4. Let F(x)=-1.(a) (15 Points) Determine The Fourier Series Of F(x) On [-1, 1].(b) (10 Points) Determine

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Understanding Fourier series is fundamental in mathematical analysis, signal processing, and numerous engineering applications. In this article, we focus on a specific problem involving the function F(x) = -1, and how to determine its Fourier series expansion over the interval [-1, 1]. This problem is split into two parts: (a) finding the Fourier series of F(x) on [-1, 1], and (b) performing a related task, which will be clarified as we explore the details.

Understanding Fourier Series and Their Significance

What Is a Fourier Series?

A Fourier series is a way to represent a periodic function as a sum of simple sine and cosine waves. This decomposition allows complex functions to be analyzed and processed more easily, especially in contexts like signal processing, heat transfer, and acoustics. Formally, for a function f(x) with period 2L, the Fourier series is expressed as:

- $f(x) = a_0/2 + \Sigma\ [a_n \cos(n\pi x / L) + b_n \sin(n\pi x / L)]$ for $n=1$ to ∞

Here, the coefficients a_n and b_n determine the amplitude of the cosine and sine components, respectively.

Why Focus on the Interval [-1, 1]?

In many problems, especially those involving functions defined on a finite interval, it's necessary to consider the Fourier series over that specific domain. When the function is extended periodically outside the interval, the Fourier series provides an approximation that converges to the original function within the interval, under certain conditions.

Part (a): Determining the Fourier Series of F(x) = -1 on [-1, 1]

Step 1: Understanding the Function

The function F(x) = −1 is a constant function over the interval [−1, 1]. Since it is constant, its Fourier series will be relatively simple, consisting mainly of the average value (a_0 term) plus possibly some sine terms if the function is odd.

Step 2: Calculating the Fourier Coefficients

Given that the function is constant, the coefficients are calculated as:

- a_0 (Average or DC coefficient):

$$
a_0 = \frac{1}{L} \int_{-L}^{L} F(x) dx
$$

Since $L=1$, and $F(x) = -1$:

$$
a_0 = \frac{1}{1} \int_{-1}^{1} (-1) dx = \int_{-1}^{1} -1 dx = -1 \times (1 - (-1)) = -2
$$

- a_n (Cosine coefficients):

$$
a_n = \frac{1}{L} \int_{-L}^{L} F(x) \cos(n \pi x / L) dx
$$

$$
a_n = \int_{-1}^{1} -1 \times \cos(n \pi x) dx
$$

Since $\cos(n \pi x)$ is an even function and -1 is constant:

$$
a_n = - \int_{-1}^{1} \cos(n \pi x) dx
$$

Integrating:

$$
a_n = - \left[\frac{\sin(n \pi x)}{n \pi} \right]_{-1}^{1} = - \left(\frac{\sin(n \pi \times 1) - \sin(n \pi \times -1)}{n \pi} \right)
$$

$$
a_n = - \left(\frac{\sin(n \pi) - \sin(-n \pi)}{n \pi} \right)
$$

Recall that $\sin(n \pi) = 0$, and $\sin(-n \pi) = - \sin(n \pi) = 0$:

$$
a_n = - (0 - 0) = 0
$$

- b_n (Sine coefficients):

$$
b_n = \frac{1}{L} \int_{-L}^{L} F(x) \sin(n \pi x / L) dx
$$

$$
b_n = \int_{-1}^{1} -1 \times \sin(n \pi x) dx
$$

Since $\sin(n \pi x)$ is an odd function, and the product of an even and odd function is odd, the integral over symmetric limits is zero:

$$
b_n = 0
$$

Step 3: Write the Fourier Series

Putting it all together, the Fourier series for $F(x) = -1$ on $[-1, 1]$ is:

$$
F(x) = -1 = \frac{a_0}{2} + \sum_{n=1}^\infty \left(a_n \cos(n \pi x) + b_n \sin(n \pi x) \right)
$$

which simplifies to:

$$
F(x) = -1 = -1 + 0 + 0 + \dots
$$

Thus, the Fourier series reduces simply to:

$$
\boxed{
F(x) = -1
}
$$

since all the coefficients beyond a_0 are zero, confirming that the constant function has a Fourier series consisting solely of the average value.

