4. A Consumer With U[x,y)=min(3x,y] And Income M=12 Pays Px=2, Py =1. Compute The Cross-price Elasticity

4. A Consumer With U[x,y)=min(3x,y] And Income M=12 Pays Px=2, Py =1. Compute The Cross-price Elasticity

Understanding consumer behavior and how changes in the prices of goods influence demand is fundamental in economics. In this article, we analyze a specific utility function, U[x,y] = min(3x, y), with a consumer having an income of M = 12, facing prices Px = 2 for good x and Py = 1 for good y. Our primary goal is to compute the cross-price elasticity of demand for these goods, which measures the responsiveness of the quantity demanded for one good when the price of another good changes. This detailed examination provides insights into the nature of consumer preferences, substitution effects, and how market prices influence demand patterns.

Understanding the Utility Function and Budget Constraint

Utility Function Analysis

The utility function in question, U[x, y] = min(3x, y), indicates a specific type of consumer preference known as perfect complements. In this case, the consumer derives utility based on the minimum of two expressions: 3x and y. This implies that the consumer values x and y in fixed proportions, specifically, three units of x for each unit of y, or vice versa, depending on how the minimum is activated.

Key points about the utility function:

- It exhibits a "kink" at the point where 3x = y.
- Utility increases only along the line y = 3x.
- The consumer’s preferences are such that they derive the highest utility when the two components are balanced in the ratio y = 3x.

Budget Constraint

Given the consumer’s income and the prices:

- $M = 12$
- $P_x = 2$
- $P_y = 1$

The budget constraint is:

$$ 2x + 1 \times y \leq 12 $$

which simplifies to:

$$ y \leq 12 - 2x $$

The consumer chooses x and y to maximize utility subject to this budget constraint.

Optimal Consumption Bundle: Deriving the Demand Functions

Step 1: Understanding the Utility Maximization Condition

Since the utility function is $U[x, y] = \min(3x, y)$, the consumer maximizes utility by choosing x and y such that:

$$ y = 3x $$

This is because utility increases only when both 3x and y are increased simultaneously, maintaining the minimum as large as possible.

Step 2: Budget Constraint Substitution

Substituting $y = 3x$ into the budget constraint:

$$ y = 3x \leq 12 - 2x $$

which yields:

$$ 3x \leq 12 - 2x $$

$$ 3x + 2x \leq 12 $$

$$ 5x \leq 12 $$

$$ x \leq \frac{12}{5} = 2.4 $$

Since the consumer wants to maximize utility, and utility is increasing along $y = 3x$, the optimal x is at the maximum feasible x, which is 2.4.

Correspondingly:

$$ y = 3 \times 2.4 = 7.2 $$

Step 3: Confirming the Demand Bundle

The optimal consumption bundle is:

- $x^{} = 2.4$ units
- $y^{} = 7.2$ units

This bundle satisfies the budget constraint:

$$ 2 \times 2.4 + 1 \times 7.2 = 4.8 + 7.2 = 12 $$

which exhausts the consumer’s income, confirming it as the optimal bundle.

Calculating the Cross-Price Elasticity

Definition of Cross-price Elasticity

The cross-price elasticity of demand for good y with respect to the price of good x is defined as:

$$ E_{Py, Px} = \frac{\partial Q_y}{\partial P_x} \times \frac{P_x}{Q_y} $$

Where:

- Q_y = quantity demanded of y
- P_x = price of x
- $\partial Q_y / \partial P_x$ = partial derivative of demand for y with respect to Px

This elasticity measures how much the demand for y responds to changes in Px. A positive value indicates substitutes, while a negative suggests complements.

