Use A Graphing Utility To Graph The Polar Equation. Inner Loop Of R = 3 + 6 Cos(). Find The Area Of The

Use A Graphing Utility To Graph The Polar Equation. Inner Loop Of R = 3 + 6 Cos(). Find The Area Of The

Understanding how to analyze and graph polar equations is a vital skill in advanced mathematics, particularly in calculus and analytical geometry. The given polar equation, R = 3 + 6 Cos(θ), describes a limaçon—a type of polar curve with distinctive inner and outer loops. By employing a graphing utility, we can visualize this curve, identify significant features like loops and cusps, and calculate the area enclosed within specific regions of the graph. This article provides a comprehensive guide on how to use a graphing utility to visualize the inner loop of the polar equation R = 3 + 6 Cos(θ) and determine the area of this inner loop, with detailed explanations suitable for students and educators alike.

Understanding the Polar Equation R = 3 + 6 Cos(θ)

Basics of Polar Coordinates and Equations

Polar coordinates represent points in the plane using a radius and angle, denoted as (r, θ). The radius r indicates the distance from the origin to the point, while θ specifies the angle measured from the positive x-axis. Polar equations, such as R = 3 + 6 Cos(θ), describe the relationship between r and θ and generate curves when graphed.

In the given equation:

- The term 3 is the constant part, influencing the baseline radius.
- The term 6 Cos(θ) introduces variation depending on the angle θ, causing the curve to oscillate between different radii.

Features of the Limaçon R = 3 + 6 Cos(θ)

- The graph is symmetric with respect to the horizontal axis (polar axis).
- The curve exhibits an inner loop because the maximum and minimum radii are not always positive.
- The maximum radius occurs when Cos(θ) = 1, i.e., θ = 0°, resulting in R = 3 + 6(1) = 9.
- The minimum radius occurs when Cos(θ) = -1, i.e., θ = 180°, resulting in R = 3 + 6(-1) =

-3, which indicates the inner loop.

Understanding these features is crucial for visualization and calculating associated areas.

Using a Graphing Utility to Visualize the Curve

Choosing an Appropriate Graphing Tool

Several graphing utilities support polar equations, including:

- Desmos Graphing Calculator
- GeoGebra
- Wolfram Alpha
- Graphing calculators like TI-84 or similar

For the most interactive experience, Desmos or GeoGebra are recommended due to their user-friendly interfaces and real-time plotting capabilities.

Steps to Graph R = 3 + 6 Cos(θ)

1. Access the Graphing Utility:
- Navigate to [Desmos](https://www.desmos.com/calculator) or open GeoGebra.

2. Input the Polar Equation:
- For Desmos, select the polar graphing mode.
- Enter the equation in the form: `r = 3 + 6 cos(θ)`.

3. Adjust the Viewing Window:
- Set θ to range from 0° to 360° (or 0 to 2π radians).
- Adjust r-axis to comfortably display the maximum radius (up to 10 or more).

4. Analyze the Graph:
- Observe the inner loop, outer loop, and the overall shape.
- Use tools to identify key points, such as where the inner loop starts and ends.

5. Identify Critical Points:
- Find where the inner loop intersects or where the radius is zero.
- These points are essential for area calculations.

Mathematical Analysis of the Inner Loop

Finding the Range of θ for the Inner Loop

The inner loop occurs where the radius r becomes negative, indicating the curve crosses the origin or loops inward.

- Set R = 0 to find the angles where the curve passes through the pole (origin):

$$ 0 = 3 + 6 \cos(\theta) $$
$$ 6 \cos(\theta) = -3 $$
$$ \cos(\theta) = -\frac{1}{2} $$

- The solutions for θ are:

$$ \theta = \arccos\left(-\frac{1}{2}\right) $$

- In degrees:

$$ \theta = 120^\circ \quad \text{and} \quad 240^\circ $$

- In radians:

$$ \theta = \frac{2\pi}{3} \quad \text{and} \quad \frac{4\pi}{3} $$

Thus, the inner loop exists for θ between 120° and 240° (or $\frac{2\pi}{3}$ and $\frac{4\pi}{3}$).

