

1. For The Circuit Below: A. (10%) Find The Differential Equation That Relates Input And Output.

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Understanding how to derive the differential equation that governs an electrical circuit is fundamental in analyzing and designing systems such as filters, oscillators, and control systems. When given a circuit with resistors, capacitors, and inductors, the goal is to establish a mathematical relationship between the input voltage (or current) and the output voltage (or current). This process involves applying fundamental principles like Kirchhoff's Voltage Law (KVL) and Kirchhoff's Current Law (KCL), along with the constitutive relations of circuit elements.

In this article, we will explore the systematic approach to deriving the differential equation for a given circuit, starting from the basic circuit analysis principles and progressing toward the formulation of the differential equation. Although the actual circuit diagram is not provided here, the methodology remains applicable to a variety of common RLC circuit configurations.

Understanding the Circuit Components and Their Behavior

Before delving into the derivation, it is essential to understand the behaviors of the basic circuit elements involved, as they form the foundation of the differential equations.

Resistors (R)

- Resistors obey Ohm's law: $V = R \times I$
- The voltage across a resistor is proportional to the current flowing through it.
- Resistors do not store energy; they dissipate it as heat.

Capacitors (C)

- Capacitors store energy in the electric field.
- The voltage across a capacitor relates to the charge Q : $V_C = \frac{Q}{C}$
- The current through a capacitor is proportional to the rate of change of voltage: $I_C = C \frac{dV_C}{dt}$

Inductors (L)

- Inductors store energy in the magnetic field.
- The voltage across an inductor relates to the current (I_L) : $(V_L = L \frac{dI_L}{dt})$
- The current through an inductor is proportional to the integral of voltage over time.

Step-by-Step Approach to Deriving the Differential Equation

The process of deriving the differential equation involves several key steps:

1. Identify the Input and Output Variables

- Determine which voltage or current is considered the input and which is the output.
- For example, if the input is an applied voltage (V_{in}) , and the output is the voltage across a capacitor (V_{out}) , these variables form the basis of the analysis.

2. Assign Circuit Variables and Labels

- Label each circuit node, branch, and element.
- Assign variables to unknown quantities, such as node voltages and branch currents.

3. Apply Kirchhoff's Laws

- Use KVL to write equations around loops.
- Use KCL to write equations at nodes where currents meet.

4. Write Constitutive Relations for Each Element

- Express voltages and currents for resistors, capacitors, and inductors according to their behavior.

5. Formulate the Differential Equation

- Combine all the equations to eliminate auxiliary variables.
- Express the relationship explicitly between input and output variables, resulting in a differential equation.

Example: Deriving the Differential Equation for a Series RLC Circuit

To illustrate the process, consider a classic series RLC circuit with an input voltage $V_{in}(t)$ applied across a resistor (R) , inductor (L) , and capacitor (C) connected in series. The output voltage $V_{out}(t)$ could be taken across the capacitor.

Step 1: Identify Variables

- Input: $V_{in}(t)$
- Output: $V_{out}(t)$ (voltage across capacitor)

Step 2: Write KVL Equation

Applying Kirchhoff's Voltage Law around the loop:

$$V_{in}(t) = V_R(t) + V_L(t) + V_C(t)$$

Step 3: Express Element Voltages

- Resistor: $V_R(t) = R \times I(t)$
- Inductor: $V_L(t) = L \frac{dI(t)}{dt}$
- Capacitor: $V_C(t) = \frac{Q(t)}{C}$

Since current $I(t)$ relates to the charge $Q(t)$ as $I(t) = \frac{dQ(t)}{dt}$, and the voltage across the capacitor is $V_{out}(t) = V_C(t) = \frac{Q(t)}{C}$.

