Transform The Differential Equation Y3y 5y=sinh(at) Y(0)=1 Y=5 Into An Algebraic Equation By Taking The

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Introduction to Differential Equations and Their Transformation

Differential equations are fundamental tools in mathematical modeling, describing relationships involving functions and their derivatives. They are widely used across physics, engineering, biology, and economics to model dynamic systems. However, solving differential equations directly can be complex, especially for higher-order equations or nonlinear forms.

Transforming differential equations into algebraic equations simplifies analysis and solution processes. This approach involves applying various methods such as substitution, Laplace transforms, or other integral transforms that convert derivatives into algebraic terms. This article focuses on transforming a specific third-order differential equation into an algebraic equation by taking an appropriate method, providing detailed steps and explanations for clarity.

Understanding the Given Differential Equation and Initial Conditions

Given Differential Equation

The differential equation under consideration is:

$$Y'''(t) + 5Y(t) = \sinh(at)$$

where:
- $Y'''(t)$ represents the third derivative of $Y(t)$ with respect to t,
- $5Y(t)$ is the scaled function,
- $\sinh(at)$ is the hyperbolic sine function involving a parameter a.

Initial Conditions

The problem provides initial conditions:
- $Y(0) = 1$,
- Y at some point is 5 (likely $Y(t_0) = 5$; if a specific t value is intended, it should be clarified).

These conditions are essential for solving the differential equation and will be used in the transformation process.

Choosing the Transformation Method

To convert the differential equation into an algebraic form, the most effective and common method is the Laplace Transform because it simplifies derivatives into algebraic terms and handles initial conditions seamlessly.

Advantages of Using Laplace Transform:
- Converts differential operators into algebraic expressions.
- Easily incorporates initial conditions.
- Suitable for linear differential equations with constant coefficients.

Applying the Laplace Transform to the Differential Equation

Step 1: Recall the Laplace Transform Properties

For a function $Y(t)$, its Laplace transform is:

$$\mathcal{L}\{Y(t)\} = Y(s)$$

The transforms of derivatives are:

$$
\begin{aligned}
\mathcal{L}\{Y'(t)\} &= sY(s) - Y(0) \\
\mathcal{L}\{Y''(t)\} &= s^2Y(s) - sY(0) - Y'(0) \\
\mathcal{L}\{Y'''(t)\} &= s^3Y(s) - s^2Y(0) - sY'(0) - Y''(0)
\end{aligned}
$$

Step 2: Transform the Differential Equation

Applying the Laplace transform to both sides:

$$
\mathcal{L}\{Y'''(t) + 5Y(t)\} = \mathcal{L}\{\sinh(at)\}
$$

which becomes:

$$
s^3 Y(s) - s^2 Y(0) - s Y'(0) - Y''(0) + 5 Y(s) = \mathcal{L}\{\sinh(at)\}
$$

Given initial conditions:
- $Y(0) = 1$,
- $Y'(0)$ and $Y''(0)$ are not explicitly provided, so assume they are zero unless specified otherwise.

For simplicity, assuming $Y'(0) = 0$ and $Y''(0) = 0$, the equation simplifies to:

$$
s^3 Y(s) - s^2 \times 1 + 5 Y(s) = \mathcal{L}\{\sinh(at)\}
$$

Transforming the Forcing Function: $\sinh(at)$

Laplace Transform of $\sinh(at)$

The Laplace transform of hyperbolic sine is:

$$
\mathcal{L}\{\sinh(at)\} = \frac{a}{s^2 - a^2}
$$

This is valid for $s > |a|$.

Formulating the Algebraic Equation

Substituting the transforms into the equation:

\[

$$
s^3 Y(s) - s^2 + 5 Y(s) = \frac{a}{s^2 - a^2}
$$

Rearranged as:

$$
(s^3 + 5) Y(s) = s^2 + \frac{a}{s^2 - a^2}
$$

Therefore,

$$
Y(s) = \frac{s^2}{s^3 + 5} + \frac{a}{(s^2 - a^2)(s^3 + 5)}
$$

This algebraic expression in s is the transformed form of the original differential equation.

Solving the Algebraic Equation for $Y(s)$

Step 1: Simplify the Expression

Express $Y(s)$ as:

$$
Y(s) = \frac{s^2}{s^3 + 5} + \frac{a}{(s^2 - a^2)(s^3 + 5)}
$$

Further simplification may involve partial fractions or other algebraic techniques depending on the goal, such as inverse Laplace transform.

Step 2: Find the Inverse Laplace Transform

To revert back to the original function $Y(t)$, apply the inverse Laplace transform:

$$
Y(t) = \mathcal{L}^{-1}\{Y(s)\}
$$

which may involve decomposing the expression into simpler parts and using known inverse transforms.

Additional Considerations and Boundary Conditions

- The initial condition $Y(0) = 1$ was used in the transformation. If additional derivatives at zero are specified, they should be incorporated into the algebraic equation.
- If the problem involves boundary conditions at other points or more complex initial data, the transformation process would include those adjustments.
- For the boundary condition $Y = 5$, clarity is needed on whether this refers to a specific point in t or a value at a different point; otherwise, it may be used to verify the solution after inverse transformation.

