

$2\pi(9) + 6\pi h = 66$

$18\pi + 6\pi h = 66$

Divide through by π:

$18 + 6h = 66 / \pi \approx 66 / 3.1416 \approx 21.0$

Now, solve for h:

$6h = 21 - 18 = 3$

$h = 3 / 6 = 0.5$ cm

Thus, at r = 3 cm, h ≈ 0.5 cm.

# Finding the Rate of Change of the Height (dh/dt)

Recall the differentiated surface area equation:

$2\pi\ dr/dt\ (2r + h) + 2\pi\ r\ dh/dt = 0$

Plugging in the known values:

- r = 3 cm
- h = 0.5 cm
- dr/dt = 0.4 cm/sec

Calculate:

$2\pi\ 0.4\ (23 + 0.5) + 2\pi\ 3\ dh/dt = 0$

Simplify:

$2\pi\ 0.4\ (6 + 0.5) + 6\pi\ dh/dt = 0$

Calculate inside parentheses:

$6 + 0.5 = 6.5$

Calculate the first term:

$2\pi\ 0.4\ 6.5 = 2\ 3.1416\ 0.4\ 6.5$

First, $2\ 3.1416 = 6.2832$

Then, $6.2832\ 0.4 = 2.51328$

Finally, $2.51328\ 6.5 \approx 16.3363$

So, the first term ≈ 16.3363

The equation becomes:

$16.3363 + 6\pi\ dh/dt = 0$

Now, solve for dh/dt:

$6\pi \, dh/dt = -16.3363$

$dh/dt = -16.3363 / (6\pi)$

Calculate denominator:

$6 \; 3.1416 \approx 18.8496$

Thus:

$dh/dt \approx -16.3363 / 18.8496 \approx -0.866$ cm/sec

Interpretation: The negative sign indicates that the height is decreasing at approximately 0.866 cm/sec at this instant.

## Summary of Key Steps to Solve Related Rates Problems for Cylinders

To effectively solve similar problems involving the surface area of a cylinder and changing dimensions, follow these steps:

- Write down the known quantities and what needs to be found.
- Recall and write the formula relating the surface area, radius, and height.
- Differentiate the formula with respect to time to relate the rates of change.
- Substitute known values into the differentiated equation at the instant you're analyzing.
- Solve for the unknown rate, such as dh/dt or dr/dt.

## Additional Tips for Solving Cylinder Surface Area Problems

- Always verify units and convert if necessary.
- When given a constant surface area, differentiate and set the derivative to zero.
- Use algebraic substitution to find missing dimensions if needed.
- Understand the physical implications of your results, such as whether the height is increasing or decreasing.

# Conclusion: Mastering Related Rates in Cylinder Geometry

Problems involving the surface area of cylinders and rates of change are fundamental in calculus and real-world applications such as engineering, manufacturing, and physics. The key to solving these problems lies in understanding the formulas, differentiating correctly, and substituting known values accurately. By practicing with various scenarios, you'll develop the intuition needed to approach more complex related rates problems confidently. Whether the radius or height is changing, the methodology remains similar—set up the formula, differentiate, and plug in known values to find the unknown rate. Remember, mastering these concepts enhances your problem-solving skills and deepens your understanding of geometry and calculus interactions.

# Frequently Asked Questions

## What is the formula for the surface area of a cylinder?

The surface area of a cylinder is given by the formula: Surface Area = $2\pi r(h + r)$, where r is the radius and h is the height.

## Given the surface area of a cylinder is 66 cm², how do you find its height if the radius is known?

Using the formula $2\pi r(h + r) = 66$, you can solve for h: $h = (66 / (2\pi r)) - r$.

## If the radius of a cylinder is increasing at a rate of 0.4 cm/sec, how is this rate related to the change in surface area?

The rate of change of the surface area depends on both the radius and height. Using differentiation, $dA/dt = 2\pi(h + 2r)\, dr/dt$, you can find how surface area changes over time.

## How do you find the rate of change of the surface area of a cylinder when the radius is increasing at 0.4 cm/sec?

Differentiate the surface area formula with respect to time: $dA/dt = 2\pi(h + r)\, dr/dt + 2\pi r\, dh/dt$. If height is constant or related to radius, substitute accordingly to find dA/dt.

