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Understanding electrostatics and the behavior of point charges is fundamental in physics. When three point charges are arranged at the corners of an equilateral triangle, interesting phenomena occur due to the interplay of Coulomb's law and symmetry. This article provides a comprehensive analysis of the problem, including the calculation of forces, potential energy, and equilibrium conditions. Whether you're a student preparing for an exam or a physics enthusiast, this guide will deepen your understanding of the electrostatic interactions in such configurations.

Introduction to Electrostatics and Point Charges

Fundamentals of Coulomb's Law

Coulomb's law describes the force between two point charges. Mathematically, it is expressed as:

$$
F = k_e \frac{|q_1 q_2|}{r^2}
$$

Where:
- F is the magnitude of the force between charges,
- k_e is Coulomb's constant ($8.988 \times 10^9\, \mathrm{Nm^2/C^2}$),
- q_1 and q_2 are the magnitudes of the charges,
- r is the distance between the charges.

The force is attractive if the charges are opposite in sign and repulsive if they are of the same sign.

Configuration of Charges in an Equilateral Triangle

Consider three point charges q_1, q_2, and q_3 placed at the corners of an equilateral triangle with side length a. Due to symmetry, the analysis simplifies significantly, especially when charges are identical or follow specific arrangements.

Problem Statement and Objective

Suppose three point charges are fixed at the corners of an equilateral triangle as depicted in Figure P15.13 (not shown here). The problem is to determine:

- The net force acting on each charge due to the other two,
- The potential energy of the system,
- Conditions under which the system is in equilibrium,

- The nature of interactions if the charges are of different signs or magnitudes.

The goal is to derive expressions for these quantities and analyze their physical implications.

Assumptions and Notations

Before proceeding with calculations, clarify assumptions:

- The charges are point charges.
- The triangle is perfectly equilateral with side length a.
- The charges are fixed at the corners (no movement or equilibrium considerations unless specified).
- Charges can be of any magnitude and sign.

Notations:
- q_1, q_2, q_3: magnitudes of charges at vertices A, B, and C respectively.
- $r_{AB} = r_{BC} = r_{CA} = a$: distances between charges.

Calculating the Forces Between Charges

Force on a Single Charge

To compute the net force on a particular charge, sum the vector contributions from the other two charges.

For example, the force on q_1 due to q_2 and q_3:

$$
\vec{F}_{1} = \vec{F}_{12} + \vec{F}_{13}
$$

Where:

$$
\vec{F}_{12} = k_e \frac{q_1 q_2}{a^2} \hat{r}_{12}
$$

$$
\vec{F}_{13} = k_e \frac{q_1 q_3}{a^2} \hat{r}_{13}
$$

The vectors $\hat{r}_{12}$ and $\hat{r}_{13}$ point along the lines connecting the charges, with directions depending on the signs of the charges.

Symmetry in Equilateral Triangle

Given the symmetry:

- The magnitudes of the forces between each pair are equal if the charges are identical.
- The directions are separated by 60° angles.

Use vector addition principles to sum forces, considering their directions.

Example: Identical Charges

Suppose all charges are equal ($q_1 = q_2 = q_3 = q$):

$$
F = k_e \frac{q^2}{a^2}
$$

The net force on each charge involves adding two vectors separated by 60°, which results in:

$$
F_{net} = 2F \cos(30^\circ) = 2 \times k_e \frac{q^2}{a^2} \times \frac{\sqrt{3}}{2} = k_e \frac{q^2}{a^2} \sqrt{3}
$$

Directionally, the net force can be determined based on vector components.

Potential Energy of the System

Calculating Total Electrostatic Potential Energy

The electrostatic potential energy U of a system of point charges is the sum over all unique pairs:

\[
U = \sum_{i