5. A Jar Containing 15 Marbles Of Which 5 Are Blue, 8 Are Red And 2 Are Yellow, If Two Marbles Are Drawn

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Understanding probability is essential in solving many real-world problems, especially those involving chance and randomness. In this article, we explore the probability scenarios associated with drawing two marbles from a jar containing a total of 15 marbles, categorized by color: 5 blue, 8 red, and 2 yellow. We will analyze various cases, calculate probabilities, and discuss the implications of drawing marbles without replacement. This comprehensive guide aims to enhance your understanding of basic probability concepts through practical examples.

Overview of the Marble Jar Composition

Before diving into probability calculations, it's crucial to understand the composition of the jar:

- Total Marbles: 15
- Blue Marbles: 5
- Red Marbles: 8
- Yellow Marbles: 2

This distribution influences the likelihood of drawing specific colors and combinations. When drawing two marbles without replacement, the total number of possible outcomes and the probabilities of various events can be precisely calculated.

Fundamental Concepts of Probability in the Marble Drawing Scenario

What is Probability?

Probability measures the likelihood of an event occurring. It is expressed as a number between 0 and 1, where:

- 0 indicates impossibility
- 1 indicates certainty

- Values in between represent chances

Basic Probability Formula

For any event A:

$$
P(A) = \frac{\text{Number of favorable outcomes}}{\text{Total number of possible outcomes}}
$$

Drawing Without Replacement

In this scenario, once a marble is drawn, it is not returned to the jar, affecting the probabilities for subsequent draws.

Total Number of Outcomes When Drawing Two Marbles

The total number of ways to draw 2 marbles from 15 is calculated using combinations:

$$
\text{Total outcomes} = \binom{15}{2} = \frac{15 \times 14}{2 \times 1} = 105
$$

Thus, there are 105 possible unordered pairs of marbles that can be drawn.

Probabilities of Specific Events

1. Drawing Two Blue Marbles

Number of ways to select 2 blue marbles:

$$
\binom{5}{2} = \frac{5 \times 4}{2} = 10
$$

Probability:

$$
P(\text{both blue}) = \frac{10}{105} = \frac{2}{21} \approx 0.0952

\]

2. Drawing Two Red Marbles

Number of ways to select 2 red marbles:

$$
\binom{8}{2} = \frac{8 \times 7}{2} = 28
$$

Probability:

$$
P(\text{both red}) = \frac{28}{105} = \frac{4}{15} \approx 0.2667
$$

3. Drawing Two Yellow Marbles

Number of ways to select 2 yellow marbles:

$$
\binom{2}{2} = 1
$$

Probability:

$$
P(\text{both yellow}) = \frac{1}{105} \approx 0.0095
$$

Probabilities of Drawing Marbles of Different Colors

1. One Blue and One Red

Number of favorable outcomes:

- Blue first, then Red: $5 \times 8 = 40$
- Red first, then Blue: $8 \times 5 = 40$

Total favorable outcomes:

$$
40 + 40 = 80
$$

Probability:

$$
P(\text{blue and red}) = \frac{80}{105} \approx 0.7619
$$

2. One Blue and One Yellow

Number of favorable outcomes:

- Blue first, then Yellow: $5 \times 2 = 10$
- Yellow first, then Blue: $2 \times 5 = 10$

Total:

$$
10 + 10 = 20
$$

Probability:

$$
P(\text{blue and yellow}) = \frac{20}{105} \approx 0.1905
$$

3. One Red and One Yellow

Number of favorable outcomes:

- Red first, then Yellow: $8 \times 2 = 16$
- Yellow first, then Red: $2 \times 8 = 16$

Total:

$$
16 + 16 = 32
$$

Probability:

$$
P(\text{red and yellow}) = \frac{32}{105} \approx 0.305
$$

Step-by-Step Calculation of Conditional Probabilities

Conditional probability helps understand the likelihood of an event given that another has occurred. For example, what is the probability of drawing a red marble second, given that a blue marble was drawn first?

