Three Vectors Are So Related That A +C = 5+j15 And A + 2B = 0. Where B Is The Conjugate Of C, Determine

Three Vectors Are So Related That A + C = 5 + j15 And A + 2B = 0. Where B Is The Conjugate Of C, Determine

Understanding the relationships among vectors in complex form is a fundamental aspect of advanced mathematics and engineering. When given specific equations involving vectors A, B, and C, especially with conditions involving conjugates, it becomes possible to determine unknown vectors and analyze their properties. This article explores how to approach the problem where three vectors are related through the equations A + C = 5 + j15 and A + 2B = 0, with the additional condition that B is the conjugate of C. By breaking down these relationships step-by-step, we can derive the explicit forms of the vectors involved.

Understanding the Given Relationships

1. The Main Equations

The problem provides two key equations:

- A + C = 5 + j15
- A + 2B = 0

Additionally, it states that B is the conjugate of C, which can be written as:

- B = conjugate of C = C

These relationships form the basis of our analysis.

2. Significance of Conjugate Vectors

In complex vector analysis, the conjugate of a complex number or vector involves changing the sign of the imaginary component. If C = x + jy, then its conjugate C = x - jy. Recognizing this property allows us to connect B and C explicitly.

Step-by-Step Solution Approach

1. Express B in terms of C

Since B is the conjugate of C:

- $B = \bar{C} = x - jy$

Suppose C is expressed as:

- $C = x + jy$

where x and y are real numbers.

2. Rewrite the equations in terms of x and y

Using the first equation $A + C = 5 + j15$, we can express A as:

- $A = (5 + j15) - C = (5 + j15) - (x + jy) = (5 - x) + j(15 - y)$

Similarly, from the second equation $A + 2B = 0$, and substituting $B = \bar{C}$:

- $A + 2\bar{C} = 0$

Expressed in terms of x and y:

- $A + 2(x - jy) = 0$

Since A is already expressed in terms of x and y, we substitute $A = (5 - x) + j(15 - y)$:

- $(5 - x) + j(15 - y) + 2x - 2jy = 0$

Combine like terms:

- $(5 - x + 2x) + j(15 - y - 2y) = 0$
- $(5 + x) + j(15 - 3y) = 0$

For this complex expression to be zero, both the real and imaginary parts must be zero:

- Real part: $5 + x = 0 \rightarrow x = -5$
- Imaginary part: $15 - 3y = 0 \rightarrow y = 5$

Result:
$x = -5$ and $y = 5$.

Determining the Vectors A, B, and C

1. Find C

Recall that C was expressed as:

- $C = x + jy = -5 + j5$

2. Find B

Since B is the conjugate of C:

- $B = C = -5 - j5$

3. Find A

Recall that:

- $A = (5 - x) + j(15 - y)$

Substitute x = -5 and y = 5:

- A = (5 - (-5)) + j(15 - 5) = (5 + 5) + j(10) = 10 + j10

Final vectors:

- **A = 10 + j10**
- **B = -5 - j5**
- **C = -5 + j5**

Summary:
Through algebraic manipulation and understanding the properties of complex conjugates, we have explicitly determined the vectors A, B, and C based on the given relationships.

Applications and Significance of Vector Relationships in Engineering and Physics

1. Signal Processing

Complex vectors are fundamental in representing signals, especially in Fourier analysis. Understanding how vectors relate through conjugates helps in analyzing phase and amplitude relationships within signals.

2. Electromagnetic Theory

In electromagnetics, complex vectors describe wave propagation, with conjugates representing reflected or conjugate fields. Recognizing how vectors relate aids in the design of antennas and waveguides.

3. Control Systems

Complex analysis is critical in control system stability analysis through root locus and frequency response methods. Relationships among vectors provide insights into system behavior.

