

The Terminal Side Of An Angle in Standard Position Passes Through The Point(3,4). Use The Figure To Find

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Understanding the geometric and trigonometric principles behind angles in standard position is fundamental in mathematics, especially when analyzing points on the coordinate plane. When a terminal side of an angle in standard position passes through a specific point like (3,4), it opens doors to exploring concepts such as reference angles, sine, cosine, tangent, and the radius or hypotenuse associated with the point. This comprehensive guide will delve into how to interpret such a scenario, how to utilize a figure to find key trigonometric values, and how these principles apply across various mathematical contexts.

What Does It Mean for an Angle to Be in Standard Position?

Definition of Standard Position

An angle in standard position is an angle whose vertex is at the origin (0,0) of the coordinate plane, and its initial side coincides with the positive x-axis. The terminal side of the angle is determined by rotating from the initial side, either clockwise or counterclockwise, to reach the position of the terminal side.

Implications of Standard Position

- The angle's measure is typically expressed in degrees or radians.
- The orientation of the terminal side relative to the x-axis determines the angle's sign and value.
- Angles in standard position are used to define the trigonometric functions for any point on the terminal side.

Analyzing The Point (3,4) and Its Significance

The Coordinates as a Point on the Terminal Side

When the terminal side of an angle passes through the point (3,4), it indicates that this point lies somewhere along the ray (or line) that forms the terminal side of the angle. The key is understanding how this point relates to the angle's measure and its associated trigonometric functions.

Calculating the Distance from the Origin

The distance from the origin to the point (3,4) is critical in finding the hypotenuse of the right triangle formed. This distance, often called the radius or hypotenuse (r), is calculated using the Pythagorean theorem:

$$r = \sqrt{x^2 + y^2} = \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5$$

This length, $r = 5$, becomes a fundamental value in determining sine, cosine, and tangent ratios.

Constructing the Right Triangle and Using the Figure to Find Trigonometric Ratios

Visualizing the Triangle

Drawing a right triangle with:

- The origin (0,0) as the vertex
- The point (3,4) lying on the terminal side
- The horizontal leg along the x-axis from (0,0) to (3,0)
- The vertical leg from (3,0) to (3,4)

This triangle helps in visualizing the relationships between the sides and the angles.

Key Components of the Triangle

- Adjacent side (along x-axis): 3 units

- Opposite side (along y-axis): 4 units
- Hypotenuse (r): 5 units

Calculating Trigonometric Functions

Using the sides of the triangle, the primary trigonometric functions for the angle θ (whose terminal side passes through (3,4)) are:

1. **Sine:** $\sin \theta = \text{opposite} / \text{hypotenuse} = 4 / 5 = 0.8$
2. **Cosine:** $\cos \theta = \text{adjacent} / \text{hypotenuse} = 3 / 5 = 0.6$
3. **Tangent:** $\tan \theta = \text{opposite} / \text{adjacent} = 4 / 3 \approx 1.333$

These ratios are fundamental in solving various trigonometric problems involving the point (3,4).

Determining the Angle Measure

Using the Coordinates to Find θ

To find the measure of the angle θ in degrees, given the point (3,4), use the inverse tangent function:

$$\theta = \arctangent (\text{opposite} / \text{adjacent}) = \arctangent (4 / 3) \approx 53.13^\circ$$

Alternatively, in radians:

$$\theta \approx 0.927 \text{ radians}$$

This angle is in the first quadrant because both x and y are positive.

Understanding the Quadrant

Since the point (3,4) is in the first quadrant, the angle θ is between 0° and 90° , simplifying the interpretation of the trigonometric ratios.

Using the Figure to Find Other Related Values

Reference Angle and Its Significance

The reference angle is the acute angle between the terminal side and the x-axis. For the point (3,4), this is $\theta \approx 53.13^\circ$.

Finding the Cotangent, Secant, and Cosecant

Beyond sine, cosine, and tangent, other trigonometric functions can be calculated:

- **Cosecant:** $\csc \theta = 1 / \sin \theta = 5 / 4 = 1.25$
- **Secant:** $\sec \theta = 1 / \cos \theta = 5 / 3 \approx 1.6667$
- **Cotangent:** $\cot \theta = 1 / \tan \theta = 3 / 4 = 0.75$

These functions are useful in various applications, including solving triangles, wave functions, and more.

Applications of Knowing the Terminal Side Passes Through (3,4)

Solving Trigonometric Equations

Knowing these ratios allows for solving equations involving angles and their trigonometric functions, especially in physics and engineering.

