Use The Given Conditions To Find The Exact Values Of Sin(2), Cos(2), And Tan(20) Using The Double-angle

Use The Given Conditions To Find The Exact Values Of Sin(2), Cos(2), And Tan(20) Using The Double-angle

Understanding how to determine the exact values of trigonometric functions such as sine, cosine, and tangent is fundamental in mathematics, especially in the study of angles and their properties. The double-angle formulas are powerful tools that help simplify and solve problems involving these functions. In this article, we will explore how to use the double-angle identities to find the exact values of sin(2), cos(2), and tan(20) degrees, given specific conditions or known values. By mastering these techniques, students and enthusiasts can enhance their problem-solving skills and deepen their understanding of trigonometry.

Understanding Double-Angle Formulas in Trigonometry

Double-angle formulas are identities that express trigonometric functions of double angles in terms of functions of the original angles. These formulas are vital because they allow us to simplify complex expressions and solve for unknown values when certain conditions are known.

Key Double-Angle Identities

The primary double-angle formulas are as follows:

1. Sine Double-Angle Formula:

$$
\sin(2\theta) = 2 \sin \theta \cos \theta
$$

2. Cosine Double-Angle Formula:

$$
\cos(2\theta) = \cos^2 \theta - \sin^2 \theta
$$

Alternatively, expressible as:

$$
\cos(2\theta) = 2 \cos^2 \theta - 1
$$

or

$$
\cos(2\theta) = 1 - 2 \sin^2 \theta
$$

3. Tangent Double-Angle Formula:

$$
\tan(2\theta) = \frac{2 \tan \theta}{1 - \tan^2 \theta}
$$

These identities are essential in deriving exact values of trigonometric functions for double angles when the value of the original angle is known or can be deduced.

Applying Conditions to Find Exact Values

Let's assume we are given specific conditions or known values related to certain angles, which enable us to determine the exact values of $\sin 2^\circ$, $\cos 2^\circ$, and $\tan 20^\circ$ using these double-angle formulas.

Using Known Values to Find $\sin 2^\circ$ and $\cos 2^\circ$

Suppose we are provided with the value of $\sin 1^\circ$ or $\cos 1^\circ$, or perhaps the sine or cosine of a related angle, and we want to find $\sin 2^\circ$ and $\cos 2^\circ$. Here's a step-by-step approach:

Step 1: Use the Double-Angle Formulas

Recall:

$$
\sin(2^\circ) = 2 \sin 1^\circ \cos 1^\circ
$$

$$
\cos(2^\circ) = \cos^2 1^\circ - \sin^2 1^\circ
$$

or equivalently,

$$
\cos(2^\circ) = 2 \cos^2 1^\circ - 1
$$

Step 2: Find $\sin 1^\circ$ and $\cos 1^\circ$

While these are not standard angles with exact radical forms, in some cases, known approximate values or identities can help. Alternatively, if the problem provides $\sin 1^\circ$ or $\cos 1^\circ$ explicitly, they can be substituted directly.

Step 3: Calculate $\sin 2^\circ$ and $\cos 2^\circ$

Using the known or given values, compute the double-angle values:

- For example, if $\sin 1^\circ \approx 0.017452$ and $\cos 1^\circ \approx 0.999848$:

$$
\sin 2^\circ = 2 \times 0.017452 \times 0.999848 \approx 0.0349
$$

$$
\cos 2^\circ = 2 \times (0.999848)^2 - 1 \approx 2 \times 0.999696 - 1 = 1.999392 - 1 = 0.999392
$$

Note: These are approximate; for exact values, algebraic expressions or radical forms are needed, which are often complicated for non-standard angles like 1° or 2°.

Finding $\tan 20^\circ$ Using the Double-Angle Formula

The tangent function can be expressed in terms of sine and cosine:

$$
\tan \theta = \frac{\sin \theta}{\cos \theta}
$$

To find $\tan 20^\circ$, one approach is to relate it to the double angle of a known angle, say 10°, since:

$$
20^\circ = 2 \times 10^\circ
$$

Step 1: Find $\tan 10^\circ$

If $\tan 10^\circ$ is known or can be approximated, then:

$$
\tan 20^\circ = \frac{2 \tan 10^\circ}{1 - \tan^2 10^\circ}
$$

This is derived directly from the tangent double-angle identity.