Conclusion and Summary

Transforming a third-order differential equation such as:

$$Y'''(t) + 5Y(t) = \sinh(at)$$

into an algebraic form involves applying the Laplace transform, which converts derivatives into algebraic terms with respect to the variable s. The key steps include:

1. Applying the Laplace transform to both sides of the differential equation.
2. Incorporating initial conditions to simplify the expression.
3. Transforming the forcing function $\sinh(at)$ into its algebraic form.
4. Solving the resulting algebraic equation for $Y(s)$.
5. Using inverse Laplace transform techniques to find $Y(t)$.

This process effectively reduces the complex differential equation into a manageable algebraic equation, facilitating easier analysis, solution, and interpretation of the behavior of the system modeled by the original differential equation.

Final Remarks

Understanding and applying the transformation of differential equations into algebraic equations is a fundamental skill in engineering and applied mathematics. Mastery of Laplace transforms, initial condition management, and inverse transforms enables scientists and engineers to analyze complex systems efficiently. The approach demonstrated here can be adapted to various types of differential equations, making it a versatile tool in mathematical problem-solving.

Frequently Asked Questions

What is the first step in transforming the differential equation Y''' + 5Y = sinh(at) with initial condition Y(0) = 1 into an algebraic equation?

The first step is to take the Laplace transform of both sides of the differential equation to convert it from a differential equation to an algebraic equation in the Laplace domain.

How do you handle the initial condition Y(0) = 1 when applying the Laplace transform to the differential equation?

You incorporate the initial condition by using the Laplace transform properties of derivatives, which include initial value terms, ensuring the algebraic equation accounts for Y(0) = 1.

What is the Laplace transform of the third derivative Y'''?

The Laplace transform of Y''' is s^3 L{Y(t)} - s^2 Y(0) - s Y'(0) - Y''(0), assuming initial conditions for derivatives are zero unless specified otherwise.

How do you find the Laplace transform of sinh(at) in this context?

The Laplace transform of sinh(at) is a / (s^2 - a^2), which is used to convert the right-hand side of the differential equation into an algebraic expression.

Once the Laplace transform is applied, what does the resulting algebraic equation look like?

It becomes a polynomial in s times L{Y(t)} minus initial terms equals the transformed sinh(at), typically written as (s^3 + 5) L{Y(t)} - initial terms = a / (s^2 - a^2).

How can you solve for L{Y(t)} after obtaining the algebraic equation?

Rearrange the algebraic equation to isolate L{Y(t)} by dividing both sides by the polynomial expression (s^3 + 5) and including the initial condition terms.

What is the final step to obtain Y(t) from its Laplace transform?

Apply the inverse Laplace transform to L{Y(t)} to find the explicit expression for Y(t), which may involve partial fraction decomposition.

Why is transforming the differential equation into an

algebraic equation helpful in solving it?

Transforming to an algebraic equation simplifies the problem, making it easier to solve for Y(t) explicitly, especially using standard inverse Laplace transform techniques.

Additional Resources

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Introduction

In the realm of differential equations, transforming a complex, often nonlinear, differential equation into an algebraic form is a powerful technique that simplifies analysis and solution. This approach enables mathematicians and engineers to tackle problems that initially seem intractable by reducing them to manageable algebraic expressions. Today, we explore a specific yet illustrative example: transforming the differential equation

$$ Y'''(t) + 5Y(t) = \sinh(at) $$

with initial conditions $Y(0) = 1$ and $Y(5) = ?$ (assuming the second condition aims for $Y(5) = 5$), into an algebraic equation through the method of taking the Laplace transform. This process highlights a fundamental technique in applied mathematics, bridging the gap between differential calculus and algebraic simplicity.

Understanding the Differential Equation

The given differential equation:

$$
Y'''(t) + 5Y(t) = \sinh(at)
$$

is a linear nonhomogeneous differential equation of third order. It involves the third derivative of $Y(t)$, a constant coefficient, and a hyperbolic sine function as the forcing term.

Key components:
- Third derivative $Y'''(t)$: Represents the rate of change of acceleration or the curvature of the function.
- Constant term $5Y(t)$: Suggests a proportional feedback of the function itself.
- Nonhomogeneous term $\sinh(at)$: Acts as an external input or forcing function.

Initial conditions:
- $Y(0) = 1$
- $Y(5) = 5$ (assuming the second point is a boundary condition)

The Rationale for Transforming

Transforming the differential equation into an algebraic one serves several purposes:

- Simplification: Differential operators become algebraic symbols, making the equation easier to manipulate.
- Solution derivation: Once transformed, algebraic techniques can be used to find the solution in the transform domain.
- Inverse transformation: Applying the inverse transform retrieves the solution $Y(t)$ in the original domain.
- Handling initial conditions: The Laplace transform incorporates initial conditions naturally.

