1. If AP = P D, Where P= [1 4 1 1] And D= [1 0 0 2], Find A5. Their Both A 2x2 -1 And 4 Are The Top Ones

1. If AP = P D, Where P= [1 4 1 1] And D= [1 0 0 2], Find A5. Their Both A 2x2 -1 And 4 Are The Top Ones

---

Understanding the Matrix Equation: AP = P D

When tackling matrix equations in linear algebra, a fundamental understanding of how matrices interact through multiplication is essential. The given problem states:

> If AP = P D, where P = $\begin{bmatrix} 1 & 4 \\ 1 & 1 \end{bmatrix}$ and D = $\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$, find A^5.

Additionally, the problem notes that:

- Both A is a 2x2 matrix with -1 and 4 as the top elements.
- The context suggests a focus on eigenvalues, eigenvectors, and matrix powers, common in advanced linear algebra.

This problem combines several key concepts:
- Matrix multiplication and properties
- Diagonalization
- Powers of matrices
- Eigenvalues and eigenvectors

Understanding these concepts is critical for resolving the problem efficiently and accurately.

Breaking Down the Problem: Key Concepts

1. Matrix Equation AP = P D

The equation AP = P D indicates that A is similar to D, with P being the similarity transformation matrix.

- D is a diagonal matrix, representing the eigenvalues of A.
- P contains the eigenvectors of A as its columns.

2. Eigenvalues and Eigenvectors

- The diagonal entries of D (1 and 2) are the eigenvalues (λ_1 and λ_2) of A.
- The columns of P are the corresponding eigenvectors.

3. Finding Matrix A

Since AP = P D, A can be expressed as:

$$
A = P D P^{-1}
$$

Once A is known, calculating A^5 involves raising A to the fifth power, which is straightforward if A is diagonalizable:

$$
A^5 = P D^5 P^{-1}
$$

where:

$$
D^5 = \text{diagonal matrix with entries } \lambda_1^5, \lambda_2^5
$$

Step-by-Step Solution

Step 1: Verify the matrices

$$
P = \begin{bmatrix} 1 & 4 \\ 1 & 1 \end{bmatrix}
$$
$$
D = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}
$$

Step 2: Compute P^{-1}

To find $A = P D P^{-1}$, we need the inverse of P.

Determinant of P:

$$
\det(P) = (1)(1) - (4)(1) = 1 - 4 = -3
$$

Inverse of P:

$$
P^{-1} = \frac{1}{\det(P)} \begin{bmatrix} 1 & -4 \\ -1 & 1 \end{bmatrix} = -\frac{1}{3} \begin{bmatrix} 1 & -4 \\ -1 & 1 \end{bmatrix}
$$

Simplify:

$$
P^{-1} = \begin{bmatrix} -\frac{1}{3} & \frac{4}{3} \\ \frac{1}{3} & -\frac{1}{3} \end{bmatrix}
$$

Step 3: Calculate $A = P D P^{-1}$

First, compute $P D$:

$$
P D = \begin{bmatrix} 1 & 4 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} (1)(1) + (4)(0) & (1)(0) + (4)(2) \\ (1)(1) + (1)(0) & (1)(0) + (1)(2) \end{bmatrix} = \begin{bmatrix} 1 & 8 \\ 1 & 2 \end{bmatrix}
$$

Next, compute $A = (P D) P^{-1}$:

$$
A = \begin{bmatrix} 1 & 8 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} -\frac{1}{3} & \frac{4}{3} \\ \frac{1}{3} & -\frac{1}{3} \end{bmatrix}
$$

Calculate element-wise:

- First row, first column:

$$
(1)(-\frac{1}{3}) + (8)(\frac{1}{3}) = -\frac{1}{3} + \frac{8}{3} = \frac{7}{3}
$$

- First row, second column:

$$
(1)(\frac{4}{3}) + (8)(-\frac{1}{3}) = \frac{4}{3} - \frac{8}{3} = -\frac{4}{3}
$$

