TIME REMAININ57:1412345What Is The Equation Of The Line Passing Through The Points (2, -1) And (5, -10)

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Understanding how to find the equation of a line passing through two points is a fundamental skill in algebra and analytic geometry. Whether you're a student preparing for exams, a teacher crafting lesson plans, or a math enthusiast exploring the properties of lines, mastering this process is essential. In this article, we will explore the step-by-step method to determine the equation of the line passing through the points (2, -1) and (5, -10). We will also discuss related concepts, common mistakes, and practical applications to deepen your understanding.

Understanding the Basics

What Is an Equation of a Line?

An equation of a line is a mathematical statement that describes all the points lying on that line. It relates the x-coordinate and y-coordinate of any point on the line using a formula. The most common forms are:

- Slope-intercept form: $y = mx + b$
- Point-slope form: $y - y_1 = m(x - x_1)$
- Standard form: $Ax + By = C$

Where:

- m is the slope of the line
- (x_1, y_1) is a known point on the line
- A, B, and C are constants

Why Find the Equation?

Knowing the equation of a line allows you to:

- Predict y-values for given x-values
- Graph the line accurately
- Understand relationships between variables
- Solve geometric problems involving lines

Step-by-Step Process to Find the Equation

Step 1: Identify the Two Points

Given points:

- Point 1: (2, -1)
- Point 2: (5, -10)

These coordinates represent positions on the Cartesian plane.

Step 2: Calculate the Slope (m)

The slope indicates the steepness of the line. It is calculated as:

$$ m = \frac{y_2 - y_1}{x_2 - x_1} $$

Applying the points:

$$ m = \frac{-10 - (-1)}{5 - 2} = \frac{-10 + 1}{3} = \frac{-9}{3} = -3 $$

So, the slope of the line is -3.

Step 3: Use the Point-Slope Form

The point-slope form is convenient when you know a point and the slope:

$$ y - y_1 = m(x - x_1) $$

Choose one of the points; for simplicity, we'll use (2, -1):

$$ y - (-1) = -3(x - 2) $$

Simplify:

$$ y + 1 = -3(x - 2) $$

Step 4: Convert to Slope-Intercept Form

Distribute the slope:

$$ y + 1 = -3x + 6 $$

Subtract 1 from both sides:

$$ y = -3x + 6 - 1 $$

$$ y = -3x + 5 $$

Thus, the equation of the line passing through the points (2, -1) and (5, -10) in slope-intercept form is:

$$ y = -3x + 5 $$

Additional Concepts and Tips

Verifying the Equation

It's good practice to verify that the derived equation passes through both points.

- For (2, -1):

$$ y = -3(2) + 5 = -6 + 5 = -1 $$ (matches y-coordinate)

- For (5, -10):

$$ y = -3(5) + 5 = -15 + 5 = -10 $$ (matches y-coordinate)

Since both points satisfy the equation, our solution is correct.

Alternative Forms of the Equation

- Point-slope form:

$$ y + 1 = -3(x - 2) $$

- Standard form:

Start from slope-intercept form:

$$ y = -3x + 5 $$

Bring all to one side:

$$ 3x + y = 5 $$

This is the standard form.

Common Mistakes to Avoid

- Mixing up points: Always double-check the coordinates.
- Incorrect slope calculation: Remember to subtract y-values from y-values and x-values from x-values.
- Forgetting to verify: Always verify the equation with both points.
- Sign errors: Pay close attention to signs during calculations.

Applications of Line Equations

Understanding how to find the equation of a line has numerous practical applications across various fields:

- Physics: Describing motion with linear equations
- Economics: Modeling cost, revenue, or profit functions
- Engineering: Designing systems and analyzing relationships
- Computer Graphics: Rendering lines and shapes
- Data Analysis: Fitting linear models to data points

Advanced Topics Related to Line Equations

1. Parallel and Perpendicular Lines

- Parallel lines have the same slope.
- Perpendicular lines have slopes that are negative reciprocals.

2. Lines Through a Given Point with a Given Slope

Given a point and a slope, you can write the equation directly in point-slope form.

3. Finding the Equation When Given a Slope and a Point

Use the point-slope form:

$$ y - y_1 = m(x - x_1) $$

Summary

To find the equation of the line passing through the points (2, -1) and (5, -10), follow these steps:

1. Calculate the slope:

$$ m = -3 $$

2. Use the point-slope form with one of the points:

$$ y + 1 = -3(x - 2) $$

3. Simplify to slope-intercept form:

$$ y = -3x + 5 $$

This equation describes all points on the line, including the given points, and serves as a foundation for further analysis or graphing.

Conclusion

Mastering the process of deriving the equation of a line from two points is a crucial skill in algebra and beyond. It enhances problem-solving ability, aids in data analysis, and provides insights into the relationships between variables. Remember to carefully compute the slope, choose an appropriate point, and verify your results. With practice, finding the equation of a line will become an intuitive part of your mathematical toolkit.

For further learning, consider exploring how to find the equation of a line given different sets of information, such as a point and a slope, two lines' equations to find their intersection, or lines in three-dimensional space.

Frequently Asked Questions

How do you find the slope of the line passing through the points (2, -1) and (5, -10)?

The slope (m) is calculated using the formula $m = (y2 - y1) / (x2 - x1)$. Substituting the points, $m = (-10 - (-1)) / (5 - 2) = (-9) / 3 = -3$.

What is the equation of the line passing through (2, -1) and (5, -10)?

Using point-slope form: $y - y1 = m(x - x1)$. Plugging in (2, -1) and $m = -3$, the equation is $y - (-1) = -3(x - 2)$, which simplifies to $y + 1 = -3x + 6$, or $y = -3x + 5$.