Example: Probability of Drawing a Red Marble Second, Given First is Blue

- First draw: Blue (probability $\frac{5}{15} = \frac{1}{3}$)
- Remaining marbles after first draw: 14
- Remaining red marbles: 8 (since blue was removed)

Probability of second draw being red:

$$
P(\text{red second} \mid \text{blue first}) = \frac{8}{14} = \frac{4}{7}
$$

Joint probability:

$$
P(\text{blue first and red second}) = P(\text{blue first}) \times P(\text{red second} \mid \text{blue first}) = \frac{1}{3} \times \frac{4}{7} = \frac{4}{21} \approx 0.1905
$$

This matches the earlier probability of drawing blue then red in the two-draw sequence.

Expected Values and Variance in the Drawing Process

Expected Number of Blue Marbles in Two Draws

Since each draw is dependent, the expected value can be calculated considering the probability of drawing a blue marble in each position.

- Probability of drawing blue first:

$$
P(\text{blue first}) = \frac{5}{15} = \frac{1}{3}
$$

- Probability of drawing blue second:

- If blue was not drawn first, the probability of drawing blue second:

$$
\text{If first is not blue:} \quad P(\text{blue second} \mid \text{not blue first}) = \frac{5}{14} \quad (\text{assuming blue not drawn first})
$$

But to find the expected number, we consider the total expected value over both draws:

$$
E(\text{number of blue marbles in two draws}) = 2 \times P(\text{drawing a blue marble in a single draw})
$$

Since probability changes after each draw, the precise expected value involves more complex calculations, but a simplified approximation is:

$$
E \approx 2 \times \left(\frac{5}{15} \right) = 2 \times \frac{1}{3} = \frac{2}{3} \approx 0.6667
$$

Similarly, the variance can be computed based on probabilities, but this level of detail is often beyond the scope of introductory probability discussions.

Practical Applications of the Marble Drawing Problem

Understanding the probability of drawing specific colors from a jar is not just a theoretical exercise; it has practical implications in various fields, including:

- Games and Gambling: Calculating odds to strategize.
- Quality Control: Random sampling to detect defects.
- Decision Making: Assessing risks in uncertain scenarios.
- Education: Teaching fundamental probability concepts.

Strategies for Solving Similar Probability Problems

When approaching problems involving drawing items from a collection, consider these steps:

1. Identify the total number of items and categories.
2. Determine the total outcomes using combinations or permutations.
3. Define the specific event and count favorable outcomes.
4. Calculate the probability as the ratio of favorable outcomes to total outcomes.
5. Consider whether the problem involves replacement or not, as it affects calculations.
6. Use conditional probability for dependent events.
7. Validate your results by checking that total probabilities sum to 1 when considering all possible events.

Conclusion

Drawing two marbles from a jar containing 15 marbles of different colors illustrates fundamental probability principles, including combinations, conditional probability, and dependent events. By understanding the distribution of marbles—5 blue, 8 red, and 2 yellow—and applying basic formulas, we can accurately determine the likelihood of various outcomes. These concepts are widely applicable, from games of chance to real-world decision-making scenarios. Mastery of such problems enhances analytical thinking and provides a foundation for more advanced probability and statistics topics.

Summary of Key Probabilities

Event	Probability
Both marbles are blue	$\frac{2}{21}$ (~9.52%)
Both marbles are red	$\frac{4}{15}$ (~26.67%)
Both marbles are yellow	$\frac{1}{105}$ (~0.95%)

Frequently Asked Questions

What is the probability of drawing two blue marbles from the jar without replacement?

The probability is (5/15) (4/14) = (1/3) (2/7) = 2/21.

What is the probability of drawing one red and one yellow marble in any order?

The probability is (8/15)(2/14) + (2/15)(8/14) = (8/15)(1/7) + (2/15)(4/7) = (8/105) + (8/105) = 16/105.

What is the probability of drawing two marbles of different colors?

Total probability of two different colors = 1 - Probability of both same color. Calculations: Same color: Blue-Blue: (5/15)(4/14)=2/21; Red-Red: (8/15)(7/14)=4/15; Yellow-Yellow: (2/15)(1/14)=1/105. Sum: 2/21 + 4/15 + 1/105 = 10/105 + 28/105 + 1/105= 39/105=13/35. Therefore, different colors: 1 - 13/35 = 22/35.

What is the probability of drawing two yellow marbles?

