What Is The Equation Of The Line That Is Parallel To The Line 5x + 2y = 12 And Passes Through The Point

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Understanding how to find the equation of a line parallel to a given line and passing through a specific point is a fundamental skill in coordinate geometry. Whether you're a student preparing for exams, a teacher designing lesson plans, or someone interested in mathematical problem-solving, grasping this concept is essential. This article will guide you through the process step-by-step, explaining key concepts, formulas, and methods to determine the desired line's equation.

Understanding the Basics of Line Equations

Before diving into the problem, it's important to understand the foundational concepts related to line equations, slopes, and parallel lines.

Standard Form of a Line Equation

The line given in the problem is expressed as:

$$ 5x + 2y = 12 $$

This is known as the standard form of a line. In general, the standard form is:

$$ Ax + By = C $$

where:

- $A, B,$ and C are constants,
- x and y are variables representing points on the line.

Slope of a Line

The slope of a line indicates its steepness and direction. It is typically denoted as m.

- For a line in the form $y = mx + b$, the slope is directly given as m.
- For lines in standard form $Ax + By = C$, the slope can be found by rearranging the equation into slope-intercept form:

$$y = -\frac{A}{B}x + \frac{C}{B}$$

Thus, the slope m is $-\frac{A}{B}$.

Parallel Lines and Their Slopes

Two lines are parallel if they have identical slopes but different y-intercepts. Therefore:

- Parallel lines share the same slope.
- The only difference between such lines is their position in the plane.

Step 1: Find the Slope of the Given Line

Given the line:

$$5x + 2y = 12$$

Let's find its slope.

Rearranging to Slope-Intercept Form

To find the slope, solve for y:

$$
\begin{aligned}
5x + 2y &= 12 \\
2y &= -5x + 12 \\
y &= -\frac{5}{2}x + 6
\end{aligned}
$$

From this, the slope of the initial line is:

$$m = -\frac{5}{2}$$

Step 2: Determine the Slope of the Parallel Line

Since parallel lines have the same slope:

$$ m_{\text{parallel}} = -\frac{5}{2} $$

This is a crucial step because the slope directly influences the form of the new line's equation.

Step 3: Use the Point-Slope Form to Find the Equation

Suppose the line must pass through a specific point, say (x_1, y_1). The general point-slope form is:

$$ y - y_1 = m(x - x_1) $$

- (x_1, y_1) is the given point.
- m is the slope of the line.

Example: Let's assume the point given is $(x_1, y_1) = (x_0, y_0)$. For illustration, suppose the point is $(3, 4)$. (Note: Replace with the actual point provided in the problem.)

The equation becomes:

$$ y - 4 = -\frac{5}{2}(x - 3) $$

Step 4: Write the Equation in Slope-Intercept or Standard Form

To express the line's equation neatly, expand and simplify:

\[
\begin{aligned}
y - 4 &= -\frac{5}{2}x + \frac{15}{2} \\
y &= -\frac{5}{2}x + \frac{15}{2} + 4 \\
y &= -\frac{5}{2}x + \frac{15}{2} + \frac{8}{2} \\
y &= -\frac{5}{2}x + \frac{23}{2}

\end{aligned}
\]

This is the slope-intercept form of the line passing through \((3, 4) \) and parallel to the original line.

Alternatively, in standard form:

\[
\begin{aligned}
y &= -\frac{5}{2}x + \frac{23}{2} \\
2y &= -5x + 23 \\
5x + 2y &= 23
\end{aligned}
\]

Additional Considerations and Variations

Depending on the specific point provided, the calculations will vary slightly. The key steps remain:

- Find the slope of the original line.
- Use the point-slope form with the given point.
- Convert the resulting equation into the desired form.

Sample Problems and Solutions

Problem 1:

Find the equation of the line parallel to \(5x + 2y = 12 \) that passes through the point \((4, 7) \).

Solution:

1. Find the slope of the given line:

\[
y = -\frac{5}{2}x + 6 \Rightarrow m = -\frac{5}{2}
\]

2. Use the point-slope form:

$$
y - 7 = -\frac{5}{2}(x - 4)
$$

3. Expand:

$$
y - 7 = -\frac{5}{2}x + 10
$$

4. Solve for y:

$$
y = -\frac{5}{2}x + 10 + 7 = -\frac{5}{2}x + 17
$$

Final equation:

$$
y = -\frac{5}{2}x + 17
$$

or in standard form:

$$
5x + 2y = 34
$$

Problem 2:

Find the equation of the line parallel to $5x + 2y = 12$ passing through $(-2, 5)$.