Modeling Periodic Phenomena

Since sine and cosine functions model oscillations, understanding angles passing through specific points aids in analyzing wave behavior.

Coordinate Geometry and Graphing

Understanding how points relate to angles in standard position helps in graphing trigonometric functions and analyzing their properties.

Real-World Problem Solving

For example, in navigation, robotics, and physics, calculating angles and distances based on coordinate points is essential.

Summary: Key Takeaways

- An angle in standard position has its vertex at the origin with the initial side along the positive x-axis.
- The point (3,4) lies on the terminal side of such an angle.
- The hypotenuse (distance from origin to point) is 5.
- Trigonometric ratios are derived directly from the coordinates:
 - $\sin \theta = 4/5$
 - $\cos \theta = 3/5$
 - $\tan \theta \approx 4/3$
- The angle θ is approximately 53.13° , located in the first quadrant.
- Additional ratios like secant, cosecant, and cotangent provide further tools for analysis.

Conclusion

Understanding how to analyze a point like (3,4) in the context of an angle in standard position is crucial in trigonometry and coordinate geometry. By constructing the right triangle and applying fundamental ratios, one can accurately determine the measure of the angle and its related trigonometric functions. This knowledge not only enhances problem-solving skills but also provides a foundation for advanced applications in science and engineering.

Keywords for SEO Optimization:

- Angle in standard position
- Terminal side passing through point (3,4)
- Trigonometric ratios from coordinates
- How to find angle from point (3,4)
- Reference angle and hypotenuse
- Coordinate geometry and trigonometry
- Calculating sine, cosine, tangent for point (3,4)
- Trigonometry applications in real life
- Right triangle from coordinate points
- Understanding angles and points in the coordinate plane

Frequently Asked Questions

What is the significance of the terminal side of an angle passing through the point (3,4)?

It helps determine the angle's measure in standard position by analyzing its location relative to the coordinate axes.

How do you find the angle in standard position whose terminal side passes through (3,4)?

Use the inverse tangent function: $\theta = \arctangent(y/x) = \arctangent(4/3)$.

What is the value of the angle in degrees, given the point (3,4)?

$\theta \approx \arctangent(4/3) \approx 53.13^\circ$.

In which quadrant does the terminal side passing through (3,4) lie?

Since both x and y are positive, the terminal side lies in the first quadrant.

How can the coordinates (3,4) be used to find the sine and cosine of the angle?

Calculate the hypotenuse $r = \sqrt{3^2 + 4^2} = 5$. Then, $\text{sine} = y/r = 4/5$, $\text{cosine} = x/r = 3/5$.

What is the significance of the hypotenuse in this context?

The hypotenuse represents the distance from the origin to the point (3,4), used to find the trigonometric ratios.

Can this method be applied to any point (x, y) to find the angle in standard position?

Yes, by using $\arctan(y/x)$ and considering the signs of x and y to determine the correct quadrant, this method applies broadly.

Additional Resources

Understanding the Terminal Side of an Angle in Standard Position Passing Through the Point (3, 4): An In-Depth Exploration

When we delve into the fascinating world of coordinate geometry and trigonometry, one of the foundational concepts is the representation of angles in the coordinate plane, particularly those in standard position. The question of determining the terminal side of an angle passing through a specific point — in this case, (3, 4) — is not just an academic exercise but a crucial skill with applications spanning physics, engineering, computer graphics, and beyond. This article aims to provide an expert-level, comprehensive analysis of this problem, including the geometric intuition, algebraic procedures, and the underlying principles involved.

Understanding the Foundations: Angles in Standard Position

Before we analyze the specific problem, it's vital to understand what it means for an angle to be in standard position.

Definition and Significance

An angle is said to be in standard position if:

- Its vertex is at the origin (0,0).
- Its initial side coincides with the positive x-axis.

- Its terminal side is some ray starting from the origin, rotating counterclockwise (for positive angles) or clockwise (for negative angles).

This setup provides a consistent reference point to measure and analyze angles, especially when working with the unit circle or coordinate points.

Representing Angles Geometrically and Algebraically

Angles in standard position can be characterized by their measure (in degrees or radians), and their terminal sides can be described by equations of lines or rays in the coordinate plane.

- The measure of the angle corresponds to the rotation from the initial side (positive x-axis) to the terminal side.
- The terminal side is a ray extending from the origin, passing through points in the plane, such as the point $(3, 4)$ in our case.

The Point $(3, 4)$: Geometric and Algebraic Implications

Given the point $(3, 4)$, our goal is to determine the specific angle in standard position whose terminal side passes through this point.