## What additional information is needed to determine the rate at which the surface area of the cylinder is

## changing?

You need to know either the height of the cylinder or how the height varies with respect to the radius (dh/dt) to accurately compute the rate of change of the surface area.

## How can the given data be used to find the current height of the cylinder?

Using the surface area formula $2\pi r(h + r) = 66$ and the known radius, solve for h: $h = (66 / (2\pi r)) - r$, plugging in the current radius value.

# Additional Resources

Understanding the Surface Area of a Cylinder and the Impact of Changing Radius

Mathematics, especially geometry and calculus, often presents us with intriguing problems that combine basic formulas with dynamic rates of change. One such problem involves analyzing how the surface area of a cylinder varies with respect to its dimensions, particularly when the radius is changing over time. This exploration not only deepens our understanding of geometric properties but also demonstrates the practical application of calculus in real-world problems.

In this detailed review, we will dissect the problem: "The Surface Area Of A Cylinder Is 66 Cm. If Its Radius Is Increasing At The Rate Of 0.4 Cms-1, Find The ..." and expand upon the concepts involved, providing clarity on the steps required to solve such problems and the underlying principles that govern them.

---

# Introduction to Cylinder Surface Area

A cylinder is a three-dimensional shape characterized by two parallel circular bases connected by a curved surface. The surface area (SA) of a cylinder encompasses the total area covering both the lateral surface and the two circular bases.

Key components:

- Radius (r): The distance from the center to the edge of the circular base.
- Height (h): The perpendicular distance between the bases.
- Lateral surface area (LSA): The area of the side wrapping around the cylinder.
- Total surface area (SA): Sum of the lateral surface area and the areas of the two bases.

The formulas are:

- Lateral Surface Area (LSA): \( 2\pi r h \)
- Area of one base: \( \pi r^2 \)
- Total Surface Area (SA): \( 2\pi r h + 2\pi r^2 \)

---

# Given Data and Problem Breakdown

Let's carefully analyze the problem:

- The surface area (SA) of the cylinder is 66 cm.
- The radius (r) is increasing at a rate of 0.4 cm/sec (i.e., $\frac{dr}{dt} = 0.4 \, \text{cm/sec}$).
- The problem asks us to find what—the specific quantity or parameter is likely missing here, such as the height (h) or the rate of change of the surface area.

Note: Since the original prompt is incomplete, we will assume the typical scenario where the problem asks to find the rate of change of the surface area or the height given the data. For the sake of comprehensive understanding, we will consider two common interpretations:

1. Find the height (h) of the cylinder when the surface area is 66 cm, given the radius and its rate of increase.
2. Find the rate of change of the surface area when the radius increases at 0.4 cm/sec.

---

# Understanding the Surface Area Formula in Context

The total surface area of a cylinder:

$$
SA = 2\pi r h + 2 \pi r^2
$$

To analyze the problem effectively, we need to understand the relationship between these variables and how they change over time.

---

# Case 1: Finding the Height When Surface Area Is 66 Cm

Suppose we are asked to find the height $h$ of the cylinder when the surface area is 66 cm, with a known radius $r$.

Step 1: Express $h$ in terms of known quantities

Rearranged from the surface area formula:

$$
h = \frac{SA - 2\pi r^2}{2\pi r}
$$

Given:

$$
SA = 66 \, \text{cm}
$$

Assuming the radius is known or given, for example, suppose $r = 3\, \text{cm}$, then:

$$
h = \frac{66 - 2 \pi (3)^2}{2 \pi \times 3}
$$

Calculations:

$$
2 \pi (3)^2 = 2 \pi \times 9 = 18 \pi \approx 56.55
$$

So,

$$
h = \frac{66 - 56.55}{6 \pi} \approx \frac{9.45}{18.85} \approx 0.5\, \text{cm}
$$

This provides the height corresponding to the given surface area and radius.

---

## Case 2: Calculating the Rate of Change of Surface Area

Alternatively, if the problem is to find how fast the surface area is changing when the radius increases at 0.4 cm/sec, we can use related rates — a key concept in calculus.

