# 2. If The Utility Function Is U (x1, X2) = 10(x12+2x1x2+ X22)-50, Then We Say "commodity 1 And Commodity

**2. If The Utility Function Is U (x1, X2) = 10(x1^2 + 2x1x2 + x2^2) - 50, Then We Say "commodity 1 And Commodity" in terms of consumer preferences, utility analysis, and economic decision-making.**

## Understanding the Utility Function: U (x1, x2) = 10(x1^2 + 2x1x2 + x2^2) - 50

In microeconomics, utility functions serve as mathematical representations of consumer preferences over commodities or goods. The given utility function, *U(x1, x2) = 10(x1^2 + 2x1x2 + x2^2) - 50*, encapsulates the consumer's satisfaction derived from consuming quantities *x1* and *x2* of two commodities. Understanding this function provides insights into consumer behavior, preferences, and how they make choices under constraints.

## Mathematical Structure of the Utility Function

### Breaking Down the Function

- **Quadratic Form:** The utility function is quadratic in both *x1* and *x2*.
- **Symmetry:** The terms *x1^2* and *x2^2* are symmetric, and the cross term *2x1x2* indicates interaction between commodities.
- **Constant Term:** The constant -50 shifts the utility function vertically but does not affect preferences' shape.

### Rewriting the Function

The utility function can be expressed as:

```
U(x1, x2) = 10[x1^2 + 2x1x2 + x2^2] - 50
```

Notice that *x1^2 + 2x1x2 + x2^2* resembles the expansion of the square of a sum:

```
(x1 + x2)^2 = x1^2 + 2x1x2 + x2^2
```

Thus, the utility function simplifies to:

```
U(x1, x2) = 10(x1 + x2)^2 - 50
```

# Interpreting the Utility Function

## Implications of the Simplified Form

Since *U(x1, x2) = 10(x1 + x2)^2 - 50*, the utility depends solely on the sum *x1 + x2*. This indicates:

- Consumers derive the same utility from different combinations of *x1* and *x2* as long as their sum remains constant.
- The utility function is monotonically increasing in *x1 + x2* because the coefficient 10>0, implying higher total quantities lead to higher utility.

## Preferences and Indifference Curves

Given the utility function, consumer preferences can be visualized through indifference curves. Since utility depends on *x1 + x2*, the indifference curves are straight lines of the form:

```
x1 + x2 = constant
```

All points along such a line provide the same level of utility.

# Analyzing Consumer Behavior Based on the Utility Function

## Optimal Consumption Choices

Consumers aim to maximize their utility subject to budget constraints. The key steps involve:

1. Identifying feasible combinations of *x1* and *x2* based on income and prices.
2. Choosing the combination on the budget line where the highest indifference curve is tangent to the budget constraint.

## Marginal Utility and Substitution

Since the utility depends on $x1 + x2$, the marginal utility with respect to each commodity is:

```
MU_{x1} = ∂U/∂x1 = 20(x1 + x2)
MU_{x2} = ∂U/∂x2 = 20(x1 + x2)
```

Both marginal utilities are equal, reflecting symmetry. The consumer is willing to substitute between $x1$ and $x2$ as long as their sum remains the same.

# Economic Insights and Practical Implications

## Commodity Substitutes and Complementarity

Given the utility depends solely on the sum $x1 + x2$, commodities act as perfect substitutes in consumption. The consumer perceives the combination of the two commodities as interchangeable regarding utility gains.

## Effect of Price Changes

Changes in the prices of commodities influence the consumer's optimal bundle, but since preferences depend on the sum, the consumer will adjust quantities to maintain the total sum, considering their budget constraints.

## Consumer Satisfaction and Utility Maximization

Maximizing utility involves choosing the highest possible $x1 + x2$ within the budget. The shape of the indifference lines suggests that consumers prefer to allocate their resources to increase the total sum, rather than favoring one commodity over the other.

# Graphical Representation of the Utility Function

## Indifference Curves

- Since $U(x1, x2) = 10(x1 + x2)^2 - 50$, indifference curves are straight lines with slope -1 in the (x1, x2) plane.
- Higher utility corresponds to lines further away from the origin.

## Budget Constraints and Optimal Points

Plotting budget lines and indifference curves allows visualization of consumer choices. The optimal point occurs where the budget line is tangent to an indifference line, which, given the linear nature, occurs along the line $x1 + x2 = constant$.