- Second row, first column:

$$
(1)(-\frac{1}{3}) + (2)(\frac{1}{3}) = -\frac{1}{3} + \frac{2}{3} = \frac{1}{3}
$$

- Second row, second column:

$$
(1)(\frac{4}{3}) + (2)(-\frac{1}{3}) = \frac{4}{3} - \frac{2}{3} = \frac{2}{3}
$$

Thus,

$$
A = \begin{bmatrix} \frac{7}{3} & -\frac{4}{3} \\ \frac{1}{3} & \frac{2}{3} \end{bmatrix}
$$

Calculating A^5: Raising the Matrix to the Power

1. Diagonalization Approach

Since $A = P D P^{-1}$, then:

$$
A^n = P D^n P^{-1}
$$

where:

$$
D^n = \begin{bmatrix} \lambda_1^n & 0 \\ 0 & \lambda_2^n \end{bmatrix}
$$

Given eigenvalues:

$$
\lambda_1 = 1,\quad \lambda_2 = 2
$$

So,

$$
D^5 = \begin{bmatrix} 1^5 & 0 \\ 0 & 2^5 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 32 \end{bmatrix}
$$

2. Compute A^5

$$
A^5 = P D^5 P^{-1}
$$

Recall:

$$
P = \begin{bmatrix} 1 & 4 \\ 1 & 1 \end{bmatrix}
$$
$$
P^{-1} = \begin{bmatrix} -\frac{1}{3} & \frac{4}{3} \\ \frac{1}{3} & -\frac{1}{3} \end{bmatrix}
$$
$$
D^5 = \begin{bmatrix} 1 & 0 \\ 0 & 32 \end{bmatrix}

\]

Calculate \(P D^5\):

\[
P D^5 = \begin{bmatrix} 1 & 4 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 32 \end{bmatrix} = \begin{bmatrix} (1)(1)+(4)(0) & (1)(0)+(4)(32) \\ (1)(1)+(1)(0) & (1)(0)+(1)(32) \end{bmatrix} = \begin{bmatrix} 1 & 128 \\ 1 & 32 \end{bmatrix}
\]

Finally, compute:

\[
A^5 = (P D^5) P^{-1} = \begin{bmatrix} 1 & 128 \\ 1 & 32 \end{bmatrix} \begin{bmatrix} -\frac{1}{3} & \frac{4}{3} \\ \frac{1}{3} & -\frac{1}{3} \end{bmatrix}
\]

Calculate element-wise:

- First row, first column:

\[
(1)(-\frac{1}{3}) + (128)(\frac{1}{3}) = -\frac{1}{3} + \frac{128}{3} = \frac{127}{3}
\]

- First row, second column:

\[
(1)(\frac{4}{3}) + (128)(-\frac{1}{3}) = \frac{4

Frequently Asked Questions

Given that AP = P D, with P = [1 4; 1 1] and D = [1 0; 0 2], how do you find the matrix A?

To find A, use the relation A = P D P^{-1}. First, compute P^{-1}, then multiply P D by P^{-1}. The inverse of P is calculated, and multiplying P D with P^{-1} gives A.

What is the explicit form of the matrix A when P = [1 4; 1 1] and D = [1 0; 0 2]?

Calculating P^{-1} and then A = P D P^{-1}, the resulting matrix A is [[-2, 8], [1, -3]].

Given that both A and P are 2x2 matrices with specific top

entries, how do the entries 2 and 4 in the matrices relate to the problem?

The entries 2 and 4 are the top-left elements of matrices A and P respectively, indicating the specific positions of these key values in the matrices, which are used in calculations or to identify the matrices' structure.

How do the eigenvalues relate to the diagonal matrix D in the equation AP = P D?

The diagonal matrix D contains the eigenvalues of matrix A. Since AP = P D, the columns of P are eigenvectors of A corresponding to the eigenvalues on the diagonal of D.

What is the significance of the matrices A and P being 2x2 with specific entries like 2 and 4?

The 2x2 matrices with entries 2 and 4 indicate the size and particular structure of the matrices involved, which simplifies computations and helps in understanding eigenvalues and eigenvectors in the context of matrix diagonalization.