---

### Step-by-step Approach to Related Rates

Step 1: Identify known quantities

- $\frac{dr}{dt} = 0.4\, \text{cm/sec}$
- $SA = 66\, \text{cm}$ (at the particular instant)

Step 2: Write the surface area formula as a function of $r$ and $h$:

$$
SA = 2\pi r h + 2\pi r^2
$$

Step 3: Determine if $h$ is constant or changing

- If $h$ is constant, the problem simplifies.
- If $h$ is changing, then we need to consider $\frac{dh}{dt}$.

Let's explore both scenarios.

---

## Scenario A: Height is constant

If $h$ is constant, then:

$$
SA = 2 \pi r h + 2 \pi r^2
$$

Differentiating both sides with respect to time $t$:

$$
\frac{d(SA)}{dt} = 2 \pi h \frac{dr}{dt} + 4 \pi r \frac{dr}{dt}
$$

Rearranged:

$$
\frac{d(SA)}{dt} = 2 \pi \left( h + 2 r \right) \frac{dr}{dt}
$$

Since $h$ and $r$ are known or can be calculated, this formula allows us to find the rate at which the surface area is changing.

Example:

Suppose at the moment when $r=3\, \text{cm}$, the surface area matches 66 cm (from previous calculation). Then:

$$

\frac{d(SA)}{dt} = 2 \pi (h + 2 \times 3) \times 0.4
\]

But we need \(h\). From earlier, when \(r=3\, \text{cm}\):

\[
h \approx 0.5\, \text{cm}
\]

So,

\[
\frac{d(SA)}{dt} = 2 \pi (0.5 + 6) \times 0.4 = 2 \pi \times 6.5 \times 0.4
\]

Calculations:

\[
2 \pi \times 6.5 \times 0.4 \approx 2 \times 3.1416 \times 6.5 \times 0.4 \approx 6.2832 \times 6.5 \times 0.4
\]

\[
6.2832 \times 6.5 \approx 40.84
\]

Then,

\[
40.84 \times 0.4 \approx 16.34 \, \text{cm}^2/\text{sec}
\]

This means the surface area is increasing at approximately 16.34 cm$^2$/sec at that moment.

---

## Scenario B: Height is changing

If the height \(h\) is also changing over time, then the problem becomes more complex.

Step 1: Express \(h\) as a function of \(t\).

Step 2: Differentiate the surface area formula with respect to \(t\):

\[
\frac{d(SA)}{dt} = 2 \pi r \frac{dh}{dt} + 2 \pi h \frac{dr}{dt} + 4 \pi r \frac{dr}{dt}
\]

Step 3: Use known values to solve for the unknowns, such as \( \frac{dh}{dt} \).

---

# Deep Dive into Calculus Concepts: Related Rates and Differentiation

The core mathematical principle underpinning this problem is related rates, a technique in calculus used to find the rate at which one quantity changes with respect to another, especially when multiple variables depend on time.

Fundamental ideas:

- When variables are related through an equation, their rates of change are connected via differentiation.
- Implicit differentiation allows us to differentiate equations where variables are intertwined.
- The chain rule is central in expressing derivatives of composite functions.

Example:

Given the surface area formula:

\[
SA(r, h) = 2 \pi r h + 2 \pi r^2
\]

If both \(r\) and \(h\) depend on \(t\), then:

\[
\frac{d(SA)}{dt} = 2 \pi \left( r \frac{dh}{dt} + h \frac{dr}{

## The Surface Area Of A Cylinder Is 66 Cm If Its Radius Is Increasing At The Rate Of 0 4 Cms 1 Find The

The Surface Area Of A Cylinder Is 66 Cm If Its Radius Is Increasing At The Rate Of 0 4 Cms 1 Find The

## Related Articles

- The Skin Is Not Able To Receive Stimuli Because The Cells Of The Epidermis Are Not Living And Therefore
- Tam Company Is Negotiating For The Purchase Of Equipment That Would Cost $100,000, With The Expectation
- Studies Show That Among Chinese College Students (in Both The People's Republic And Taiwan), One Of The