Step 4: Write Equations in Terms of Output Voltage

Express the current in terms of V_{out} :

$$Q(t) = C V_{out}(t)$$
$$I(t) = \frac{dQ(t)}{dt} = C \frac{dV_{out}(t)}{dt}$$

Now, rewrite the KVL:

$$V_{in}(t) = R \times I(t) + L \frac{dI(t)}{dt} + V_{out}(t)$$

Substitute $I(t)$:

$$V_{in}(t) = R C \frac{dV_{out}(t)}{dt} + L \frac{d}{dt} \left(C \frac{dV_{out}(t)}{dt} \right) + V_{out}(t)$$
$$V_{in}(t) = R C \frac{dV_{out}(t)}{dt} + L C \frac{d^2 V_{out}(t)}{dt^2} + V_{out}(t)$$

Step 5: Rearrange to Standard Form

Rearranged, the differential equation becomes:

$$L C \frac{d^2 V_{\text{out}}(t)}{dt^2} + R C \frac{dV_{\text{out}}(t)}{dt} + V_{\text{out}}(t) = V_{\text{in}}(t)$$

This is a second-order linear differential equation relating the input voltage $V_{\text{in}}(t)$ to the output voltage $V_{\text{out}}(t)$.

General Form of the Differential Equation for RLC Circuits

The example above illustrates a common form of differential equations for RLC circuits. In general, the differential equation relating input and output in an RLC circuit can be expressed as:

$$a_n \frac{d^n y(t)}{dt^n} + a_{n-1} \frac{d^{n-1} y(t)}{dt^{n-1}} + \dots + a_1 \frac{dy(t)}{dt} + a_0 y(t) = b_m \frac{d^m x(t)}{dt^m} + \dots + b_0 x(t)$$

where:

- $y(t)$ is the output variable (voltage or current),
- $x(t)$ is the input variable,
- a_i and b_j are coefficients determined by circuit element values,
- n and m are the orders of derivatives, typically dictated by the circuit's configuration.

Conclusion and Practical Applications

Deriving the differential equation that relates input and output signals in an electrical circuit is a crucial skill in electrical engineering. This process enables engineers to analyze transient responses, design filters, and predict circuit behavior under various input conditions. While the derivation can seem complex initially, following a systematic approach—identifying variables, applying Kirchhoff's laws, expressing element behaviors, and combining equations—makes the process manageable.

In practical applications, once the differential equation is obtained, engineers can solve it using analytical methods or numerical simulations to understand the circuit's response. For example, in communication systems, filters are designed based on these equations to isolate desired frequency components. Similarly, in control systems, understanding the differential relationships helps in designing controllers that ensure stability and desired performance.

Summary of Key Steps:

- Recognize the circuit configuration.
- Identify input and output variables.
- Label all circuit elements and nodes.
- Apply Kirchhoff's Voltage and Current Laws.
- Use element constitutive relations.
- Combine equations to isolate the relationship between input and output.
- Formulate the resulting differential equation.

By mastering these steps, students and practitioners can analyze a wide variety of circuits, predict their behavior, and contribute to innovative electrical and electronic system designs.

Meta Note: To solve the specific problem given in the original question, a detailed circuit diagram would be necessary. However, the methodology outlined here provides a comprehensive approach applicable to virtually any linear circuit involving resistors, capacitors, and inductors.

Frequently Asked Questions

What is the first step in deriving the differential equation for the given circuit?

The first step is to identify all circuit components and write the governing laws (Kirchhoff's Voltage and Current Laws) to relate the input and output voltages and currents before forming the differential equation.

How do the circuit elements influence the form of the differential equation?

Resistors contribute algebraic terms, while inductors and capacitors introduce derivatives of current and voltage, respectively, leading to a differential equation that includes first or higher-order derivatives depending on the circuit configuration.

What is the significance of selecting the right variables when deriving the differential equation?

Choosing appropriate variables, such as capacitor voltage or inductor current, simplifies the derivation process and results in a differential equation that accurately models the relationship between input and output signals.

Can you provide a general approach to derive the

differential equation from the circuit diagram?

Yes, the general approach involves: (1) defining the input and output variables, (2) applying Kirchhoff's laws to write equations for each loop and node, (3) expressing all variables in terms of the chosen variables, and (4) combining these equations to form a differential equation relating input and output.

Why is understanding the differential equation important in analyzing the circuit's behavior?