Step 1: Expressing Q_y as a Function of Px

From the previous section, the demand for y depends on the maximum x affordable, which is:

$$ x^ = \frac{M}{5} \quad \text{when} \quad P_x = 2 $$

More generally, for any Px:

$$ 5x = M \Rightarrow x = \frac{M}{5} $$

But considering the general case where Px varies, the budget constraint becomes:

$$ y = M - P_x \times x $$

and utility maximization still occurs when:

$$ y = 3x $$

Substituting into the budget constraint:

$$ 3x = M - P_x x $$

$$ 3x + P_x x = M $$

$$ x (3 + P_x) = M $$

$$ x = \frac{M}{3 + P_x} $$

then,

$$ y = 3x = 3 \times \frac{M}{3 + P_x} = \frac{3M}{3 + P_x} $$

Therefore, the demand for y as a function of Px is:

$$ Q_y (P_x) = \frac{3M}{3 + P_x} $$

Step 2: Computing the Partial Derivative $\frac{\partial Q_y}{\partial P_x}$

Given:

$$ Q_y (P_x) = \frac{3M}{3 + P_x} $$

The derivative with respect to Px is:

$$ \frac{\partial Q_y}{\partial P_x} = - \frac{3M}{(3 + P_x)^2} $$

Plugging in the known values:

$$ M = 12, \quad P_x = 2 $$

we get:

$$ \frac{\partial Q_y}{\partial P_x} = - \frac{3 \times 12}{(3 + 2)^2} = - \frac{36}{25} = -1.44 $$

Step 3: Calculating the Cross-price Elasticity

Recall:

$$ E_{Py, Px} = \frac{\partial Q_y}{\partial P_x} \times \frac{P_x}{Q_y} $$

At Px = 2:

$$ Q_y = \frac{3 \times 12}{3 + 2} = \frac{36}{5} = 7.2 $$

Substituting the derivatives and values:

$$ E_{Py, Px} = -1.44 \times \frac{2}{7.2} = -1.44 \times \frac{1}{3} = -0.48 $$

Interpreting the Cross-price Elasticity Result

The calculated cross-price elasticity:

$$ E_{Py, Px} = -0.48 $$

implies that the demand for y is inelastic with respect to the price of x, and the negative sign indicates that x and y are complements in consumption. When the price of x increases, the demand for y decreases, but the magnitude less than 1 shows inelasticity — demand responds less proportionally to price changes.

This aligns with the nature of the utility function, which models perfect complements, where goods are used together in fixed proportions. An increase in Px reduces the consumer's purchasing power and consequently decreases the demand for both goods.

Summary and Conclusion

In this analysis, we've examined a consumer with a utility function $U[x, y] = \min(3x, y)$, income of 12, and prices $P_x = 2$ and $P_y = 1$. We derived the consumer’s optimal consumption bundle, confirming that the consumer chooses $x = 2.4$ and $y = 7.2$. Using this setup, we calculated the cross-price elasticity of demand for y with respect to Px and found it to be approximately -0.48.

This negative elasticity indicates that x and y are complementary goods in the consumer’s preferences: as the price of x rises, demand for y falls. The inelastic magnitude suggests that demand responds less than proportionally to price changes, typical for goods that are used together in fixed proportions.

Understanding cross-price elasticities is vital for businesses and policymakers. For example, knowing that goods are complements can influence pricing strategies, marketing, and regulation policies. If a company considers increasing the price of a complementary product, they should anticipate a decrease in demand for related goods.

Additional Insights and Applications

- Market Strategy: Firms selling complementary goods should coordinate pricing strategies to avoid negative impacts on demand.
- Policy Implications: Taxation or subsidies on one good can indirectly influence the demand for related goods.
- Consumer Behavior Analysis: Recognizing the utility structure helps in

Frequently Asked Questions

What is the utility function given for the consumer?

The utility function is U[x,y] = min(3x, y).

What are the prices of goods x and y?

The price of good x (Px) is 2, and the price of good y (Py) is 1.

What is the consumer's income (M)?

The consumer's income is $M = 12$.

How do you find the optimal bundle for this utility function?

Since the utility is $\min(3x, y)$, the consumer maximizes utility by setting $3x = y$, subject to the budget constraint $2x + y = 12$.

What is the optimal consumption of x and y?

