# 2.23. Given X1t2 = 3u1t2 + Tu1t2 - 3t - 14u1t - 12 - 5u1t - 22 (a) Sketch X1t2. (b) Calculate The Values

**2.23. Given X1t2 = 3u1t2 + Tu1t2 - 3t - 14u1t - 12 - 5u1t - 22 (a) Sketch X1t2. (b) Calculate The Values**

Understanding the behavior of a function like $X_{1}(t)$ requires a thorough analysis of its components, graphical representation, and computation of specific values. In this article, we will explore the given function step-by-step, starting with its structure, then moving on to its graphical sketch, and finally calculating relevant values to interpret its behavior.

## Analyzing the Given Function

The provided function is:

$$X_{1}(t) = 3u_{1}(t) + T u_{1}(t) - 3t - 14u_{1}(t) - 12 - 5u_{1}(t) - 22$$

Before delving into the sketch and calculations, it's essential to understand the components involved, especially the units or functions represented by $u_{1}(t)$ and $T$.

## Deciphering the Components

- $u_{1}(t)$: Typically, in mathematical contexts, $u(t)$ denotes the unit step function. The subscript "1" might suggest a shifted or specific version of the unit step, possibly indicating a step at $t = 1$.

- $T$: This could signify a constant, a parameter, or an operator. For simplicity and in common practice, if $T$ is a constant, it should be specified; otherwise, it might be an operator acting on $u_{1}(t)$.

Given the context and typical notation, it's reasonable to interpret:

- $u_{1}(t)$ as the unit step function shifted to activate at $t = 1$:

$$
u_{1}(t) =
\begin{cases}
0, & t < 1 \\
1, & t \geq 1
\end{cases}
$$

- $T$ as a constant or parameter that is part of the function's composition.

For clarity, let's assume $T$ is a constant. If specific value is given, we will use it; otherwise, we will proceed symbolically.

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# Rearranging the Function for Clarity

Let's combine like terms to simplify the expression:

$$
X_{1}(t) = 3u_{1}(t) + T u_{1}(t) - 3t - 14u_{1}(t) - 12 - 5u_{1}(t) - 22
$$

Group the terms involving $u_{1}(t)$:

$$
X_{1}(t) = (3u_{1}(t) + T u_{1}(t) - 14u_{1}(t) - 5u_{1}(t)) - 3t - (12 + 22)
$$

Simplify the coefficients:

$$
X_{1}(t) = (3 + T - 14 - 5) u_{1}(t) - 3t - 34
$$

$$
X_{1}(t) = (T - 16) u_{1}(t) - 3t - 34
$$

Thus, the function simplifies to:

$$
X_{1}(t) = (T - 16) u_{1}(t) - 3t - 34
$$

This form is much more manageable for analysis and graphing.

---

# Sketching $X_{1}(t)$

## Step 1: Understand the Behavior for $t < 1$

Since $u_{1}(t) = 0$ for $t < 1$, the function reduces to:

$$
X_{1}(t) = -3t - 34
$$

This is a straight line with slope $-3$ and y-intercept at $-34$.

- At $t = 0$, $X_{1}(0) = -3(0) - 34 = -34$
- At $t = 1$, $X_{1}(1) = -3(1) - 34 = -3 - 34 = -37$

So, for $t < 1$, the graph is a line descending with slope $-3$.

---

## Step 2: Behavior for $t \geq 1$

Here, $u_{1}(t) = 1$, so:

$$
X_{1}(t) = (T - 16) - 3t - 34
$$

which simplifies to:

$$
X_{1}(t) = (T - 16) - 3t - 34 = (T - 50) - 3t
$$

This is another straight line, with the same slope $-3$, but with a different intercept depending on $T$.

- At $t = 1$:

$$
X_{1}(1) = (T - 50) - 3(1) = T - 50 - 3 = T - 53
$$

- As $t \to \infty$, the function continues decreasing linearly with slope $-3$.

---

## Step 3: Sketch Summary

- The graph for $t < 1$:

- Starting at $(0, -34)$,
- Descending with slope $-3$,
- At $t = 1$, $X_{1}(1) = -37$.

- At $t = 1$, the function jumps (or continues smoothly, depending on $T$) to:

$$
X_{1}(1) = T - 53
$$

- For $t \geq 1$:

- The line continues with slope $-3$,
- Starting from $(1, T - 53)$.

Note: The actual shape depends on $T$. For example:

- If $T = 0$, then the function jumps from $-37$ to $-53$ at $t=1$.
- If $T = 20$, then the jump is from $-37$ to $-33$.

---

# Calculating Specific Values of $X_{1}(t)$

To fully understand the behavior of $X_{1}(t)$, let's compute some key points for different $t$ values, assuming specific values for $T$ where needed.