How can you find A^5 once you've determined matrix A?

To find A^5, perform matrix multiplication of A by itself five times, or use diagonalization: A^5 = P D^5 P^{-1}, where D^5 is obtained by raising each diagonal entry of D to the fifth power.

What is the final value of A^5 based on the given matrices?

Given the diagonalization, A^5 = P D^5 P^{-1}. Since D = [[1, 0], [0, 2]], D^5 = [[1^5, 0], [0, 2^5]] = [[1, 0], [0, 32]]. Multiplying accordingly yields A^5, which results in matrix [[-62, 248], [31, -97]].

Additional Resources

Understanding the relationship between matrices, powers, and their properties is fundamental in linear algebra. In this comprehensive guide, we explore the problem: If \(AP = P D \), where \(P = \begin{bmatrix} 1 & 4 \\ 1 & 1 \end{bmatrix} \) and \(D = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} \), find \(A^5 \). This problem involves concepts such as matrix similarity, eigenvalues, eigenvectors, and matrix powers—cornerstones of advanced algebra. By analyzing this problem step-by-step, you will gain insight into how to approach similar matrix problems and understand the deeper structure of matrix transformations.

Introduction: Why Matrix Powers and Similarity Matter

Matrices are fundamental in multiple disciplines, including physics, engineering, computer science, and mathematics. They help us describe linear transformations, solve systems of equations, and analyze complex data structures. When working with matrices, understanding how to compute powers of matrices and relate them via similarity transformations is essential.

The problem we're tackling is a classic example that illustrates these concepts. It involves matrices \(P \), \(D \), and \(A \), with the key relation \(AP = P D \). This relation indicates that \(A \) is similar to \(D \), a diagonal matrix, which simplifies the computation of powers like \(A^5 \).

Breaking Down the Problem: The Key Equation \(AP = P D \)

What does \(AP = P D \) mean?

The equation \(AP = P D \) suggests that matrix \(A \) is similar to \(D \):

\[
A = P D P^{-1}
\]

This is a fundamental concept in linear algebra: similar matrices share many properties, including eigenvalues, eigenvectors, characteristic polynomial, and minimal polynomial.

Given data:

- \(P = \begin{bmatrix} 1 & 4 \\ 1 & 1 \end{bmatrix} \)
- \(D = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} \)

The goal:

- Find \(A^5 \)

Step 1: Confirming the Similarity and Computing \(A \)

Find \(P^{-1} \)

Calculating the inverse of \(P \):

\[
P = \begin{bmatrix} 1 & 4 \\ 1 & 1 \end{bmatrix}
\]

Determinant:

\[
\det(P) = (1)(1) - (4)(1) = 1 - 4 = -3
\]

Inverse:

\[

P^{-1} = \frac{1}{\det(P)} \begin{bmatrix} 1 & -4 \\ -1 & 1 \end{bmatrix} = -\frac{1}{3}
\begin{bmatrix} 1 & -4 \\ -1 & 1 \end{bmatrix}
\]

Simplified:

\[
P^{-1} = \begin{bmatrix} -\frac{1}{3} & \frac{4}{3} \\ \frac{1}{3} & -\frac{1}{3} \end{bmatrix}
\]

Step 2: Computing \(A \)

Using the similarity relation:

\[
A = P D P^{-1}
\]

First, multiply \(P \) and \(D \):

\[
P D = \begin{bmatrix} 1 & 4 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} =
\begin{bmatrix} (1)(1) + (4)(0) & (1)(0) + (4)(2) \\ (1)(1) + (1)(0) & (1)(0) + (1)(2) \end{bmatrix}
= \begin{bmatrix} 1 & 8 \\ 1 & 2 \end{bmatrix}
\]

Next, multiply this result by \(P^{-1} \):

\[
A = \begin{bmatrix} 1 & 8 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} -\frac{1}{3} & \frac{4}{3} \\
\frac{1}{3} & -\frac{1}{3} \end{bmatrix}
\]

Calculating each entry:

- Entry (1,1):

\[
(1)(-\frac{1}{3}) + (8)(\frac{1}{3}) = -\frac{1}{3} + \frac{8}{3} = \frac{7}{3}
\]

- Entry (1,2):

\[
(1)(\frac{4}{3}) + (8)(-\frac{1}{3}) = \frac{4}{3} - \frac{8}{3} = -\frac{4}{3}
\]

- Entry (2,1):

$$
(1)(-\frac{1}{3}) + (2)(\frac{1}{3}) = -\frac{1}{3} + \frac{2}{3} = \frac{1}{3}
$$

- Entry (2,2):

$$
(1)(\frac{4}{3}) + (2)(-\frac{1}{3}) = \frac{4}{3} - \frac{2}{3} = \frac{2}{3}
$$

Thus,

$$
A = \begin{bmatrix} \frac{7}{3} & -\frac{4}{3} \\ \frac{1}{3} & \frac{2}{3} \end{bmatrix}
$$

Step 3: Computing A^5 Using Similarity

Since A is similar to D, and similar matrices have the same eigenvalues, the key property is:

$$
A^n = P D^n P^{-1}
$$

Thus,

$$
A^5 = P D^5 P^{-1}
$$

Because D is diagonal, raising it to the 5th power is straightforward:

$$
D^5 = \begin{bmatrix} 1^5 & 0 \\ 0 & 2^5 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 32 \end{bmatrix}
$$

Step 4: Final Calculation of A^5

Compute $P D^5$:

$$
P D^5 = \begin{bmatrix} 1 & 4 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 32 \end{bmatrix} = \begin{bmatrix} (1)(1) + (4)(0) & (1)(0) + (4)(32) \\ (1)(1) + (1)(0) & (1)(0) + (1)(32) \end{bmatrix} = \begin{bmatrix} 1 & 128 \\ 1 & 32 \end{bmatrix}
$$

Now, multiply this result by P^{-1}:

$$
A^5 = \begin{bmatrix} 1 & 128 \\ 1 & 32 \end{bmatrix} \begin{bmatrix} -\frac{1}{3} & \frac{4}{3} \\ \frac{1}{3} & -\frac{1}{3} \end{bmatrix}
$$

Calculations:

- Entry (1,1):

$$
(1)(-\frac{1}{3}) + (128)(\frac{1}{3}) = -\frac{1}{3} + \frac{128}{3} = \frac{127}{3}
$$

- Entry (1,2):

$$
(1)(\frac{4}{3}) + (128)(-\frac{1}{3}) = \frac{4}{3} - \frac{128}{3} = -\frac{124}{3}
$$

- Entry (2,1):

$$
(1)(-\frac{1}{3}) + (32)(\frac{1}{3}) = -\frac{1}{3} + \frac{32}{3} = \frac{31}{3}
$$

- Entry (2,2):

$$
(1)(\frac{4}{3}) + (32)(-\frac{1}{3}) = \frac{4}{3} - \frac{32}{3} = -\frac{28}{3}
$$

Final answer:

$$
\boxed{
A^5 = \begin{bmatrix} \frac{127}{3} & -\frac{124}{3} \\ \frac{31}{3} & -\frac{28}{3} \end{bmatrix}
}
$$

Additional Insights: The Significance of the Top Ones and Negative Values

The problem mentions, "They're Both A 2x2 -1 And 4 Are The Top Ones," which hints at the eigenvalues and the structure of D. The eigenvalues \(

1 If Ap P D Where P 1 4 1 1 And D 1 0 0 2 Find A5 Their Both A 2x2 1 And 4 Are The Top Ones

1 If Ap P D Where P 1 4 1 1 And D 1 0 0 2 Find A5 Their Both A 2x2 1 And 4 Are The Top Ones

Related Articles

- A Vending Machine Containing Jellybeans Will Only Dispense One Jellybean At A Time. Inside The Container
- 1. Adichie Claims That Power Affects Which Stories Are Told, How They Are Told, And Which Stories Become
- An Amount Of $15,000 Is Borrowed From The Bank At An Annual Interest Rate Of 12%. A. Calculate The Equal

Back to Home