Part (b): Additional Tasks or Clarifications

While the problem statement stops at "Determine", implying a further task, common extensions include:

- Reconstructing F(x) using the Fourier series and analyzing convergence,
- Finding the Fourier series of related functions or modifications,
- Determining the Fourier coefficients for other boundary conditions.

Since the original function is constant, the Fourier series is straightforward, but for more complex functions, the process involves similar integral calculations, often requiring integration by parts or Fourier coefficient formulas.

Applications of Fourier Series in Engineering and Science

Understanding how to compute Fourier series for simple functions like a constant function serves as the foundation for analyzing more complex signals and waveforms. In signal processing, Fourier series allow engineers to decompose signals into harmonic components, facilitating filtering, compression, and analysis.

Some notable applications include:

- Designing filters in electronics
- Analyzing vibrations in mechanical systems
- Signal compression algorithms like MP3 and JPEG
- Studying heat transfer and wave propagation in physics

Conclusion

In this article, we explored how to determine the Fourier series of the constant function \(F(x) = -1\) over the interval \([-1, 1]\). The process involved calculating the Fourier coefficients, which revealed that the series reduces primarily to the average value of the function. This exercise underscores the simplicity of Fourier series for constant functions and highlights the importance of integral calculus in deriving Fourier coefficients. Whether you're a student or a professional in engineering or physics, mastering these fundamental techniques lays the groundwork for analyzing more complex periodic functions and signals.

By understanding the Fourier series of basic functions like \(F(x) = -1\), you build the foundation for tackling more intricate problems involving waveforms, signals, and systems analysis — essential skills in modern science and engineering.

Frequently Asked Questions

What is the process to find the Fourier Series of the function F(x) = -1 on the interval [-1, 1]?

To find the Fourier Series of F(x) = -1 on [-1, 1], you compute the Fourier coefficients a0, an, and bn using the integral formulas over the interval, then sum the series. Since F(x) is constant, the series simplifies to a single term.

What is the Fourier series expansion for the constant function F(x) = −1 on [−1, 1]?

The Fourier series of F(x) = −1 on [−1, 1] reduces to its average value, which is a0/2, since all sine and cosine terms cancel out. Thus, the series is simply F(x) = −1.

How do you compute the Fourier coefficients a0, an, and bn for F(x) = −1?

a0 is computed as (1/1) times the integral of F(x) over [−1, 1], which yields a0 = −2. The coefficients an are zero because the integral of an odd function times cos(nπx) over symmetric limits is zero. Similarly, bn are zero because the integral of an odd function times sin(nπx) over symmetric limits is zero.

Is the Fourier series of a constant function like F(x) = −1 simply the constant itself?

Yes, for a constant function, the Fourier series reduces to the constant value, since all other coefficients (sine and cosine terms) are zero. Therefore, F(x) = −1 is represented solely by its a0 term.

What is the significance of the Fourier series for the constant function F(x) = −1?

The Fourier series confirms that the function is constant and can be represented by a single term, demonstrating that Fourier series can represent constant functions exactly with just the average value term.

How does the Fourier series of F(x) = −1 relate to its Fourier coefficients?

Since F(x) = −1 is constant, its Fourier series contains only the a0/2 term, with all other coefficients being zero, reflecting the function's uniform value across the interval.

Does the Fourier series of F(x) = −1 converge to the function everywhere on [−1, 1]?

Yes, the Fourier series converges to F(x) = −1 at all points in the interval, as the function is continuous and constant.

How would you extend this problem to find the Fourier series of a non-constant step function?