## Example: Let $T = 0$

- At $t = 0$:

$$
X_{1}(0) = -3(0) - 34 = -34
$$

- At $t = 0.5$:

$$
X_{1}(0.5) = -3(0.5) - 34 = -1.5 - 34 = -35.5
$$

- At $t = 1$:

$$
X_{1}(1) = -3(1) - 34 = -3 - 34 = -37

\]

- At \( t = 2 \):

\[
X_{1}(2) = (0 - 50) - 3(2) = -50 - 6 = -56
\]

- At \( t = 3 \):

\[
X_{1}(3) = -50 - 9 = -59
\]

## Example: Let \( T = 20 \)

- At \( t = 0 \):

\[
X_{1}(0) = -3(0) - 34 = -34
\]

- At \( t = 1 \):

\[
X_{1}(1) = (20 - 50) - 3(1) = -30 - 3 = -33
\]

- At \( t = 2 \):

\[
X_{1}(2) = -30 - 6 = -36
\]

- At \( t = 3 \):

\[
X_{1}(3) = -30 - 9 = -39
\]

These calculations illustrate how the parameter \( T \) influences the function's value at \( t = 1 \) and beyond.

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# Applications and Significance of $X_{1}(t)$

Understanding functions like $X_{1}(t)$, especially those involving step functions and linear components, is crucial in various fields such as control systems, signal processing, and electrical engineering.

Practical Applications:

- Signal Analysis: The step function models sudden changes or switches in signals, and analyzing the resulting functions helps in system response evaluation.

- Control Systems: Piecewise functions are common in control logic, where system

# Frequently Asked Questions

## What is the purpose of sketching the function X1t2 in the given problem?

Sketching X1t2 helps visualize its behavior over time, identify key features like intercepts and asymptotes, and gain insights into its overall trend based on the given expression.

## How can I approach sketching the function X1t2 = 3u1t2 + Tu1t2 - 3t - 14u1t - 12 - 5u1t - 22?

Start by simplifying the expression if possible, identify the types of functions involved (e.g., step functions u1t2, u1t, etc.), and plot each component separately to understand their contributions before combining them into the overall sketch.

## What does the notation u1t2 and u1t represent in the expression for X1t2?

u1t2 and u1t are likely unit step functions (Heaviside functions) that switch on at specific points (t=2 and t=1 respectively), modeling sudden changes or signal activation at those points.

## How do I interpret the phrase 'Calculate The Values' in the context of this problem?

It suggests computing the numerical values of X1t2 at specific points or over a range of t-values, possibly to analyze the function's behavior or verify the sketch with actual data points.

## What steps should I follow to compute values of X1t2 at specific t-values?

Identify the values of t, evaluate the step functions u1t2 and u1t at those points, substitute into the expression, and perform the arithmetic to find the corresponding X1t2 values.

# Additional Resources

Understanding the Mathematical Expression and Its Significance

In the realm of mathematical analysis, equations serve as the foundational language that describes relationships between variables and constants, enabling us to model, interpret, and predict real-world phenomena. The given expression, "2.23. Given X1t2 = 3u1t2 + Tu1t2 - 3t - 14u1t - 12 - 5u1t - 22," appears complex at first glance, but a systematic approach can unravel its structure and implications. This comprehensive review aims to decode this expression, explore its graphical representation, and perform detailed calculations to elucidate its behavior.

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# Deciphering the Expression: Breaking Down the Components

## Understanding the Notation and Variables

The expression in question is:

X1t2 = 3u1t2 + Tu1t2 - 3t - 14u1t - 12 - 5u1t - 22

At first glance, the notation suggests a relationship involving multiple variables and functions:

- X1t2: Likely a function of variables t and possibly other parameters, representing a specific quantity or state.
- u1t2: Possibly a function or variable dependent on t, perhaps denoting a state or input.
- Tu1t2: The notation 'T' preceding u1t2 may imply a transformation or operator acting on u1t2.
- Constants: Numerical values such as -3, -14, -12, -22, indicating fixed parameters influencing the relationship.

Given the notation, it's reasonable to interpret:

- u1t2 as a function u1(t,2), but the '2' may be part of the notation or an index.
- Tu1t2 as an operator T acting on u1t2.
- The presence of multiple terms involving u1t2 suggests a combination of functions and transformations.

Clarification of Variables:

- t: The primary independent variable.
- u1t2: A function of t, possibly representing an input, control variable, or state.
- T: An operator or transformation applied to u1t2, possibly differentiation, integration, or linear

transformation.

Assumption for Analysis:

To proceed analytically, we will assume the following:

- u1(t, 2) simplifies to u1(t), i.e., a function of t.
- T is a linear operator, possibly differentiation with respect to t, i.e., T(u1(t)) = du1/dt.
- The constants are coefficients or offsets affecting the overall function.

This interpretation aligns with common mathematical conventions, especially in differential equations and dynamic systems analysis.

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# Part A: Graphical Sketch of X1t2

## Methodology for Sketching

To sketch X1t2, the first step involves expressing it explicitly in terms of the variable t, assuming specific forms for u1(t) and the operator T. Since the original expression is somewhat abstract, we will consider plausible forms:

- Let u1(t) be a simple function, such as u1(t) = sin(t) or u1(t) = t.
- If T is differentiation, then T(u1(t)) = du1/dt.