Concluding Remarks

The problem of determining vectors A, B, and C based on their relationships and conjugate properties demonstrates the importance of complex algebra in mathematical analysis. By systematically expressing the vectors in terms of real components, substituting into given equations, and applying properties of conjugates, it becomes straightforward to resolve unknowns. This approach not only reinforces fundamental algebraic techniques but also underscores the practical relevance of these concepts in engineering disciplines like signal processing, electromagnetics, and control systems.

Understanding such relationships enhances problem-solving skills and deepens comprehension of complex vector interactions, making it a valuable exercise for students and professionals alike.

Frequently Asked Questions

What is the given vector relation involving vectors A and C?

The given relation is A + C = 5 + j15.

How is vector B related to vector C in the problem?

Vector B is the conjugate of vector C.

What is the second vector relation provided in the problem?

The second relation is A + 2B = 0.

What is the goal of the problem involving vectors A, B, and C?

To determine the vectors A, B, and C based on the given relations.

How can the conjugate relationship between B and C be expressed mathematically?

If B is the conjugate of C, then B = C, where C denotes the complex conjugate of C.

Given the relation A + C = 5 + j15, how can we express A in terms of C?

A = (5 + j15) - C.

Using the relation A + 2B = 0 and B = C, how can we find A in terms of C?

Substitute B = C into the second relation: A + 2C = 0, thus A = -2C.

How do we use both relations to find the vector C?

Set (5 + j15) - C = -2C and solve for C by expressing C in terms of its real and imaginary parts.

What is the general approach to solve for C given the complex vector relations?

Express C as x + jy, substitute into the equations, and solve for x and y to find the specific vectors.

Additional Resources

Three Vectors Are So Related That A + C = 5 + j15 And A + 2B = 0. Where B Is The Conjugate Of C, Determine

Understanding the relationships among vectors in complex planes and their algebraic manipulations is fundamental in advanced mathematics and engineering disciplines such as signal processing, electromagnetism, and vector calculus. The problem presented involves three vectors, A, B, and C, interconnected through algebraic equations that include complex conjugation and addition. Analyzing such relationships requires a careful breakdown of the given equations, understanding the properties of conjugates, and systematically solving for the unknown vectors. This article provides a comprehensive review of the problem, exploring the mathematical principles involved, the step-by-step solution, and the broader implications of such vector relationships.

Understanding the Problem Setup

The problem states:

> Three vectors are so related that $A + C = 5 + j15$ and $A + 2B = 0$, where B is the conjugate of C. Determine the vectors.

At first glance, the problem involves three vectors A, B, and C, with two key equations linking them:

1. $A + C = 5 + j15$

2. $A + 2B = 0$

Additionally, it is specified that:

- B is the conjugate of C, i.e., $B = \overline{C}$

This setup suggests a complex number framework, where vectors are represented as complex numbers. The conjugate relationship indicates that B and C are complex conjugates, which has profound implications for their real and imaginary parts.

Mathematical Foundations and Key Concepts

Before delving into the solution, it's essential to clarify some fundamental concepts:

Complex Vectors and Conjugation

- Complex Number Representation of Vectors: Vectors in the plane can be represented as complex numbers $z = x + jy$, where x and y are the real components.

- Complex Conjugate: For a complex number $z = x + jy$, the conjugate is $\overline{z} = x - jy$. Geometrically, conjugation reflects the point across the real axis.

- Properties of Conjugates:
- $\overline{z_1 + z_2} = \overline{z_1} + \overline{z_2}$
- $\overline{z_1 z_2} = \overline{z_1} \, \overline{z_2}$
- $\overline{\overline{z}} = z$

Vector Equations and Their Complex Forms

- The equations relate vectors via addition and conjugation, which can be expressed as operations on their complex representations.

- The problem reduces to solving for complex numbers A, B, C satisfying the given equations.

Step-by-Step Solution Approach

The goal is to find the explicit forms of A, B, and C. The steps involve expressing known relationships, substituting conjugates, and solving for the unknowns systematically.

Step 1: Express A in terms of C

From the first equation:

$$A + C = 5 + j15$$

$$
A = (5 + j15) - C
$$

This expresses A directly in terms of C.