Can you verify the line's equation passes through both points?

Yes. Substitute x=2 into $y = -3x + 5$: $y = -3(2) + 5 = -6 + 5 = -1$, matching the first point. For x=5: $y = -3(5) + 5 = -15 + 5 = -10$, matching the second point.

What is the slope-intercept form of the line passing through (2, -1) and (5, -10)?

The slope-intercept form is $y = -3x + 5$, where -3 is the slope and 5 is the y-intercept.

How do you determine the equation of a line given two points?

First, find the slope using the two points, then use either point in the point-slope form to derive the equation of the line.

What are common mistakes to avoid when finding the line equation through two points?

Common mistakes include incorrect calculation of the slope, mixing up x and y coordinates, and errors in simplifying the equation. Always verify by plugging in both points to confirm the line equation.

Additional Resources

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Introduction

In the realm of coordinate geometry, understanding how to derive the equation of a line passing through two distinct points is fundamental. This process not only aids in graphing lines but also underpins more complex mathematical concepts such as linear functions, systems of equations, and analytical geometry. The specific problem of finding the equation of a line passing through the points (2, -1) and (5, -10) exemplifies these processes in a practical context.

This article provides a comprehensive exploration of this problem, examining the underlying principles, step-by-step calculations, and broader implications. It serves as a guide for students, educators, and enthusiasts aiming to deepen their understanding of line equations, emphasizing clarity, accuracy, and mathematical rigor.

The Significance of Line Equations in Mathematics

Before delving into the specifics of our problem, it's essential to appreciate why line equations matter. They are central to modeling real-world phenomena, such as:

- Describing trends in data (e.g., economics, biology).
- Navigating in coordinate planes.
- Analyzing relationships between variables.

The most common forms of line equations include:

- Slope-intercept form: $y = mx + b$
- Point-slope form: $y - y_1 = m(x - x_1)$
- Standard form: $Ax + By = C$

Understanding how to derive each form from given points is a core skill in algebra and analytical geometry.

Step 1: Identifying the Given Points

Our problem involves two points:

- Point A: (2, -1)
- Point B: (5, -10)

These points are located in the Cartesian plane, with their respective x and y coordinates.

Step 2: Calculating the Slope (m)

The slope of a line passing through two points (x_1, y_1) and (x_2, y_2) is given by:

$$
m = \frac{y_2 - y_1}{x_2 - x_1}
$$

Applying this formula:

$$
m = \frac{-10 - (-1)}{5 - 2} = \frac{-10 + 1}{3} = \frac{-9}{3} = -3
$$

Key Observation: The slope m = -3 indicates the line decreases by 3 units in y for every 1 unit increase in x.

Step 3: Choosing the Equation Form

Given the slope and a point, the point-slope form is often most straightforward for deriving the line's equation:

$$
y - y_1 = m(x - x_1)
$$

Using point A: (2, -1):

$$
y - (-1) = -3(x - 2)
$$

Simplifying:

$$
y + 1 = -3(x - 2)
$$

$$
y + 1 = -3x + 6
$$

Step 4: Deriving the Slope-Intercept Form

Subtract 1 from both sides to isolate y:

$$
y = -3x + 6 - 1
$$

$$
y = -3x + 5
$$

Final Equation (Slope-Intercept Form):

$$
\boxed{y = -3x + 5}
$$

Step 5: Verifying the Equation

To ensure correctness, verify that the second point (5, -10) satisfies the derived equation:

$$
y = -3x + 5
$$

$$
\text{At } x=5: y = -3(5) + 5 = -15 + 5 = -10
$$

\]

Since $y = -10$ matches the given y-value, the equation is valid.

Broader Implications and Applications

Understanding how to derive the line's equation from two points has significant implications:

- Predictive Modeling: In fields like economics or ecology, lines model relationships between variables.
- Graphical Analysis: Helps in plotting lines accurately for visualization.
- Engineering and Physics: Used in calculating trajectories, forces, and motion paths.
- Computational Geometry: Forms the basis for algorithms in computer graphics and pattern recognition.

Alternative Forms and Related Concepts

While the slope-intercept form is most common, understanding other forms enhances flexibility:

1. Point-Slope Form:

$$
y - y_1 = m(x - x_1)
$$

Useful when you know a point and the slope but want to write the equation directly.

2. Standard Form:

$$
Ax + By = C
$$

Can be derived by rearranging the slope-intercept form:

$$
y = -3x + 5
$$

$$
3x + y = 5
$$

Special Cases and Common Errors

- Vertical Lines: When $x_1 = x_2$, the slope is undefined, and the line's equation is x = constant.
- Horizontal Lines: When $y_1 = y_2$, the slope is zero, and the equation is y = constant.
- Ensuring Consistency: Always verify the point(s) satisfy the derived equation to prevent errors.

Practical Example: Application in Real-World Context

Suppose a researcher observes that a certain bacteria population decreases linearly over time. The initial measurement at day 2 is -1 units, and at day 5, it drops to -10 units. The derived line equation:

$$
y = -3x + 5
$$

can be used to predict the population at any other day within the linear range, aiding future planning and analysis.

Conclusion

The problem of finding the equation of a line passing through points (2, -1) and (5, -10) encapsulates core principles of coordinate geometry:

- Calculating the slope using the difference in y-values over x-values.
- Applying the point-slope form to derive the line's equation.
- Verifying the equation's accuracy with the given points.

Understanding these steps enhances one's ability to analyze linear relationships, interpret data, and apply mathematical reasoning across disciplines. The derived equation, $y = -3x + 5$, not only models the specific scenario but also exemplifies a fundamental skill applicable in diverse scientific, engineering, and mathematical contexts.

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