You would compute the Fourier coefficients by integrating the step function over the interval, considering its piecewise definitions, and sum the resulting series to approximate the function.

What is the role of symmetry in simplifying the

Fourier series calculation for $F(x) = -1$?

Since $F(x) = -1$ is an even, constant function, the sine terms (which are odd functions) vanish, simplifying the Fourier series to only cosine terms and the constant term.

What is the main takeaway from deriving the Fourier series of a constant function like $F(x) = -1$?

The main takeaway is that constant functions have simple Fourier series representations consisting solely of their average value, illustrating the series' ability to represent even the simplest functions exactly.

Additional Resources

Fourier Series of $F(x) = -1$ on $[-1, 1]$

The task of determining the Fourier series of a function is fundamental in analyzing periodic functions, especially in fields such as signal processing, physics, and engineering. In this review, we focus on the specific function $F(x) = -1$ defined on the interval $[-1, 1]$ and explore its Fourier series representation in detail. We will analyze both parts of the problem: (a) deriving the Fourier series for $F(x)$ on $[-1, 1]$, and (b) understanding the implications or related aspects as indicated in the original question.

Understanding the Function $F(x) = -1$ on $[-1, 1]$

Before delving into the Fourier series, it is crucial to understand the nature of the function itself.

- Constant Function: $F(x) = -1$ is a constant function across the interval $[-1, 1]$.
- Symmetry: The function is an even function because $F(-x) = F(x) = -1$.
- Periodicity: To find a Fourier series, the function is typically extended periodically outside the interval $[-1, 1]$. The periodic extension depends on the context (e.g., period 2, 4, etc.).

This simplicity in the function's form makes the Fourier series straightforward to compute, but it also serves as an excellent example for understanding the Fourier expansion of constant functions.

Part (a): Determining the Fourier Series of $F(x) = -1$ on $[-1, 1]$

Step 1: Recall the Fourier Series Formula

For a function defined on the interval [-L, L], the Fourier series is given by:

$$
F(x) = \frac{a_0}{2} + \sum_{n=1}^\infty \left(a_n \cos \frac{n \pi x}{L} + b_n \sin \frac{n \pi x}{L} \right)
$$

where the Fourier coefficients are:

$$
a_n = \frac{1}{L} \int_{-L}^L F(x) \cos \frac{n \pi x}{L} \, dx
$$
$$
b_n = \frac{1}{L} \int_{-L}^L F(x) \sin \frac{n \pi x}{L} \, dx
$$
$$
a_0 = \frac{1}{L} \int_{-L}^L F(x) \, dx
$$

Given our interval [-1, 1], we have L = 1.

Step 2: Compute the Fourier Coefficients

Since F(x) = -1 is constant:

- Compute a_0:

$$
a_0 = \frac{1}{1} \int_{-1}^1 (-1) \, dx = \int_{-1}^1 -1 \, dx = - \left[x \right]_{-1}^1 = - (1 - (-1)) = -2
$$

- Compute a_n:

$$
a_n = \int_{-1}^1 -1 \cdot \cos(n \pi x) \, dx
$$

Since the integrand is an even function ($\cos(n \pi x)$ is even and the constant -1), the integral over [-1, 1] simplifies to:

$$
a_n = 2 \int_{0}^1 -1 \cdot \cos(n \pi x) \, dx = -2 \int_0^1 \cos(n \pi x) \, dx
$$

Evaluate:

$$
\int_0^1 \cos(n \pi x) \, dx = \left[\frac{\sin(n \pi x)}{n \pi} \right]_0^1 = \frac{\sin(n \pi)}{n \pi} - 0 = 0
$$

$$

because $\sin(n \pi) = 0$ for all integers n.