Suppose:

- u1(t) = sin(t)

Then:

- T u1(t) = du1/dt = cos(t)

Assuming these, the expression becomes:

X1(t) = 3 sin(t) + cos(t) - 3t - 14 sin(t) - 12 - 5 sin(t) - 22

Simplify step-by-step:

1. Combine like terms involving u1(t):

- 3 sin(t) - 14 sin(t) - 5 sin(t) = (3 - 14 - 5) sin(t) = -16 sin(t)

2. Sum constant terms:

- (-12 - 22) = -34

3. Now, the expression reduces to:

$X1(t) = -16 \sin(t) + \cos(t) - 3t - 34$

This is a tractable function combining sinusoidal, linear, and constant components.

Graphical Behavior:

- The sinusoidal component ($-16 \sin(t) + \cos(t)$) oscillates between certain bounds, with amplitude roughly $\sqrt{(-16)^2 + 1^2} = \sqrt{256 + 1} \approx 16.03$, indicating significant oscillations.
- The linear term ($-3t$) introduces a decreasing trend as t increases.
- The constant term (-34) shifts the entire function downward.

Sketching Approach:

- Plot the sinusoidal component: oscillates between approximately ±16.
- Add the linear trend: a downward sloping line with slope -3.
- Adjust for the constant shift: shift the graph downward by 34 units.

Qualitative Description:

The graph of X1(t) is a sinusoidal wave with large amplitude, superimposed on a downward linear trend. As t increases, the oscillations persist, but the overall trend declines steadily, crossing the x-axis multiple times depending on the amplitude and slope.

Implications and Interpretation:

Such a function could model phenomena with oscillatory behavior subject to a decay or decline over time, such as damping in mechanical systems, or signal processing with decreasing amplitude.

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## Alternative Assumptions for u1(t) and T

If different functions are chosen (e.g., $u1(t) = t^2$, T as integration), the shape of X1(t) would differ accordingly. For instance:

- $u1(t) = t^2 \Rightarrow u1(t) = t^2$, T u1(t) = derivative, 2t.
- Resulting X1(t) would combine quadratic, linear, and constant terms, producing a parabola-like shape with oscillations.

The key takeaway is that the specific form of u1(t) and the operator T significantly influence the graph of X1(t).

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# Part B: Calculations and Numerical Analysis

## Calculating Specific Values of X1(t)

To evaluate X1(t) numerically, we need to specify:

- The form of u1(t)
- The interpretation of T
- The specific value of t

Example Calculation:

Assuming:

- u1(t) = sin(t)
- T u1(t) = du1/dt = cos(t)

And using the simplified form:

X1(t) = -16 sin(t) + cos(t) - 3t - 34

Let's compute X1(t) at specific points:

1. t = 0:

X1(0) = -16 0 + 1 - 0 - 34 = 1 - 34 = -33

2. t = π/2 ≈ 1.5708:

X1(π/2) = -16 1 + 0 - 3(1.5708) - 34

= -16 + 0 - 4.7124 - 34 = (-16 - 4.7124 - 34) ≈ -54.7124

3. t = π ≈ 3.1416:

X1(π) = -16 0 + (-1) - 33.1416 - 34

= 0 -1 - 9.4248 - 34 ≈ -44.4248

4. t = 2π ≈ 6.2832:

X1(2π) = -16 0 + 1 - 36.2832 - 34

= 0 + 1 - 18.8496 - 34 ≈ -51.8496

Observations:

- The values oscillate with the sinusoidal terms.

- The linear term dominates for larger t, making the function increasingly negative.
- These calculations help understand the behavior over a range of t.

Applications:

Such evaluations are crucial in systems modeling, where specific values inform stability, oscillation amplitude, or decay rate.

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## Extended Numerical Analysis

For more comprehensive insights, consider:

- Plotting X1(t) over an interval, e.g., $t \in [0, 10]$, to observe oscillation patterns and trend.
- Computing derivatives to find maxima, minima, and inflection points.
- Integrating over an interval to find total accumulated effect or energy.

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# Concluding Insights and Broader Implications

Summary of Findings:

- The expression X1t2, under plausible assumptions, combines sinusoidal and linear components, resulting in a function that oscillates while trending downward.
- Graphical sketches reveal the dominant oscillatory behavior modulated by a linear decline, indicative of systems with damping or decaying signals.
- Numerical calculations at specific points validate the theoretical model, demonstrating the function's behavior over various t values.

Broader Context:

Understanding such functions is vital across disciplines:

- Engineering: analyzing oscillatory systems, control responses.
- Physics: modeling wave phenomena with damping.
- Economics: representing cyclical trends with decline.
- Data Science: signal processing and noise filtering.

Limitations and Future Directions:

- Precise modeling requires clarity on the definitions of u1t2 and T.
- Future work could involve fitting actual data to such models or exploring different forms of u1(t) and T

# 2 23 Given X1t2 3u1t2 Tu1t2 3t 14u1t 12 5u1t 22 A Sketch X1t2 B Calculate The Values

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