Locating the Point in the Coordinate Plane

The point $(3, 4)$ is located in the first quadrant, where both x and y are positive. Its position relative to the origin provides clues about the measure and orientation of the terminal side of the angle.

Significance of the Point's Coordinates

- The x -coordinate (3) indicates the horizontal distance from the origin.
- The y -coordinate (4) indicates the vertical distance from the origin.
- The ratio y/x gives the tangent of the angle (more on that shortly).

Determining the Terminal Side: Methodology and Calculations

The crux of this problem is to find the angle in standard position such that its terminal side passes through (3, 4). We approach this systematically.

Step 1: Visualizing the Ray

Imagine a ray originating at (0, 0) and passing through (3, 4). The direction of this ray defines our terminal side. To find the measure of this angle, we analyze the geometric and algebraic relationships involved.

Step 2: Calculating the Radius (Distance from Origin to the Point)

First, determine the distance r from the origin to the point (3, 4):

$$r = \sqrt{x^2 + y^2} = \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5$$

This length, often called the radius, is crucial because it relates to the unit circle and trigonometric ratios.

Step 3: Using Trigonometric Ratios to Find the Angle

In the context of the coordinate plane, the angle θ in standard position that corresponds to the point (3, 4) can be found using the tangent function:

$$\tan \theta = \frac{y}{x} = \frac{4}{3}$$

This ratio provides the tangent of the angle between the positive x-axis and the terminal side passing through (3, 4).

Step 4: Computing the Angle θ

Using the inverse tangent function:

$$\theta = \arctan \left(\frac{4}{3} \right)$$

Calculating this:

$$\theta \approx \arctan(1.3333) \approx 53.13^\circ$$

or in radians:

$$\theta \approx 0.927 \text{ radians}$$

This value represents the measure of the principal angle in standard position whose terminal side passes through (3, 4).

Step 5: Confirming the Quadrant

Since both x and y are positive, the point is in the first quadrant, and this angle directly corresponds to the measure between 0° and 90° , confirming our calculation.

Additional Considerations and Applications

While the core calculation yields a specific angle, understanding the broader context is essential for comprehensive expertise.

Angles Greater Than 360° or Negative Angles

- If asked for the coterminal angles, you can add or subtract multiples of 360° .
- For negative angles, the rotation is clockwise from the positive x-axis, but the terminal side still passes through the same line, just in the opposite direction.

Use in Trigonometry: Finding Exact Values

- The ratio $\frac{4}{5}$ is reminiscent of a 3-4-5 right triangle, which is a Pythagorean triple. This offers an opportunity to find the exact trigonometric values:
- $\sin \theta = \frac{y}{r} = \frac{4}{5}$
- $\cos \theta = \frac{x}{r} = \frac{3}{5}$

- $\tan \theta = \frac{4}{3}$

Having these exact ratios is invaluable for solving more complex problems where precise values are required.

Application in Real-World Contexts

- Navigation and Bearings: Determining the direction of a line passing through a point.
- Physics: Calculating angles of force vectors.
- Computer Graphics: Rotating objects or determining the orientation of elements.

Visualizing the Result: The Conceptual Framework

A diagram would typically depict:

- The coordinate plane with axes labeled.
- The origin point (0,0).
- The point (3,4) plotted in the first quadrant.
- A ray starting at (0,0) and passing through (3,4), representing the terminal side.
- The initial side along the positive x-axis.
- The angle $\theta \approx 53.13^\circ$ between the initial and terminal sides.

This visualization solidifies understanding and confirms the mathematical calculations.

Summary of Key Takeaways

- The point (3, 4) lies on a ray originating from the origin, defining the terminal side of an angle in standard position.
- The angle θ can be found using $\tan \theta = \frac{y}{x}$, leading to $\theta \approx 53.13^\circ$.
- The length of the radius from the origin to the point is 5, corresponding to the hypotenuse of a 3-4-5 triangle.
- Exact trigonometric ratios are derived from the point's coordinates, facilitating precise calculations.
- This process exemplifies the deep connection between algebraic methods and geometric intuition.

Conclusion

Determining the terminal side of an angle passing through a specific point like (3, 4) exemplifies the elegant interplay between coordinate geometry and trigonometry. By leveraging fundamental ratios, inverse functions, and geometric visualization, we gain a comprehensive understanding of how angles are represented and analyzed in the coordinate plane. This knowledge not only deepens mathematical fluency but also empowers applications across various scientific and engineering disciplines, illustrating the enduring relevance of these foundational concepts.

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