Thus,

$$
a_n = -2 \times 0 = 0
$$

- Compute b_n:

$$
b_n = \int_{-1}^1 -1 \cdot \sin(n \pi x) \, dx
$$

The integrand is an odd function ($\sin(n \pi x)$ is odd, and multiplied by a constant -1 keeps it odd). The integral over symmetric limits of an odd function is zero:

$$
b_n = 0
$$

Step 3: Write the Fourier Series

Putting it all together:

$$
F(x) = \frac{a_0}{2} + \sum_{n=1}^\infty \left(a_n \cos n \pi x + b_n \sin n \pi x \right)
$$

which simplifies to:

$$
F(x) = -1 + 0 + 0 = -1
$$

Thus, the Fourier series of $F(x) = -1$ on $[-1, 1]$ is simply:

$$
\boxed{
F(x) = -1
}
$$

This is expected, as the Fourier series of a constant function is the constant itself, with no higher harmonic terms needed.

Part (b): Related Aspects and Implications

While the original question's part (b) is incomplete, typically such problems ask for further analysis such as:

- Fourier Series on Different Periods: Extending the function periodically with a certain period, such as 2, 4, etc.
- Fourier Series for Modified Functions: For example, F(x) = -1 for x in [-1, 1], extended periodically.
- Fourier Series of Related Functions: For instance, considering the odd or even extensions or the effect of shifting the function.

Assuming the intent is to understand the Fourier series of the periodic extension of F(x), we can analyze the periodic extension of F(x) = -1 as a square wave:

- Features of the periodic extension:

- It repeats the constant value -1 over each period.
- Since the function is constant, its Fourier series remains constant across the entire domain.

- Implications:

- The Fourier series converges uniformly to the function everywhere.
- The simplicity of the Fourier series reflects the function's uniformity.

Features and Pros/Cons of Fourier Series Representation

Features:

- Simplicity for constant functions: The Fourier series reduces to the constant term, making it straightforward.
- Orthogonality: The basis functions (sines and cosines) are orthogonal, simplifying coefficient calculations.
- Convergence: For continuous functions, Fourier series converge uniformly; for discontinuous functions, convergence may be pointwise with Gibbs phenomena near jump discontinuities.

Pros:

- Efficient Representation: Constant functions are represented by a single term, reducing computational complexity.
- Analytical Clarity: The Fourier series provides insights into the frequency components, even for simple functions.

Cons:

- Limited for non-periodic functions: Fourier series inherently assume periodicity; for non-periodic functions, Fourier transforms are more appropriate.
- Gibbs Phenomenon: For functions with discontinuities, oscillations near

jumps can occur in partial sums, although this is not relevant for constant functions.

Conclusion and Final Remarks

The Fourier series of the constant function F(x) = -1 on the interval [-1, 1] is remarkably straightforward. The derivation confirms that the series reduces to the constant itself, with all sine and cosine coefficients (except the average term) being zero. This exemplifies the power of Fourier analysis in decomposing functions into harmonic components, even in the simplest cases. The implications extend to understanding how constant or simple functions are represented in the frequency domain, which is foundational for more complex analyses involving signals, waveforms, and periodic phenomena.

In application, recognizing that a constant function's Fourier series is itself emphasizes the importance of symmetry, periodicity, and the nature of harmonic analysis. It also provides a baseline for understanding more complicated Fourier expansions, including those involving discontinuities, non-constant functions, or functions defined on different intervals or with different periodic extensions.

In summary:

- The Fourier series of F(x) = -1 on [-1, 1] is simply -1, with no sine or cosine terms.
- This exemplifies the Fourier series of a constant function, illustrating its straightforward nature.
- Extending this understanding to periodic extensions and more complex functions reveals the broad utility of Fourier analysis in mathematical and engineering contexts.

If further clarification or additional exploration of related Fourier series topics is desired, such as convergence behavior, Fourier transforms, or application examples, these can be elaborated upon in subsequent discussions.

4 Let F X 1 A 15 Points Determine The Fourier Series Of F X On 1 1 B 10 Points Determine

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