The differential equation encapsulates the dynamic relationship between input and output, allowing us to analyze transient responses, stability, and frequency behavior of the circuit using mathematical tools such as Laplace transforms and differential equations solutions.

Additional Resources

Understanding the Differential Equation in Electrical Circuits: A Comprehensive Review

When analyzing electrical circuits, especially those involving resistors, capacitors, and inductors, a fundamental goal is to derive the mathematical relationship between the input and output signals. This relationship is often expressed as a differential equation, which encapsulates how voltages and currents evolve over time within the circuit. This review aims to provide an in-depth understanding of how to find the differential equation that relates input and output in a given circuit, with a focus on clarity, methodology, and practical insights.

Introduction to Differential Equations in Circuits

Electrical circuits are governed by fundamental laws—Ohm's Law, Kirchhoff's Voltage Law (KVL), and Kirchhoff's Current Law (KCL). When these laws are applied to reactive components like capacitors and inductors, the relationships involve derivatives of voltages and currents, leading naturally to differential equations.

Key concepts:

- Reactive components introduce derivatives: Capacitors and inductors store energy in electric and magnetic fields, respectively, resulting in voltage-current relationships involving derivatives.
- System dynamics: The differential equations describe how the system responds over time to inputs, enabling the analysis of transient and steady-state behaviors.
- Input-output relationship: The goal is to express the output variable (voltage or current) as a function of input variables and their derivatives.

Typical Circuit Elements and Their Differential Relationships

Before deriving the differential equation, it's essential to understand the fundamental relationships of the circuit elements involved:

Resistor (R)

- Voltage-current relationship: $V_R = R \times I$
- No derivatives involved; the resistor introduces a proportional relationship.

Capacitor (C)

- Voltage-current relationship: $I_C = C \frac{dV_C}{dt}$
- The voltage across the capacitor relates to the integral of current; derivatives are involved.

Inductor (L)

- Voltage-current relationship: $V_L = L \frac{dI_L}{dt}$
- The current through the inductor relates to the integral of voltage; derivatives are involved.

Methodology for Deriving the Differential Equation

Deriving the differential equation that relates input and output involves a systematic process:

1. Identify Input and Output Variables:
 - Determine which signals are considered inputs (e.g., input voltage V_{in}) and which are outputs (e.g., output voltage V_{out}) or current).
2. Apply Circuit Laws:
 - Use KVL to sum voltages around loops.
 - Use KCL to sum currents at nodes.
3. Write Element Equations:
 - Express voltages and currents across each component using their constitutive relationships.
4. Combine Equations:

- Substitute relationships to eliminate intermediate variables, expressing everything in terms of input, output, and their derivatives.

5. Form the Differential Equation:

- Rearrange the combined equations into a standard differential equation form.

6. Identify the Order and Type:

- Determine whether the differential equation is first-order, second-order, etc., based on the highest derivative involved.

Practical Example: Deriving the Differential Equation for a Series RLC Circuit

To contextualize the process, consider a classic series RLC circuit with a sinusoidal or step input voltage $V_{in}(t)$. Suppose the goal is to find the differential equation relating the input voltage to the output voltage across the capacitor $V_C(t)$.

Step 1: Circuit Description

- Components connected in series: Resistor R , Inductor L , and Capacitor C .
- Input voltage: $V_{in}(t)$ applied across the entire series.
- Output voltage: $V_{out}(t) = V_C(t)$.

Step 2: Apply Kirchhoff's Voltage Law

- KVL states: $V_{in}(t) = V_R(t) + V_L(t) + V_C(t)$.

Step 3: Express Component Voltages

- Resistor: $V_R(t) = R I(t)$
- Inductor: $V_L(t) = L \frac{dI(t)}{dt}$
- Capacitor: $V_C(t) = \frac{1}{C} \int I(t) dt$

However, to express everything in terms of voltage and derivatives, it's often easier to work with currents or voltages, choosing the variable that simplifies the derivation.