Calculating the Area of the Inner Loop

The area enclosed by a polar curve from θ = a to θ = b is given by:

$$ A = \frac{1}{2} \int_{a}^{b} r^2 \, d\theta $$

In this case:

- $r = 3 + 6 \cos(\theta)$
- The bounds are from $\theta = \frac{2\pi}{3}$ to $\theta = \frac{4\pi}{3}$

Therefore:

$$ A_{inner} = \frac{1}{2} \int_{2\pi/3}^{4\pi/3} (3 + 6 \cos(\theta))^2 \, d\theta $$

Step-by-Step Calculation of the Inner Loop Area

Expanding the Integrand

$$
(3 + 6 \cos \theta)^2 = 9 + 36 \cos \theta + 36 \cos^2 \theta
$$

Thus, the area formula becomes:

$$
A_{inner} = \frac{1}{2} \int_{2\pi/3}^{4\pi/3} \left(9 + 36 \cos \theta + 36 \cos^2 \theta \right) d\theta
$$

Splitting the Integral

$$
A_{inner} = \frac{1}{2} \left[\int_{2\pi/3}^{4\pi/3} 9\, d\theta + \int_{2\pi/3}^{4\pi/3} 36 \cos \theta\, d\theta + \int_{2\pi/3}^{4\pi/3} 36 \cos^2 \theta\, d\theta \right]
$$

Calculate each term separately.

Integral of the Constant Term

$$
\int 9\, d\theta = 9\theta
$$

Evaluate from $2\pi/3$ to $4\pi/3$:

$$
9 \left(\frac{4\pi}{3} - \frac{2\pi}{3} \right) = 9 \times \frac{2\pi}{3} = 9 \times \frac{2\pi}{3} = 6\pi
$$

Integral of the Cosine Term

$$
\int 36 \cos \theta\, d\theta = 36 \sin \theta
$$

Evaluate from $2\pi/3$ to $4\pi/3$:

$$
36 \left(\sin \frac{4\pi}{3} - \sin \frac{2\pi}{3} \right)
$$

Recall:

- $\sin \frac{2\pi}{3} = \frac{\sqrt{3}}{2}$
- $\sin \frac{4\pi}{3} = -\frac{\sqrt{3}}{2}$

So:

$$
36 \left(-\frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2} \right) = 36 \left(-\frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2} \right) = 36 \left(-\sqrt{3} \right) = -36 \sqrt{3}
$$

Integral of the $\cos^2 \theta$ Term

Use the double-angle identity:

$$
\cos^2 \theta = \frac{1 + \cos 2\theta}{2}
$$

Thus:

$$
36 \cos^2 \theta = 36 \times \frac{1 + \cos 2\theta}{2} = 18 (1 + \cos 2\theta)
$$

The integral becomes:

$$
\int 18 (1 + \cos 2\theta) d\theta = 18 \int 1\, d\theta + 18 \int \cos 2\theta\, d\theta
$$

Calculate separately:

- $\int 1\, d\theta = \theta$
- \(\int \cos 2\theta\, d\

Frequently Asked Questions

What is the polar equation of the inner loop in R = 3 + 6 Cos(θ)?

The given equation is R = 3 + 6 Cos(θ). To find the inner loop, analyze the values of R for different θ; the inner loop occurs where R becomes negative or forms a loop shape, typically for certain ranges of θ.

How do you use a graphing utility to graph the polar equation R = 3 + 6 Cos(θ)?

Set the graphing utility to polar mode, input the equation R = 3 + 6 Cos(θ), and adjust the viewing window to appropriately display the entire graph, including the inner loop.

What steps are involved in finding the area enclosed by the inner loop of the polar graph R = 3 + 6 Cos(θ)?

Identify the θ-values where the inner loop occurs, then set up the integral for area: (1/2) ∫ R(θ)^2 dθ over the interval covering the inner loop, and evaluate the integral.

How do you determine the limits of integration for the inner loop of R = 3 + 6 Cos(θ)?

Solve R = 0 to find the θ-values where the loop begins and ends. These θ-values serve as the limits of integration for calculating the inner loop's area.

What is the formula for calculating the area of a region bounded by a polar curve?

The area A is given by A = (1/2) ∫ from θ1 to θ2 of R(θ)^2 dθ, where θ1 and θ2 are the bounds of the region.

Can you explain why the inner loop of R = 3 + 6 Cos(θ) appears in the graph?

The inner loop appears because for certain θ values, the radius R becomes negative, which when plotted in polar coordinates results in a loop inside the main shape due to the symmetry of the cosine function.

