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Understanding the behavior of point charges in electrostatics is fundamental to the study of electric fields and forces. When three identical charges are arranged at the vertices of an equilateral triangle, the symmetry of the configuration provides a rich ground for analyzing the forces, potential energies, and resultant electric fields. In this article, we explore the specifics of such a setup, delve into the calculations involved, and discuss the physical implications of placing three identical point charges at the corners of an equilateral triangle with side length 90 cm.

Introduction to the Setup

Basic Parameters of the Configuration

- Charges: Q1 = Q2 = Q3 = 3.0×10^{-7} C
- Shape: Equilateral triangle
- Side length: 90 cm = 0.9 meters
- Environment: Assumed vacuum or air (permittivity $\varepsilon_0 \approx 8.854 \times 10^{-12}$ $C^2/N{\cdot}m^2$)

This configuration involves three identical point charges placed precisely at the vertices of an equilateral triangle. The symmetry simplifies the analysis of forces and potential energies, as all sides are equal and each charge experiences similar influences.

Electrostatic Forces in the Triangle

Calculating the Force Between Two Charges

Using Coulomb's law, the magnitude of the force F between any two charges (say Q1 and Q2) separated by distance r is given by:

$$
F = \frac{k \, |Q_1 Q_2|}{r^2}
$$

where:
- $k = \frac{1}{4 \pi \varepsilon_0} \approx 8.988 \times 10^9 \ \text{N·m}^2/\text{C}^2$
- $Q_1 = Q_2 = 3.0 \times 10^{-7} \ \text{C}$

- $r = 0.9 \ \text{m}$

Substituting the values:

$$
F = \frac{8.988 \times 10^9 \times (3.0 \times 10^{-7})^2}{(0.9)^2}
$$

$$
F = \frac{8.988 \times 10^9 \times 9.0 \times 10^{-14}}{0.81}
$$

$$
F \approx \frac{8.088 \times 10^{-4}}{0.81} \approx 1.0 \times 10^{-3} \ \text{N}
$$

Thus, each pair of charges exerts an attractive or repulsive force of approximately 1 millinewton (assuming positive charges repel).

Force Distribution and Symmetry

- Due to the equilateral triangle symmetry, each charge experiences forces from the other two charges.
- The forces are vector quantities, and their directions depend on the relative positions.

Net Force on Each Charge

Force Components and Vector Summation

Considering a single charge, for example Q1, it experiences:

- The force $\vec{F}_{12}$ due to Q2.
- The force $\vec{F}_{13}$ due to Q3.

Since all charges are equal and the triangle is equilateral:

- The magnitudes of these forces are equal ($\sim 1 \times 10^{-3}$ N).
- The directions are along the lines connecting the charges.

To find the net force on Q1, we:

1. Resolve $\vec{F}_{12}$ and $\vec{F}_{13}$ into components.
2. Use vector addition considering the angles between forces.

Because the triangle is equilateral, the angle between the lines from Q1 to Q2 and Q1 to Q3 is 60°.

Calculating the Resultant Force

- The two forces $\vec{F}_{12}$ and $\vec{F}_{13}$ are equal in magnitude and separated by 60°.
- The resultant $\vec{F}_{\text{net}}$ on Q1 is obtained via the law of cosines or vector addition:

$$
|\vec{F}_{\text{net}}| = \sqrt{F^2 + F^2 + 2 F^2 \cos 60^\circ} = \sqrt{2F^2 + 2F^2 \times 0.5} = \sqrt{2F^2 + F^2} = \sqrt{3} F
$$

Substituting $F = 1 \times 10^{-3} \ \text{N}$:

$$
|\vec{F}_{\text{net}}| \approx \sqrt{3} \times 1 \times 10^{-3} \approx 1.732 \times 10^{-3} \ \text{N}
$$

This net force acts along the bisector of the angle between the two forces, directed outward or inward depending on whether the charges repel or attract.

Potential Energy of the System

Calculating the Total Electrostatic Potential Energy

The total potential energy U of a system of point charges is:

\[
U = \sum_{i