Step 2: Use known value or approximation of $\tan 10^\circ$

Approximate value:

$$
\tan 10^\circ \approx 0.1763
$$

Then,

$$
\tan 20^\circ \approx \frac{2 \times 0.1763}{1 - (0.1763)^2} = \frac{0.3526}{1 - 0.0311} = \frac{0.3526}{0.9689} \approx 0.364
$$

Note: For exact calculations, radicals or algebraic expressions involving special angles are necessary, but for most practical purposes, approximate decimal values suffice.

Exact Values of Trigonometric Functions for Special Angles

In many cases, exact values for certain angles are well known due to their special properties and constructions. For example:

- $\sin 30^\circ = \frac{1}{2}$
- $\cos 60^\circ = \frac{1}{2}$
- $\sin 45^\circ = \frac{\sqrt{2}}{2}$
- $\cos 45^\circ = \frac{\sqrt{2}}{2}$
- $\sin 60^\circ = \frac{\sqrt{3}}{2}$
- $\cos 30^\circ = \frac{\sqrt{3}}{2}$

However, for angles like 20°, 2°, or 1°, exact radical forms are more complex and often involve nested radicals or are expressed through their sine and cosine values derived from multiple-angle identities.

Practical Examples and Problem-Solving Strategies

To effectively apply the double-angle formulas, consider the following strategies:

Example 1: Find $\sin 2^\circ$ given $\sin 1^\circ$

- Use $\sin(2^\circ) = 2 \sin 1^\circ \cos 1^\circ$.
- If $\sin 1^\circ$ and $\cos 1^\circ$ are known or approximated, substitute and compute.

Example 2: Find $\cos 2^\circ$ given $\cos 1^\circ$

- Use $\cos(2^\circ) = 2 \cos^2 1^\circ - 1$.
- Substitute the known or approximated value to find $\cos 2^\circ$.

Example 3: Find $\tan 20^\circ$ from $\tan 10^\circ$

- Use $\tan 20^\circ = \frac{2 \tan 10^\circ}{1 - \tan^2 10^\circ}$.
- If $\tan 10^\circ$ is known, compute directly.

Conclusion

Using the given conditions and the double-angle formulas effectively allows us to find the exact or approximate values of $\sin 2^\circ$, $\cos 2^\circ$, and $\tan 20^\circ$. The key steps involve understanding the identities, substituting known values, and simplifying expressions accordingly. While for some angles exact radical forms are complex, approximations provide practical solutions in most scenarios. Mastery of these identities enhances problem-solving efficiency in trigonometry and supports more advanced mathematical explorations.

Additional Tips for Trigonometric Calculations

- Always verify whether the angle is in degrees or radians before applying formulas.
- Use known special angles where possible to simplify calculations.
- When approximate values are acceptable, utilize calculator values with appropriate precision.
-

Frequently Asked Questions

How can I use the double-angle formulas to find sin(2), cos(2), and tan(20) if I know certain initial conditions?

You can apply the double-angle formulas: $\sin(2\theta) = 2\sin\theta\cos\theta$, $\cos(2\theta) = \cos^2\theta - \sin^2\theta$, and $\tan(2\theta) = 2\tan\theta / (1 - \tan^2\theta)$. By knowing the values of $\sin\theta$ and $\cos\theta$ or $\tan\theta$ for a specific angle, you can compute the exact double-angle values accordingly.

If given the values of sin(θ) and cos(θ), how do I find sin(2θ), cos(2θ), and tan(2θ)?

Use the double-angle formulas: $\sin(2\theta) = 2\sin\theta\cos\theta$, $\cos(2\theta) = \cos^2\theta - \sin^2\theta$, and $\tan(2\theta) = 2\tan\theta / (1 - \tan^2\theta)$. Substitute the known values of $\sin\theta$, $\cos\theta$, or $\tan\theta$ into these formulas to find the exact double-angle values.

What are the double-angle formulas for sine, cosine, and tangent, and how do they help in finding the exact values?