What are the key features of the graph of R = 3 + 6 Cos(θ)?

The graph is a limaçon with an inner loop, symmetric about the polar axis, with maximum and minimum radii at specific θ values, illustrating the inner loop caused by the cosine term.

How do you interpret the area calculation in terms of the graph's shape?

The area calculation focuses on the region enclosed by the inner loop, providing insight into the size of the loop relative to the entire graph and is obtained by integrating over the appropriate θ range.

What tools or features should be used in a graphing utility to accurately plot and analyze R = 3 + 6 Cos(θ)?

Use the polar mode, set appropriate window parameters, utilize the trace or calculate features to identify key points, and perform definite integrals for area calculations.

What is the final step to find the area of the inner loop of the polar graph R = 3 + 6 Cos(θ)?

Calculate the definite integral of (1/2) R(θ)^2 between the θ-values where R = 0, to obtain the precise area of the inner loop.

Additional Resources

Use A Graphing Utility To Graph The Polar Equation. Inner Loop Of R = 3 + 6 Cos(θ). Find The Area Of The Inner Loop.

Graphing polar equations can be a fascinating way to visualize complex curves and understand their properties better. In particular, the polar equation R = 3 + 6 Cos(θ) presents an interesting case with its inner loop feature, creating a shape that is both mathematically intriguing and visually captivating. Using a graphing utility to plot this equation allows students and mathematicians alike to explore the shape dynamically, analyze its features, and ultimately compute the area of its inner loop efficiently. This article offers a comprehensive guide to using graphing tools for this specific equation, explaining the process step-by-step, discussing the mathematical background, and providing insights into calculating the area of the inner loop.

Understanding the Polar Equation R = 3 + 6 Cos(θ)

Before diving into graphing and calculations, it's essential to understand the nature of the given polar equation.

Polar Coordinates and Equations

Polar coordinates describe points in the plane using a radius R (distance from the origin) and an angle θ (measured from the positive x-axis). Equations in polar form, such as $R = f(\theta)$, define the radius as a function of the angle, and plotting these equations reveals curves with diverse shapes.

The Shape of R = 3 + 6 Cos(θ)

This particular equation is a type of limacon, characterized by its inner loop due to the coefficients involved. The general form $R = a + b \cos(\theta)$ produces various shapes depending on the ratio of a to b:

- If $|b| > a$, the limacon has an inner loop.
- If $|b| = a$, the limacon exhibits a cardioid shape.
- If $|b| < a$, the shape resembles a dimpled or convex limacon without an inner loop.

In our case, since $6 > 3$, the graph will feature an inner loop, which is the focus of this exploration.

Using a Graphing Utility to Plot the Equation

Graphing utilities, such as Desmos, GeoGebra, or graphing calculator apps, are invaluable for visualizing polar equations. They help verify the shape, identify key features, and facilitate further calculations like area.

Steps to Graph R = 3 + 6 Cos(θ)

1. Select a Graphing Tool:
- Desmos (online and free)
- GeoGebra (online or desktop)
- Graphing calculator (e.g., TI-83/84, Casio)

2. Input the Equation:
- For Desmos:
- Use the polar mode and input `r = 3 + 6 cos(θ)` directly.
- Ensure the θ variable is set in radians; most tools default to radians.

3. Adjust the Viewing Window:
- Set θ from 0 to 2π (0 to approximately 6.28 radians).
- Set r to a range that comfortably includes the maximum and minimum radii, e.g., from -6 to 9.

4. Analyze the Graph:

- Observe the inner loop clearly.
- Note the symmetry about the polar axis.
- Identify key points where the curve intersects the origin or reaches maximum/minimum radii.

5. Refine for Clarity:
- Use gridlines and labels.
- Zoom in/out as needed to capture the entire shape.

Mathematical Analysis of the Inner Loop

Understanding the inner loop’s properties helps in calculating its area accurately.

Finding the Range of R

Since R = 3 + 6 Cos(θ), the maximum and minimum values of R occur at specific θ:

- Maximum R: When Cos(θ) = 1,
- $R_{max} = 3 + 6(1) = 3 + 6 = 9$
- Minimum R: When Cos(θ) = -1,
- $R_{min} = 3 + 6(-1) = 3 - 6 = -3$

Negative R values indicate points plotted in the direction opposite to the angle θ, which is typical in polar graphs and signifies the inner loop.