Setting $3x = y$ and using the budget constraint: $2x + 3x = 12$, which gives $5x = 12$, so $x = 12/5 = 2.4$ and $y = 3\ 2.4 = 7.2$.

How is the cross-price elasticity of demand calculated?

Cross-price elasticity = $(\Delta Qx / Qx) / (\Delta Py / Py)$, measuring the responsiveness of Qx to changes in Py.

What is the effect of a change in Py on Qx in this utility function?

Since the consumer allocates spending to keep $3x = y$ and the budget constraint is linear, a change in Py affects the optimal quantities indirectly through the budget constraint, but the utility function's structure links x and y directly.

How do you compute the cross-price elasticity at the given point?

Given the linear relationship and the fixed optimal bundle, the cross-price elasticity is zero because a change in Py doesn't alter the optimal x, which is determined by the min condition and budget constraint.

What does a cross-price elasticity of zero imply about the goods?

A cross-price elasticity of zero indicates that the goods are neither substitutes nor complements; demand for x does not respond to price changes in y in this utility setup.

Additional Resources

Cross-Price Elasticity of Demand: An In-Depth Analysis of the Consumer's Utility Function

Understanding how consumers respond to changes in the prices of related goods is fundamental in microeconomics. The concept of cross-price elasticity of demand measures the responsiveness of the quantity demanded of one good to a change in the price of another. This metric is crucial for businesses and policymakers alike, offering insight into whether goods are substitutes or complements. In this article, we delve into the calculation of cross-price elasticity for a specific consumer choice scenario, characterized by a utility function $U[x,y] = \min(3x, y)$, with given income and prices.

Setting the Scene: The Consumer's Utility Function and Budget Constraints

Utility Function Analysis

The utility function under consideration is:

$$
U[x, y] = \min(3x, y)
$$

This function indicates a perfect complement-like preference structure but scaled such that the consumer derives utility based on the minimum of $3x$ and y. To interpret this:

- The consumer derives utility based on how well the quantities of goods x and y are balanced.
- Since utility depends on the minimum of $3x$ and y, the consumer prefers to consume these goods in proportions where $y = 3x$.

In essence, the consumer gains the same utility when:

$$
3x = y
$$

This shape of utility function suggests that the consumer would like to consume goods x and y in fixed proportions, with y being three times as much as x.

Budget Constraints and Prices

Given the consumer's income and prices:

- Income: $M = 12$
- Price of x: $P_x = 2$
- Price of y: $P_y = 1$

The budget constraint can be expressed as:

$$
2x + 1 \cdot y \leq 12
$$

The consumer chooses quantities x and y to maximize utility subject to this budget constraint.

Finding the Consumer's Optimal Choice

Identifying the Demand in the Context of the Utility Function

Given the utility function $U[x, y] = \min(3x, y)$, the consumer's goal is to maximize U, which is bounded by the minimum of $3x$ and y.

- To maximize utility, the consumer wants:

$$
3x = y
$$

- Substituting into the budget constraint:

$$
2x + y \leq 12
$$

Since $y = 3x$, substitute:

$$
2x + 3x \leq 12
$$

\[

5x \leq 12
\]
\[
x \leq \frac{12}{5} = 2.4
\]

- Corresponding \(y \):

\[
y = 3x = 3 \times 2.4 = 7.2
\]

- Check if this point is within the budget:

\[
2 \times 2.4 + 7.2 = 4.8 + 7.2 = 12
\]

Yes, it exactly exhausts the budget.

Optimal consumption bundle:

\[
\boxed{
x^ = 2.4, \quad y^ = 7.2
}
\]

- The maximum utility at this point:

\[
U = \min(3 \times 2.4, 7.2) = \min(7.2, 7.2) = 7.2
\]

This bundle is both feasible and utility-maximizing given the consumer's preferences and budget.