Step 2: Express B in terms of C

Given that:

$$
B = \overline{C}
$$

and from the second equation:

$$
A + 2B = 0
$$

Substitute A:

$$
(5 + j15) - C + 2 \overline{C} = 0
$$

Rearranged as:

$$
(5 + j15) = C - 2 \overline{C}
$$

This forms the core equation to solve for C.

Step 3: Express C and $\overline{C}$ in components

Let:

$$
C = x + jy
$$
$$
\overline{C} = x - jy
$$

Substitute into the equation:

$$
(5 + j15) = (x + jy) - 2(x - jy)
$$

Simplify RHS:

$$
x + jy - 2x + 2jy = (x - 2x) + (jy + 2jy) = -x + 3jy
$$

Thus:

$$
5 + j15 = -x + j3y
$$

Equating real and imaginary parts:

- Real part:

$$
5 = -x \Rightarrow x = -5
$$

- Imaginary part:

$$
15 = 3y \Rightarrow y = 5
$$

Now, we have:

$$
C = x + jy = -5 + j5
$$

and consequently:

$$
\overline{C} = -5 - j5
$$

Step 4: Find A and B

From earlier:

\[

$$
A = (5 + j15) - C
$$

Substitute $C = -5 + j5$:

$$
A = (5 + j15) - (-5 + j5) = 5 + j15 + 5 - j5 = (5 + 5) + j(15 - 5) = 10 + j10
$$

Similarly, since $B = \overline{C}$:

$$
B = -5 - j5
$$

Final Result:

$$
\boxed{
\begin{aligned}
A &= 10 + j10 \\
B &= -5 - j5 \\
C &= -5 + j5
\end{aligned}
}
$$

Verification of the Solution

Verifying the solutions ensures correctness:

- Check $A + C$:

$$
(10 + j10) + (-5 + j5) = (10 - 5) + j(10 + 5) = 5 + j15
$$

Matches the first equation.

- Check $A + 2B$:

$$
(10 + j10) + 2(-5 - j5) = 10 + j10 - 10 - j10 = 0
$$

Matches the second equation.

- Confirm B is conjugate of C:

$$
\overline{C} = -5 - j5 = B
$$

Confirmed.

Broader Implications and Applications

This problem exemplifies the power of complex numbers in solving vector equations, especially when conjugates are involved. Such techniques are widely applicable in engineering fields, including:

- Signal Processing: Complex conjugates are used in Fourier transforms and filtering operations.
- Electromagnetism: Representing oscillating fields as complex phasors simplifies calculations.
- Control Systems: Stability analysis often involves conjugate pairs of poles.

Understanding how to manipulate and interpret these relationships enhances problem-solving skills in theoretical and applied contexts.

Features, Pros, and Cons of Using Complex Numbers in Vector Problems

Features:

- Simplifies calculations involving rotations, oscillations, and wave phenomena.
- Encapsulates two-dimensional vector information into a single complex number.
- Facilitates algebraic manipulations with conjugates, magnitudes, and phases.

Pros:

- Reduces computational complexity for certain problems.
- Provides geometric interpretations, such as reflections and rotations.
- Enables straightforward solutions to conjugate-related equations.

Cons:

- Can be abstract and less intuitive for those unfamiliar with complex analysis.
- Not directly applicable in higher dimensions without extension to quaternions or vectors.
- Requires careful attention to the properties of conjugation to avoid errors.

Conclusion

The problem of determining vectors \(A\), \(B\), and \(C\) given their complex relationships exemplifies the elegant interplay between algebra and geometry. By leveraging the properties of complex conjugates, the solution becomes straightforward, revealing the underlying structure of the relationships. Such problems not only reinforce fundamental concepts in complex analysis and vector algebra but also underscore their practical utility across scientific and engineering disciplines. Mastery of these techniques empowers practitioners to approach a wide array of problems involving complex vectors with confidence and precision.

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