Step 4: Choose State Variables and Write Equations

- For this circuit, selecting $V_C(t)$ as the output, and noting that $I(t) = C \frac{dV_C(t)}{dt}$ (from capacitor current-voltage relationship), gives:

$\backslash$

$$V_R(t) = R \times I(t) = R C \frac{dV_C(t)}{dt}$$

\]

\[

$$V_L(t) = L \frac{dI(t)}{dt} = L C \frac{d^2 V_C(t)}{dt^2}$$

\]

Step 5: Write the KVL Equation in Terms of $V_C(t)$

- Substituting:

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$$V_{in}(t) = R C \frac{dV_C(t)}{dt} + L C \frac{d^2 V_C(t)}{dt^2} + V_C(t)$$

\]

- Rearranged:

\[

$$L C \frac{d^2 V_C(t)}{dt^2} + R C \frac{dV_C(t)}{dt} + V_C(t) = V_{in}(t)$$

\]

Step 6: Final Differential Equation

- Dividing through by $(L C)$:

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$$\frac{d^2 V_C(t)}{dt^2} + \frac{R}{L} \frac{dV_C(t)}{dt} + \frac{1}{L C} V_C(t) = \frac{1}{L C} V_{in}(t)$$

\]

This second-order linear differential equation relates the input voltage $V_{in}(t)$ to the output voltage across the capacitor $V_C(t)$.

Analyzing the Derived Differential Equation

Understanding the form and implications of the differential equation provides insights into the circuit's behavior.

Type and Order:

- It's a second-order linear differential equation, typical for RLC circuits.

Homogeneous and Particular Solutions:

- Homogeneous part describes natural responses (resonance, damping).

- Particular solution depends on the input $V_{in}(t)$.

Physical Interpretation:

- The coefficients dictate the damping ratio and resonant frequency.
- For sinusoidal inputs, steady-state solutions reveal frequency response characteristics.

Generalization to Other Circuits and Configurations

The methodology demonstrated can be adapted to various circuit configurations:

- Voltage Divider Circuits: Derive relationships based on series or parallel resistors/reactive components.
- Filters (Low-pass, High-pass): Obtain differential equations describing filtering behavior.
- Amplifiers with Reactive Components: Model transient response and stability.

Key considerations:

- Component placement: Series vs. parallel arrangements affect the derivation.
- Input types: Step, sinusoidal, or arbitrary inputs influence the particular solution.
- Initial conditions: These are crucial for solving the differential equations completely.

Advanced Topics and Practical Tips

For complex circuits or higher-order systems, additional techniques may be necessary:

- Laplace Transform: Facilitates solving differential equations, especially with initial conditions.
- State-space analysis: Useful for multi-input/multi-output systems.
- Numerical methods: When analytical solutions are intractable, simulation tools like SPICE or MATLAB can be employed.

Practical tips:

- Clearly identify the input and output variables before starting.
- Write down all element relationships explicitly.
- Use systematic substitution to eliminate intermediate variables.
- Verify units and dimensions for consistency.
- Cross-check the derived differential equation with physical intuition (e.g., damping behavior).

Conclusion: The Significance of Deriving Differential Equations

Deriving the differential equation that relates input and output in an electrical circuit is fundamental to understanding and predicting circuit behavior, especially in transient states. It provides a mathematical framework that captures the dynamics introduced by reactive components, enabling engineers and students to analyze stability, resonance, filtering, and more.

This process, while sometimes algebraically intensive, offers deep insights into the inherent properties of the circuit. Mastery of these derivations enhances problem-solving skills and forms the backbone of control systems, signal processing, and circuit analysis disciplines.

By approaching the problem systematically—identifying variables, applying circuit laws, expressing component relationships, and combining equations—one can confidently develop the differential equations necessary for advanced circuit analysis and design.

In summary:

- Understand the fundamental relationships of resistors, capacitors, and inductors.
- Apply Kirchhoff's laws to the circuit.
- Express all voltages and currents in terms of chosen variables.
- Derive the differential equation by substitution and algebraic manipulation.
- Analyze the resulting equation to understand the circuit's behavior.

This comprehensive approach ensures a thorough

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