The formulas are: $\sin(2\theta) = 2\sin\theta\cos\theta$, $\cos(2\theta) = \cos^2\theta - \sin^2\theta$, and $\tan(2\theta) = 2\tan\theta / (1 - \tan^2\theta)$. They allow you to compute the double-angle values directly from known single-angle values, providing a method to find exact values when certain conditions are given.

Given that cos(θ) = x and sin(θ) = y, how do I find sin(2θ), cos(2θ), and tan(2θ)?

Calculate $\sin(2\theta)$ as $2xy$, $\cos(2\theta)$ as $x^2 - y^2$, and $\tan(2\theta)$ as $(2xy) / (x^2 - y^2)$. These are direct applications of the double-angle formulas, using the known values of $\sin\theta$ and $\cos\theta$.

How do I find the exact value of tan(20°) using double-angle formulas if I know the value of tan(10°)?

Since 20° is double 10°, you can use $\tan(2\theta) = 2\tan\theta / (1 - \tan^2\theta)$. Substitute $\tan(10°)$ into this formula to find $\tan(20°)$. This method leverages the double-angle identity to obtain the exact tangent value.

Additional Resources

Use The Given Conditions To Find The Exact Values Of Sin(2), Cos(2), And Tan(20) Using The Double-angle

In the realm of trigonometry, the quest to determine exact values of sine, cosine, and tangent for specific angles is both fundamental and intriguing. These values are essential not only for theoretical mathematics but also for practical applications spanning engineering, physics, and computer science. Today, we delve into a method that leverages the power of the double-angle formulas to find the precise values of sin(2), cos(2), and tan(20). By understanding the underlying principles and applying them meticulously, we can transform complex problems into manageable calculations, unraveling the exact trigonometric values that might initially seem elusive.

Understanding the Foundations: Trigonometric Double-Angle Formulas

Before we proceed to compute the values, it is crucial to understand the core formulas that allow us to relate the sine, cosine, and tangent of an angle to those of its double.

The Double-Angle Formulas

The double-angle formulas are derived from the sum formulas of sine and cosine. They are fundamental in simplifying expressions involving angles twice as large as known quantities. The main formulas are:

- Sine double-angle formula:

$$
\sin(2\theta) = 2 \sin \theta \cos \theta
$$

- Cosine double-angle formula:

$$
\cos(2\theta) = \cos^2 \theta - \sin^2 \theta = 2 \cos^2 \theta - 1 = 1 - 2 \sin^2 \theta
$$

- Tangent double-angle formula:

$$
\tan(2\theta) = \frac{2 \tan \theta}{1 - \tan^2 \theta}
$$

These formulas are invaluable because they express the double of an angle in terms of the original angle's sine and cosine or tangent, enabling recursive calculations or simplifications based on known values.

Using Conditions to Find Sin(2) and Cos(2)

Suppose we are given specific conditions or known values related to an angle θ, which allow us to compute sin θ and cos θ, and consequently find sin(2θ) and cos(2θ).

Step 1: Identify the Known Conditions

For the purpose of this example, assume the following conditions:

- $\sin \theta = a$
- $\cos \theta = b$
- The values of a and b are known or can be derived from given conditions, such as a right triangle or algebraic equations.

Alternatively, suppose the problem states:

- $\sin \theta = \frac{3}{5}$
- $\cos \theta = \frac{4}{5}$

which are common Pythagorean triplet ratios.

Step 2: Compute Sin(2θ) and Cos(2θ)

Using the double-angle formulas:

- For $\sin(2\theta)$:

$$
\sin(2\theta) = 2 \sin \theta \cos \theta
$$

Plugging in the known values:

$$
\sin(2\theta) = 2 \times \frac{3}{5} \times \frac{4}{5} = 2 \times \frac{12}{25} = \frac{24}{25}
$$

- For $\cos(2\theta)$:

Using the formula:

$$
\cos(2\theta) = \cos^2 \theta - \sin^2 \theta
$$

Plugging in the values:

$$
\cos(2\theta) = \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2 = \frac{16}{25} - \frac{9}{25} = \frac{7}{25}
$$

Alternatively, the other forms:

$$
\cos(2\theta) = 2 \cos^2 \theta - 1 = 2 \times \frac{16}{25} - 1 = \frac{32}{25} - 1 = \frac{7}{25}
$$

which confirms our calculation.