# Conclusion

The utility function $U(x1, x2) = 10(x1^2 + 2x1x2 + x2^2) - 50$ simplifies to depend solely on the sum $x1 + x2$. This indicates that consumers view commodities 1 and 2 as perfect substitutes, prioritizing increasing the combined quantity rather than favoring one good over the other. Understanding this utility structure helps economists and marketers predict consumer behavior, design better pricing strategies, and analyze market dynamics. Recognizing the symmetry and substitution pattern embedded in the utility function is essential for making informed economic decisions and optimizing resource allocations.

# Frequently Asked Questions

## What type of utility function is U(x1, x2) = 10(x1^2 + 2x1x2 + x2^2) - 50?

It is a quadratic utility function, specifically representing a symmetric quadratic form in commodities x1 and x2.

## Does the utility function U(x1, x2) = 10(x1^2 + 2x1x2 + x2^2) - 50 indicate increasing or decreasing utility with respect to commodities?

Since the function is quadratic with positive coefficients, utility increases as the quantities of commodities increase, but the overall shape depends on the specific values of x1 and x2.

## What can be inferred about the substitutability between commodity 1 and commodity 2 from this utility function?

Given the symmetric quadratic form, the utilities suggest a certain degree of substitutability, but the exact nature depends on the marginal rates of substitution derived from the function.

## How does the constant term '-50' affect the utility levels in the function U(x1, x2)?

The '-50' shifts all utility levels downward by 50 units, serving as a baseline or fixed cost in the utility measurement.

### Based on the utility function, what is the likely shape of the indifference curves?

The indifference curves are likely to be conic sections (ellipses), given the quadratic form, representing the trade-offs between commodities x1 and x2.

## Additional Resources

If The Utility Function Is U ($x_1$, $x_2$) = 10($x_1^2$ + 2$x_1x_2$ + $x_2^2$) - 50: A Deep Dive into Consumer Preferences and Utility Theory

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Introduction

In the realm of microeconomics, understanding consumer preferences through utility functions is fundamental. Utility functions serve as mathematical representations of consumer satisfaction derived from different bundles of commodities. Among the myriad forms these functions can take, quadratic utility functions offer a rich landscape for analyzing consumer choice, marginal utilities, and optimal consumption bundles.

This article explores the utility function:

$$ U(x_1, x_2) = 10(x_1^2 + 2x_1x_2 + x_2^2) - 50 $$

and examines its implications for consumer behavior, the nature of the indifference curves it generates, and the broader economic insights it provides.

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Understanding the Utility Function: Mathematical Structure and Intuition

Deconstructing the Utility Function

The given utility function is quadratic in two commodities, $x_1$ and $x_2$:

$$ U(x_1, x_2) = 10(x_1^2 + 2x_1x_2 + x_2^2) - 50 $$

At first glance, it resembles a quadratic form that can be expressed in matrix notation:

$$
U(\mathbf{x}) = 10 \mathbf{x}^\top Q \mathbf{x} - 50
$$

where

$$
\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}
$$

and

$$
Q = \begin{bmatrix}
1 & 1 \\
1 & 1 \\
\end{bmatrix}
$$

since

$$
x_1^2 + 2x_1x_2 + x_2^2 = \begin{bmatrix} x_1 & x_2 \end{bmatrix}
\begin{bmatrix}
1 & 1 \\
1 & 1 \\
\end{bmatrix}
\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}
$$

Note that the matrix $Q$ is symmetric, which is typical for quadratic forms.

Geometric Interpretation

The quadratic form inside the utility function suggests that the shape of the indifference curves depends heavily on the properties of the matrix $Q$. Specifically:

- The positive definiteness or semi-definiteness of $Q$ determines whether the indifference curves are convex, concave, or saddle-shaped.
- The coefficient $10$ scales the quadratic form, affecting the steepness of the indifference curves.

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Analyzing the Nature of the Indifference Curves

Quadratic Forms and Conic Sections

The utility function's indifference curves are derived by setting $U(x_1, x_2) = k$, for some constant $k$:

$$
10(x_1^2 + 2x_1x_2 + x_2^2) - 50 = k
$$
$$
x_1^2 + 2x_1x_2 + x_2^2 = \frac{k + 50}{10}
$$

Let’s denote:

$$
C = \frac{k + 50}{10}
$$

The locus of points satisfying:

$$
x_1^2 + 2x_1x_2 + x_2^2 = C
$$

are conic sections—specifically, they are ellipses, hyperbolas, or parabolas, depending on the value of $C$ and the form of the quadratic.

### Simplifying the Equation

To understand the shape, complete the square or diagonalize the quadratic form.