Solution:

1. Slope:

$$
m = -\frac{5}{2}
$$

2. Point-slope form:

$$
y - 5 = -\frac{5}{2}(x + 2)
$$

3. Expand:

$$
y - 5 = -\frac{5}{2}x - 5
$$

4. Simplify:

$$
y = -\frac{5}{2}x + 0
$$

which simplifies to:

$$
y = -\frac{5}{2}x
$$

or in standard form:

$$
5x + 2y = 0
$$

Key Tips for Solving These Types of Problems

- Always start by rewriting the given line into slope-intercept form to identify its slope.
- Remember that parallel lines share the same slope.
- When using the point-slope form, plug in the point coordinates and the slope.
- To convert from point-slope to slope-intercept form, expand and simplify.
- For the standard form, clear fractions and rearrange as necessary.

Conclusion

Finding the equation of a line parallel to a given line and passing through a specific point involves understanding the slope of the original line, maintaining that slope for the new line, and then using the point-slope form to derive the equation. Mastering these steps allows you to solve a wide range of problems in coordinate geometry efficiently.

Remember:

- Identify the slope of the original line.
- Use the same slope for the new line.
- Apply the point-slope form with the given point.
- Convert the result into your preferred form (slope-intercept or standard).

With practice, determining the equation of such lines becomes straightforward, enabling you to handle complex geometric problems with confidence and precision.

Frequently Asked Questions

What is the slope of the line given by 5x + 2y = 12?

First, rewrite the equation in slope-intercept form: 2y = -5x + 12, so y = -⅟5x + 6. Therefore, the slope is -⅟5.

How do you find the equation of a line parallel to 5x + 2y = 12 passing through a specific point?

Since parallel lines have the same slope, use the slope –⅟5 and substitute the given point into the point-slope form y - y1 = m(x - x1) to find the new line's equation.

What is the general form of the equation for a line parallel to 5x + 2y = 12 passing through (x1, y1)?

The general form is y - y1 = -⅟5(x - x1). You can then rearrange to slope-intercept form if needed.

If a line passes through point (3, 4) and is parallel to 5x + 2y = 12, what is its equation?

Using the slope –⅟5 and point (3, 4), the equation is y - 4 = -⅟5(x - 3). Simplifying, y = -⅟5x + ⅟5 imes 3 + 4 = -⅟5x + frac{15}{5} + 4 = -⅟5x + 3 + 4 = -⅟5x + 7.

Can the equation of the parallel line be written in standard form?

Yes. Starting from y - y1 = -⅟5(x - x1), expand and rearrange to standard form: 5x + 2y = C, where C is a constant calculated using the point.

What is the importance of the slope in determining parallel lines?

Parallel lines have identical slopes; thus, the slope determines whether two lines are parallel or not.

How do you verify if two lines are parallel?

By comparing their slopes. If the slopes are equal, the lines are parallel.

What is the significance of the point through which the new line passes?

The point provides a specific location through which the new line must pass, allowing us to determine the particular equation of the line parallel to the given one.

Can the equation of the parallel line be different if passing through different points?

Yes. The slope remains the same, but the y-intercept changes depending on the point through which the line passes, resulting in different equations.

Additional Resources

Understanding the Equation of a Line Parallel to 5x + 2y = 12 Passing Through a Given Point

In the realm of coordinate geometry, the concept of lines and their relationships forms a foundational pillar for understanding more complex mathematical and real-world phenomena. When dealing with linear equations, one of the most intriguing topics is determining the equation of a line that is parallel to a given line and passes through a specific point. This task involves a precise understanding of the slope-intercept form, the nature of parallel lines, and algebraic manipulation to derive the desired equation. This article aims to provide a comprehensive, detailed analysis of this process, illustrating each step clearly to foster a deep understanding of the concepts involved.

Fundamental Concepts in Line Equations and Parallelism

Before delving into the specific problem, it’s essential to lay a solid foundation by understanding key concepts related to lines, their equations, and the property of parallelism.

1. The Equation of a Line in Standard and Slope-Intercept Forms

- Standard Form: The general form of a linear equation is written as $Ax + By = C$, where A, B, and C are constants, and x and y are variables representing points on the line.

- Slope-Intercept Form: The slope-intercept form is given as $y = mx + b$, where:
- 'm' is the slope of the line, indicating its steepness.
- 'b' is the y-intercept, the point where the line crosses the y-axis.

Understanding how to convert between these forms and interpret the components is crucial for analyzing line relationships.

2. The Concept of Slope and Its Significance

- The slope (m) quantifies the rate of change of y with respect to x.
- For a line in the form $Ax + By = C$, the slope can be expressed as $m = -A/B$, provided $B \neq 0$.
- Parallel lines share the same slope, which means they rise and run at the same rate but may differ in their intercepts.