The most common technique for such transformations is the Laplace transform, which converts derivatives into algebraic expressions involving the transform variable s.

The Laplace Transform: A Primer

The Laplace transform of a function $f(t)$, defined for $t \geq 0$, is:

$$
\mathcal{L}\{f(t)\} = F(s) = \int_0^{\infty} e^{-st}f(t) dt
$$

Key properties relevant for our differential equation:

- Linearity: $\mathcal{L}\{af + bg\} = aF(s) + bG(s)$
- Derivatives: $\mathcal{L}\{f^{(n)}(t)\} = s^n F(s) - s^{n-1}f(0) - \dots - f^{(n-1)}(0)$

Applying the Laplace transform to derivatives converts differential equations into algebraic equations in s.

Step-by-Step Transformation Procedure

1. Apply Laplace transform to the entire differential equation:

$$
\mathcal{L}\{Y'''(t) + 5Y(t)\} = \mathcal{L}\{\sinh(at)\}
$$

Using linearity:

$$
\mathcal{L}\{Y'''(t)\} + 5\mathcal{L}\{Y(t)\} = \mathcal{L}\{\sinh(at)\}
$$

2. Transform derivatives:

$$
\mathcal{L}\{Y'''(t)\} = s^3Y(s) - s^2Y(0) - sY'(0) - Y''(0)
$$

Assuming initial derivatives are zero (common unless specified):

$$
Y'(0) = 0, \quad Y''(0) = 0
$$

and initial value:

$$
Y(0) = 1
$$

Thus:

$$
\mathcal{L}\{Y'''(t)\} = s^3Y(s) - s^2 \times 1
$$

3. Transform the function $Y(t)$:

$$
\mathcal{L}\{Y(t)\} = Y(s)
$$

4. Transform the forcing term $\sinh(at)$:

Recall:

$$
\mathcal{L}\{\sinh(at)\} = \frac{a}{s^2 - a^2}
$$

Formulating the Algebraic Equation

Combining all, the transformed differential equation becomes:

$$
s^3Y(s) - s^2 + 5Y(s) = \frac{a}{s^2 - a^2}
$$

Rearranged:

$$

$$
(s^3 + 5)Y(s) = s^2 + \frac{a}{s^2 - a^2}
$$

Therefore, the algebraic form of the solution in the transform domain is:

$$
Y(s) = \frac{s^2 + \frac{a}{s^2 - a^2}}{s^3 + 5}
$$

Or, equivalently:

$$
Y(s) = \frac{s^2}{s^3 + 5} + \frac{a}{(s^2 - a^2)(s^3 + 5)}
$$

This algebraic expression encapsulates the original differential equation, initial conditions, and forcing function within a manageable formula.

Simplification and Inverse Transform

1. Break into simpler parts:

$$
Y(s) = \frac{s^2}{s^3 + 5} + \frac{a}{(s^2 - a^2)(s^3 + 5)}
$$

2. Partial fraction decomposition:

To find $Y(t)$, inverse Laplace transforms are typically computed via partial fraction expansion, which simplifies the complex rational functions into sums of simpler fractions.

3. Inverse Laplace transform:

After decomposition, apply the inverse Laplace transform to each term, referencing standard Laplace tables or known transforms for functions like $\frac{s^2}{s^3 + 5}$ and $\frac{a}{(s^2 - a^2)(s^3 + 5)}$.

Practical Applications and Significance

Transforming differential equations into algebraic equations is not just an academic exercise; it has real-world implications across engineering, physics, and applied mathematics.

- Control systems: Stability analysis often involves solving differential equations via Laplace transforms.
- Electrical engineering: Circuit analysis benefits from algebraic manipulation of differential equations.
- Mechanical systems: Vibrations and oscillations are modeled and analyzed through transformed equations.

- Signal processing: Filtering and system responses are derived using these transformations.

The technique streamlines the process of solving complex equations, especially when initial conditions are involved, and provides a pathway to explicit solutions.

Final Remarks

Transforming the differential equation \(Y'''(t) + 5Y(t) = \sinh(at) \) into an algebraic equation via the Laplace transform illustrates a cornerstone method in applied mathematics. By converting derivatives into algebraic terms, the problem becomes more approachable, allowing solutions to be systematically determined and then interpreted back in the original domain.

This approach exemplifies how mathematical tools bridge the gap between complex differential models and practical, solvable algebraic equations—empowering engineers, scientists, and mathematicians to analyze and design systems with precision and clarity.

Summary

- The differential equation involves third derivatives and a hyperbolic sine forcing term.
- Applying the Laplace transform converts derivatives into algebraic expressions.
- Initial conditions are incorporated naturally into the transformed equation.
- The algebraic form facilitates solution derivation via partial fractions and inverse transforms.
- The process underscores the importance of transform methods in solving real-world differential equations efficiently.

By mastering this transformation technique, practitioners can unlock solutions to a wide array of complex dynamical systems, reinforcing the profound utility of algebraic methods in mathematical analysis.

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