Since there are only 2 yellow marbles, the probability is (2/15) (1/14) = 2/210 = 1/105.

If the first marble drawn is blue, what is the probability that the second marble is red?

After drawing one blue marble, remaining marbles: 14 total, with 8 red marbles. Probability: 8/14 = 4/7.

What is the probability that at least one marble drawn is yellow?

Probability of no yellow marbles drawn (both marbles are from the other colors): (13/15) (12/14) = (13/15) (6/7) = (136)/(157) = 78/105 = 26/35. Therefore, probability of at least one yellow marble: 1 - 26/35 = 9/35.

What is the expected number of blue marbles drawn in two draws?

Expected value = number of draws probability of drawing a blue marble each time. Since draws are without replacement, the probability varies, but on average, the expected number of blue marbles is (2)(5/15) = 2/3.

Additional Resources

5. A Jar Containing 15 Marbles Of Which 5 Are Blue, 8 Are Red And 2 Are Yellow, If Two Marbles Are Drawn

In the fascinating world of probability and statistics, simple scenarios such as drawing marbles from a jar serve as fundamental examples to understand complex concepts. One such scenario involves a jar containing 15 marbles—specifically, 5 blue, 8 red, and 2 yellow marbles. When two marbles are drawn from this jar without replacement, the question naturally arises: what are the chances of various outcomes? This seemingly straightforward question opens the door to an array of probability calculations, combinatorial reasoning, and insightful interpretations. This article delves deep into the mathematical underpinnings of this scenario, exploring the probability of different events, the methods to compute these probabilities, and the broader implications for understanding randomness and chance.

Understanding the Composition of the Jar

Before diving into probability calculations, it's essential to grasp the basic composition of the jar. The total number of marbles and their distribution across colors set the stage for all subsequent analysis.

Breakdown of the Marbles

- Blue Marbles: 5
- Red Marbles: 8
- Yellow Marbles: 2

Total marbles: 15

This distribution provides the foundation for calculating the likelihood of drawing various combinations when two marbles are selected without replacement.

Fundamental Concepts in Probability and Combinatorics

To analyze this scenario accurately, one must understand key principles in probability and combinatorics.

Sample Space and Outcomes

- Sample Space (S): All possible pairs of two marbles drawn from the 15 marbles.
- Number of Outcomes in S: The total number of ways to choose 2 marbles from 15, calculated via combinations.

Combinatorial Calculations

- The number of ways to choose k items from n items, without regard to order, is given by the binomial coefficient:

$$
\binom{n}{k} = \frac{n!}{k!(n-k)!}
$$

Where:

- $n!$ is factorial of n,
- $k!$ is factorial of k.

Applying this to our problem:

- Total ways to pick 2 marbles from 15:

$$
\binom{15}{2} = \frac{15 \times 14}{2} = 105
$$

This total forms the denominator for probability calculations, representing the total possible outcomes.

Calculating Probabilities of Specific Events

With the basic setup established, the next step involves computing probabilities of various events—such as drawing two marbles of the same color, or one of each color.

Probability of Drawing Two Blue Marbles

- Number of ways to select 2 blue marbles:

$$
\binom{5}{2} = \frac{5 \times 4}{2} = 10
$$

- Probability:

$$
P(\text{both blue}) = \frac{\text{favorable outcomes}}{\text{total outcomes}} = \frac{10}{105} \approx 0.0952
$$

Probability of Drawing Two Red Marbles

- Number of ways:

$$
\binom{8}{2} = \frac{8 \times 7}{2} = 28

\]

- Probability:

$$
P(\text{both red}) = \frac{28}{105} \approx 0.2667
$$

Probability of Drawing Two Yellow Marbles

- Number of ways:

$$
\binom{2}{2} = 1
$$

- Probability:

$$
P(\text{both yellow}) = \frac{1}{105} \approx 0.0095
$$

Probability of Drawing One Blue and One Red

This involves selecting one blue and one red marble:

- Blue choices:

$$
\binom{5}{1} = 5
$$

- Red choices:

$$
\binom{8}{1} = 8
$$

- Total favorable outcomes:

$$
5 \times 8 = 40
$$

- Probability:

$$
P(\text{blue and red}) = \frac{40}{105} \approx 0.38095
$$

Similarly, for other mixed-color combinations:

- Blue and Yellow:

$$
\binom{5}{1} \times \binom{2}{1} = 5 \times 2 = 10
$$
$$
P(\text{blue and yellow}) = \frac{10}{105} \approx 0.0952
$$

- Red and Yellow:

$$
\binom{8}{1} \times \binom{2}{1} = 8 \times 2 = 16
$$
$$
P(\text{red and yellow}) = \frac{16}{105} \approx 0.1524
$$

Exploring Conditional Probabilities and Expected Outcomes

The initial calculations provide the probabilities of specific outcomes. But what if we consider conditional probabilities or expected values?

Probability of Drawing a Blue Marble First and a Red Marble Second

- First draw (blue):

$$
\frac{5}{15}
$$

- Second draw (red):

Since the marbles are drawn without replacement, after removing one blue marble:

$$
\text{Remaining marbles}: 14
$$
$$
\text{Remaining red marbles}: 8
$$

- Probability of second marble being red given the first was blue:

$$
\frac{8}{14} = \frac{4}{7}
$$

- Combined probability:

$$
P(\text{blue first, red second}) = \frac{5}{15} \times \frac{8}{14} = \frac{1}{3} \times \frac{4}{7} = \frac{4}{21} \approx 0.1905
$$

Similarly, for other ordered sequences, the calculations follow the same logic, considering the updated counts after each draw.

Expected Number of Marbles of Each Color in Two Draws

Since the draws are random, the expected number of marbles of each color can be calculated by summing the probabilities across the possible outcomes.

- Expected blue marbles:

$$
E(\text{blue}) = 2 \times P(\text{both blue}) + 1 \times P(\text{one blue}) + 0 \times P(\text{no blue})
$$

But a more straightforward approach involves calculating the probability that at least one blue marble is drawn in two draws:

- Probability of drawing at least one blue:

$$
P(\text{at least one blue}) = 1 - P(\text{no blue})
$$

\]

- Probability of no blue:

\[
\binom{10}{2} = \frac{10 \times 9}{2} = 45
\]
\[
P(\text{no blue}) = \frac{45}{105} \approx 0.4286
\]

- Therefore,

\[
P(\text{at least one blue}) \approx 1 - 0.4286 = 0.5714
\]

Similarly, expected counts can be derived using linearity of expectation, which simplifies calculations for large or complex scenarios.

Real-World Applications and Broader Implications

While the marble scenario is simple, the principles underpinning these calculations are widely applicable across disciplines.

Education and Teaching

Probability problems like this serve as excellent teaching tools, helping students develop intuition about randomness, combinations, and probability distributions.

Gaming and Gambling

Understanding odds based on combinatorial calculations informs strategies in card games, lotteries, and other chance-based activities.

Quality Control and Sampling

Sampling items from a batch and calculating probabilities of certain features aids in quality assurance processes, where decisions depend on the likelihood of defective or specific items.

Data Science and Risk Analysis

Probability models underpin predictive analytics, risk assessments, and decision-making processes in finance, health sciences, and engineering.

Limitations and Assumptions in the Scenario

While the calculations assume perfect randomness and no biases, real-world conditions may introduce variations.

- Assumption of Equal Likelihood: Each marble has an equal chance of being drawn.
- No Replacement: Once a marble is drawn, it's not replaced, affecting subsequent probabilities.
- Independence of Draws: The outcome of one draw affects the probabilities of subsequent draws.

Understanding these assumptions is crucial for applying these principles accurately in practical situations.

Conclusion: The Power of Probabilistic Reasoning

The scenario of drawing two marbles from a jar containing 15 marbles of different colors exemplifies the foundational principles of probability and combinatorics. By systematically analyzing the composition of the jar, calculating total possible outcomes, and applying combinatorial formulas, we can determine the likelihood of various events with precision. Such analyses not only deepen our appreciation for the mathematical beauty of chance but also equip us with tools applicable across diverse fields—from education and gaming to science and industry. Ultimately, these insights reinforce that even simple models can illuminate complex realities, underscoring the value of probabilistic thinking in understanding the uncertain world around us.

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