3. Parallel Lines: Definition and Properties

- Definition: Two lines are parallel if they lie in the same plane and never intersect, no matter how far extended.
- Property: Parallel lines have equal slopes.
- Implication: To find a line parallel to a given line, one must determine the slope of the original line and then find an equation with the same slope passing through a specified point.

Analyzing the Given Line: $5x + 2y = 12$

The first step in solving our problem is to analyze the given line's equation and extract its slope, which will guide us in constructing the parallel line.

1. Converting to Slope-Intercept Form

Starting with the standard form:

$5x + 2y = 12$

Solve for y:

$2y = -5x + 12$

$y = (-5/2)x + 6$

This form clearly indicates:

- Slope (m) = -5/2
- Y-intercept (b) = 6

2. Interpreting the Slope

The slope of the given line is -5/2, a negative value indicating the line descends as x increases. Since the slope is rational, the line has a consistent angle and steepness.

Formulating the Equation of the Parallel Line

Having established that parallel lines share the same slope, the next step is to determine the equation of the line that is parallel to the given line and passes through a specific point, which we will specify as (x_0, y_0). This process involves several steps:

1. Identifying the Slope of the Parallel Line

- Since the original line's slope is -5/2, the parallel line must also have:

$m_{parallel} = -5/2$

2. Using the Point-Slope Form

The point-slope form of a line’s equation is:

$y - y_0 = m(x - x_0)$

where (x_0, y_0) is a point through which the line passes, and m is the slope.

- Given point: (x_0, y_0) (the specific point will be inserted later)
- Slope: m = -5/2

The equation becomes:

$y - y_0 = (-5/2)(x - x_0)$

This form is useful because it directly incorporates the point and slope.

3. Converting to Slope-Intercept or Standard Form

To express the line in a more conventional form, such as y = mx + b or Ax + By = C, algebraic manipulation is performed.

Applying the Process with a Specific Point

Suppose we are asked to find the equation of the line parallel to 5x + 2y = 12 passing through the point (x_0, y_0). For illustration, let's choose a specific point, for example, (3, 4). The process applies equally to any point.

1. Write the Point-Slope Equation

Using the point (3, 4):

y - 4 = (-5/2)(x - 3)

2. Simplify to Slope-Intercept Form

Distribute the slope:

y - 4 = (-5/2)x + (15/2)

Add 4 to both sides:

y = (-5/2)x + (15/2) + 4

Express 4 as a fraction with denominator 2:

4 = 8/2

Thus:

y = (-5/2)x + (15/2) + (8/2) = (-5/2)x + (23/2)

The equation in slope-intercept form:

y = (-5/2)x + (23/2)

This line has the same slope as the original and passes through (3, 4).

3. Convert to Standard Form (Optional)

Multiply through by 2 to clear fractions:

2y = -5x + 23

Bring all to one side:

5x + 2y = 23

This standard form explicitly shows the relationship between x and y.

Generalized Approach for Any Point

The specific example above illustrates the method, but the same principles apply broadly:

1. Identify the slope of the given line: Convert to slope-intercept form if necessary. For 5x + 2y = 12, the slope is -5/2.

2. Use the point-slope form: $y - y_0 = m(x - x_0)$, where (x_0, y_0) is the given point.

3. Simplify to the desired form: Convert into slope-intercept or standard form based on context.

4. Verify the solution: Ensure the point lies on the line and the slope matches the original.

Significance and Applications of Parallel Lines in Geometry and Real Life

Understanding how to find the equation of a line parallel to a given line passing through a specific point is not just an academic exercise; it has numerous practical applications:

- Engineering and Architecture: Designing structures where beams, walls, or supports must run parallel to existing features.
- Navigation and Mapping: Plotting routes or boundaries that are parallel to known reference lines.
- Computer Graphics: Rendering objects with parallel edges or features.
- Robotics and Path Planning: Ensuring movement paths are parallel to obstacles or predefined routes.

This skill also underpins more advanced topics such as the study of transversals, angles, and coordinate transformations, making it a cornerstone of geometric reasoning.

Conclusion: Mastering the Equation of Parallel Lines

Determining the equation of a line parallel to a given line and passing through a specific point is a fundamental skill in coordinate geometry. By understanding the properties of lines, especially the significance of slope, and mastering the algebraic techniques to manipulate equations, students and professionals alike can confidently approach such problems. The process hinges on isolating the slope from the original line, applying the point-slope formula, and then converting the resulting expression into the desired form. As demonstrated with the example involving the line $5x + 2y = 12$ passing through (3, 4), the approach is methodical, adaptable, and essential for both academic pursuits and real-world applications, underscoring the importance of geometric literacy in diverse fields.

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