Locating the Inner Loop

- The inner loop exists where R takes on negative values; thus, for R < 0:
- 3 + 6 Cos(θ) < 0
- 6 Cos(θ) < -3
- Cos(θ) < -0.5

- The solutions for θ where Cos(θ) = -0.5 are:
- θ = 2π/3 and 4π/3

- Therefore, the inner loop occurs for θ in the interval:
- θ ∈ (2π/3, 4π/3)

This interval captures the angles where the curve loops inward, crossing through the origin and creating the inner loop structure.

Calculating the Area of the Inner Loop

The area enclosed by a polar curve R = f(θ) from θ = α to θ = β is given by:

\[A = \frac{1}{2} \int_{\alpha}^{\beta} [f(θ)]^2 dθ \]

Since the inner loop occurs between θ = 2π/3 and θ = 4π/3, the area of the inner loop is:

\[A_{inner} = \frac{1}{2} \int_{2\pi/3}^{4\pi/3} [3 + 6 \cos(θ)]^2 dθ \]

Step-by-step Calculation:

1. Expand the integrand:

\[
[3 + 6 \cos(θ)]^2 = 9 + 2 \times 3 \times 6 \cos(θ) + 36 \cos^2(θ) = 9 + 36 \cos(θ) + 36 \cos^2(θ)
\]

2. Set up the integral:

\[
A_{inner} = \frac{1}{2} \int_{2\pi/3}^{4\pi/3} \left(9 + 36 \cos(θ) + 36 \cos^2(θ) \right) dθ
\]

3. Break into separate integrals:

\[
A_{inner} = \frac{1}{2} \left[\int_{2\pi/3}^{4\pi/3} 9 dθ + \int_{2\pi/3}^{4\pi/3} 36 \cos(θ) dθ + \int_{2\pi/3}^{4\pi/3} 36 \cos^2(θ) dθ \right]
\]

4. Compute each integral:

- \(\int 9 dθ = 9θ\)

- \(\int 36 \cos(θ) dθ = 36 \sin(θ)\)

- \(\int 36 \cos^2(θ) dθ\):

Use the identity \(\cos^2(θ) = \frac{1 + \cos(2θ)}{2}\):

\[
36 \times \frac{1 + \cos(2θ)}{2} = 18 + 18 \cos(2θ)
\]

So,

\[

\int 36 \cos^2(θ) dθ = \int (18 + 18 \cos(2θ)) dθ = 18θ + 9 \sin(2θ)
\]

5. Evaluate each at the bounds:

- For θ = 4π/3:

- \(9θ = 9 \times \frac{4π}{3} = 12π\)

- \(36 \sin(θ) = 36 \sin(\frac{4π}{3}) = 36 \times -\frac{\sqrt{3}}{2} = -18 \sqrt{3}\)

- \(18θ = 18 \times \frac{4π}{3} = 24π\)

- \(9 \sin(2θ) = 9 \sin(\frac{8π}{3})\)

Since \(\sin(\frac{8π}{3}) = \sin(2π + \frac{2π}{3}) = \sin(\frac{2π}{3}) = \frac{\sqrt{3}}{2}\)

So, \(9 \times \frac{\sqrt{3}}{2} = \frac{9 \sqrt{3}}{2}\)

- For θ = 2π/3:

- \(9θ = 9 \times \frac{2π}{3} = 6π\)

- \(36 \sin(2π/3) = 36 \times \frac{\sqrt{3}}{2} = 18 \sqrt{3}\)

- \(18θ = 18 \times \frac{2π}{3} = 12π\)

- \(9 \sin(\frac{4π}{3}) = 9 \times -\frac{\sqrt{3}}{2} = -

[Use A Graphing Utility To Graph The Polar Equation Inner Loop Of R 3 6 Cos Find The Area Of The]()

Use A Graphing Utility To Graph The Polar Equation Inner Loop Of R 3 6 Cos Find The Area Of The

Related Articles

- [T/F: An Electromagnetic Radiation With A Wavelength Of 258 Nm Has A More Energy Than An Electromagnetic]()
- [The Angular Size Of The Crab Snr Is 4 2 And Its Distance From Earth Is Approxi- Mately 2000 Pc. Estimae]()
- [The Five Factor Model Of Personality Includes All Of The Following As Personality Dimensions EXCEPT:(a)]()

[Back to Home]()