1. Change of variables

Define new variables $y_1, y_2$ via a rotation that diagonalizes $Q$. Alternatively, observe that:

$$
x_1^2 + 2x_1x_2 + x_2^2 = (x_1 + x_2)^2
$$

because:

$$
(x_1 + x_2)^2 = x_1^2 + 2x_1x_2 + x_2^2
$$

This is a key insight: the quadratic form simplifies to:

$$
(x_1 + x_2)^2 = C
$$

Thus, the indifference curves are:

$$
x_1 + x_2 = \pm \sqrt{C}
$$

for $C \geq 0$.

---

## Geometric and Economic Implications

### Nature of the Indifference Curves

- If $k + 50 \geq 0$, then:

$$
x_1 + x_2 = \pm \sqrt{\frac{k + 50}{10}}
$$

which are straight lines with slope $-1$.

- For $k + 50 < 0$, the expression involves the square root of a negative number, implying no real solutions, and thus, no indifference curves at such utility levels.

Therefore, the indifference curves are a family of straight lines, specifically:

$$
x_1 + x_2 = \text{constant}
$$

This indicates perfect substitutes with a constant marginal rate of substitution (MRS) of $-1$.

Consumer Preferences

- Perfect Substitutes: The linear indifference curves suggest that the consumer views commodities 1 and 2 as perfect substitutes at a rate of 1:1. The consumer is willing to trade one unit of commodity 1 for one unit of commodity 2 without any change in utility.

- Implication of Utility Function: The quadratic form's simplification to a linear equation signifies that the utility is linearly increasing along the direction where $x_1 + x_2$ increases.

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Marginal Utilities and Consumer Choice

Calculating Marginal Utilities

The marginal utilities ($MU_1, MU_2$) are the partial derivatives of $U$:

$$
MU_1 = \frac{\partial U}{\partial x_1} = 10 \times 2x_1 + 10 \times 2x_2 = 20x_1 + 20x_2
$$

$$
MU_2 = \frac{\partial U}{\partial x_2} = 10 \times 2x_2 + 10 \times 2x_1 = 20x_2 + 20x_1
$$

which simplifies to:

$$
MU_1 = 20(x_1 + x_2)
$$
$$
MU_2 = 20(x_1 + x_2)
$$

Both marginal utilities are proportional to the sum $x_1 + x_2$, reinforcing the interpretation that the consumer’s preferences are symmetric and linear.

Marginal Rate of Substitution (MRS)

$$
MRS_{12} = - \frac{MU_1}{MU_2} = -1
$$

which indicates the consumer is always willing to substitute one unit of commodity 1 for one unit of commodity 2 at a constant rate—again, characteristic of perfect substitutes.

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Consumer Optimization and Budget Constraints

Suppose the consumer faces a budget constraint:

$$
p_1 x_1 + p_2 x_2 = I
$$

where:

- $p_1$, $p_2$: prices of commodities 1 and 2
- $I$: consumer’s income

Given the utility function's structure, the consumer will choose bundles along the line $x_1 + x_2 = \text{constant}$. The optimal choice depends on the relative prices:

- If $p_1 = p_2$, the consumer is indifferent along any point where $x_1 + x_2 = \text{max}$ given the budget.
- If $p_1 < p_2$, the consumer will prefer to buy more of commodity 1.
- If $p_2 < p_1$, the consumer will prefer to buy more of commodity 2.

Thus, the consumer’s optimal choice reduces to corner solutions, buying the commodity with the lowest price until the budget is exhausted.

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Broader Economic Insights and Implications

The Significance of Linearity in Preferences

The linear indifference curves derived from the quadratic utility function reveal perfect substitutability between commodities:

- Consumers are not willing to give up significant amounts of one good for the other.
- The marginal rate of substitution remains constant at $-1$.

This behavior contrasts sharply with typical convex indifference curves associated with more “normal” preferences, which exhibit diminishing MRS.

Utility Function as a Model of Consumer Behavior

- In practical terms, such utility functions are best suited for modeling commodities that serve similar purposes or are interchangeable—for example, different brands of the same product or similar financial instruments.
- The linear preferences imply that the consumer prioritizes the total quantity of the combined commodities, rather than their individual characteristics.

Limitations and Considerations

- The assumption of perfect substitutability simplifies analysis but may not reflect real-world preferences where goods are imperfect substitutes or complements.
- The quadratic utility function's transformation into linear indifference curves indicates that the quadratic form in this specific

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