Understanding Cross-Price Elasticity of Demand

Definition of Cross-Price Elasticity

The cross-price elasticity of demand for good \(x \) with respect to the price of good \(y \) is defined as:

\[

E_{x, P_y} = \frac{\partial x}{\partial P_y} \times \frac{P_y}{x}
\]

- It measures the percentage change in the quantity demanded of \(x \) in response to a 1% change in the price of \(y \).
- If \(E_{x, P_y} > 0 \), the goods are substitutes.
- If \(E_{x, P_y} < 0 \), they are complements.
- If \(E_{x, P_y} = 0 \), the goods are independent.

In this case, since the consumer's utility depends on both goods in a fixed proportion, the nature of the cross-price elasticity will provide insights into whether these goods are complements or substitutes.

Calculating Cross-Price Elasticity: Step-by-Step Approach

Step 1: Determine How Changes in \(P_y \) Affect \(x^ \)

Given that the optimal bundle is:

\[
x^ = \frac{12}{5} = 2.4
\]

and:

\[
y^ = 3x^ = 7.2
\]

we observe that:

- Since the optimal \(x \) depends on the budget constraint and the fixed proportion \(y = 3x \), any change in \(P_y \) affects the feasible set, and consequently, the optimal \(x \).

To analyze this, consider the impact of a change in \(P_y \) on the consumer's choice.

Step 2: Express the Budget Constraint with Variable P_y

The budget constraint:

$$
2x + P_y y \leq M
$$

At the optimum, the consumer spends all income and:

$$
y = 3x
$$

Substitute into the budget:

$$
2x + P_y \times 3x = 12
$$
$$
(2 + 3 P_y) x = 12
$$

Solve for x:

$$
x = \frac{12}{2 + 3 P_y}
$$

Similarly, y:

$$
y = 3x = \frac{36}{2 + 3 P_y}
$$

Step 3: Derive the Partial Derivative $\partial x / \partial P_y$

From the expression:

$$
x(P_y) = \frac{12}{2 + 3 P_y}
$$

Differentiate with respect to \(P_y \):

\[
\frac{\partial x}{\partial P_y} = -\frac{12 \times 3}{(2 + 3 P_y)^2} = -
\frac{36}{(2 + 3 P_y)^2}
\]

Step 4: Calculate Cross-Price Elasticity \(E_{x, P_y} \)

Recall the formula:

\[
E_{x, P_y} = \frac{\partial x}{\partial P_y} \times \frac{P_y}{x}
\]

Substitute the derivatives and expressions:

\[
E_{x, P_y} = -\frac{36}{(2 + 3 P_y)^2} \times \frac{P_y}{\frac{12}{2 + 3
P_y}} = -\frac{36 P_y}{(2 + 3 P_y)^2} \times \frac{2 + 3 P_y}{12}
\]

Simplify numerator and denominator:

\[
E_{x, P_y} = -\frac{36 P_y (2 + 3 P_y)}{12 (2 + 3 P_y)^2}
\]

Cancel common factors:

\[
E_{x, P_y} = -3 P_y \times \frac{1}{2 + 3 P_y}
\]

Therefore, the cross-price elasticity simplifies to:

\[
\boxed{
E_{x, P_y} = -\frac{3 P_y}{2 + 3 P_y}
}
\]

Interpreting the Result: Is \(y \) a Complement or Substitute to \(x \)?

Sign and Magnitude of Elasticity

The sign of \(E_{x, P_y} \):

- Since \(P_y > 0 \), the numerator \(-3 P_y \) is negative.
- The denominator \(2 + 3 P_y \) is positive.
- Thus, \(E_{x, P_y} < 0 \).

Implication: The negative cross-price elasticity indicates that \(x \) and \(y \) are complementary goods in this consumer's utility framework. As the price of \(y \) rises, the demand for \(x \) falls, consistent with the idea that these goods are consumed together in fixed proportions.

Numerical Example: Calculating at the Given Prices

Given:

\[
P_y = 1
\]

Calculate:

\[
E_{x, P_y} = -\frac{3 \times 1}{2 + 3 \times 1} = -\frac{3}{

[4 A Consumer With U X Y Min 3x Y And Income M 12 Pays Px 2 Py 1 Compute The Cross Price Elasticity]()

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