Calculating Tan(20): Applying the Double-Angle Formula

The tangent of 20 degrees (or radians, depending on the context) can be approached by considering the angle as half of 40 degrees, or by using the tangent double-angle formula if the tangent of a related angle is known.

Step 1: Recognize the Relationship

Suppose we are told that:

$$
\theta = 10^\circ
$$

and we seek $\tan(20^\circ)$.

Alternatively, if the problem involves different conditions, such as knowing $\tan \theta$ and wanting $\tan 2\theta$, then the formula:

$$
\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}
$$

becomes directly applicable.

Step 2: Apply the Double-Angle Formula

- If $\tan \theta$ is known:

For example, suppose $\tan \theta = t$. Then:

$$
\tan 2\theta = \frac{2t}{1 - t^2}
$$

- If $\theta = 10^\circ$ and $\tan 10^\circ$ is known:

Using a calculator or known tables, $\tan 10^\circ \approx 0.1763$.

Plugging into the formula:

$$
\tan 20^\circ = \frac{2 \times 0.1763}{1 - (0.1763)^2} \approx \frac{0.3526}{1 - 0.0311} = \frac{0.3526}{0.9689} \approx 0.3640
$$

which aligns with the known approximate value of $\tan 20^\circ \approx 0.3640$.

Exact Values of $\sin 20^\circ$, $\cos 20^\circ$, and \(\tan

20^\circ\)

While the previous calculations involve approximations, deriving exact values for angles like 20 degrees often requires recognizing special angle relationships or expressing the angles in terms of known exact values.

Expressing 20° Using Known Angles

- Note that $20^\circ = 60^\circ - 40^\circ$
- Or, $20^\circ = 45^\circ - 25^\circ$

However, these do not correspond to standard angles with simple radical expressions.

Alternatively, since 20° is not a standard special angle (like 30°, 45°, 60°), to derive exact values, one might:

- Use half-angle formulas, which relate $\sin 20^\circ$ and $\cos 20^\circ$ to $\sin 40^\circ$ and $\cos 40^\circ$.
- Express $\sin 20^\circ$ as:

$$
\sin 20^\circ = \sqrt{\frac{1 - \cos 40^\circ}{2}}
$$

- Similarly, $\cos 20^\circ$:

$$
\cos 20^\circ = \sqrt{\frac{1 + \cos 40^\circ}{2}}
$$

But to proceed, we need $\cos 40^\circ$.

Using Double-Angle to Find $\cos 40^\circ$

If we already know $\cos 20^\circ$, then:

$$
\cos 40^\circ = 2 \cos^2 20^\circ - 1
$$

Alternatively, if $\cos 40^\circ$ is known or can be derived, these formulas yield exact radical expressions involving nested radicals.

Note: Exact radical forms for $\sin 20^\circ$, $\cos 20^\circ$, and $\tan 20^\circ$ are complicated and typically involve solving polynomial equations derived from these identities, which often leads to expressions with roots of quadratic or quartic polynomials.

Conclusion: The Power of Double-Angle Formulas in Trigonometry

The process of determining exact values of $\sin 2$, $\cos 2$, and $\tan 20$ using double-angle formulas underscores the versatility and power of fundamental trigonometric identities. Whether starting from known angles or leveraging given conditions, these formulas provide a systematic pathway to precise calculations. They are particularly valuable when dealing with angles that are multiples or fractions of standard angles, enabling mathematicians and students alike to navigate complex problems with clarity and confidence.

In practice, the approach involves:

- Recognizing the given conditions or known values.
- Applying the double-angle formulas directly to compute \(\sin 2\theta\), \(\cos 2\

Use The Given Conditions To Find The Exact Values Of Sin 2 Cos 2 And Tan 20 Using The Double Angle

Use The Given Conditions To Find The Exact Values Of Sin 2 Cos 2 And Tan 20 Using The Double Angle

Related Articles

- Suppose The Central Bank In Interested Is Decreasing Money Supply By $2000. Assume The Money Multiplier
- Suppose That The Exchange Rate Falls From 84 Yen Per U.S. Dollar To 71 Yen Per U.S. Dollar. What Is The
- Terri Is Starting A Babysitting Service To Make Some Extra Money. She Plans To Charge A Base Fee Of $10

Back to Home