5. How Much Of An 800-gram Sample Of Potassium-40 Will Remain After 3.9 10^9 Years Of Radioactive Decay?

5. How Much Of An 800-gram Sample Of Potassium-40 Will Remain After 3.9 × 10^9 Years Of Radioactive Decay?

Understanding radioactive decay and half-life is essential for comprehending how substances like potassium-40 diminish over time. In this article, we'll explore how much of an initial 800-gram sample of potassium-40 remains after approximately 3.9 billion years, a period comparable to Earth's age. We'll delve into the concepts of half-life, decay calculations, and the scientific significance of these processes.

What Is Potassium-40 and Why Is Its Radioactive Decay Important?

Potassium-40 (K-40) is a naturally occurring isotope of potassium, making up about 0.012% of total potassium found in nature. Its significance lies in its radioactive properties and its role in geological dating techniques, such as potassium-argon dating.

Key facts about potassium-40:

- Radioactive isotope: Decays via beta decay to calcium-40 and argon-40.
- Half-life: Approximately 1.25 billion years.
- Natural abundance: About 0.012% of potassium.

Understanding how potassium-40 decays over time allows scientists to estimate the age of rocks and minerals, providing insights into Earth's history.

Understanding Radioactive Decay and Half-Life

Radioactive decay is a stochastic process where unstable isotopes spontaneously transform into more stable forms over time. The half-life is the time required for half of a given amount of a radioactive substance to decay.

Key concepts:

- Half-life ($T_{1/2}$): The characteristic time for 50% decay.

- Decay constant (λ): The probability per unit time that a nucleus will decay.
- Exponential decay: The amount of substance decreases exponentially over time according to the decay law.

The mathematical relationship governing decay is:

$$ N(t) = N_0 \times e^{-\lambda t} $$

Where:
- $N(t)$ is the remaining quantity after time t,
- N_0 is the initial quantity,
- λ is the decay constant,
- e is Euler’s number (~2.718).

Calculating the Remaining Potassium-40 After 3.9 Billion Years

Given an initial mass of potassium-40 and its half-life, we can determine how much remains after a specified time using the decay law.

Step 1: Gather Known Data

- Initial mass $N_0 = 800\, \text{grams}$
- Half-life $T_{1/2} = 1.25 \times 10^9\, \text{years}$
- Time elapsed $t = 3.9 \times 10^9\, \text{years}$

Step 2: Calculate the Decay Constant λ

The decay constant relates to half-life as:

$$ \lambda = \frac{\ln 2}{T_{1/2}} $$

$$ \lambda = \frac{0.6931}{1.25 \times 10^9} \approx 5.545 \times 10^{-10}\, \text{year}^{-1} $$

Step 3: Compute Remaining Quantity $N(t)$

Applying the decay law:

$$ N(t) = N_0 \times e^{-\lambda t} $$

$$ N(t) = 800 \times e^{- (5.545 \times 10^{-10}) \times 3.9 \times 10^9} $$

Calculate the exponent:

$$ - \lambda t = - (5.545 \times 10^{-10}) \times 3.9 \times 10^9 $$

$$ - \lambda t \approx - 2.161 $$

Now, compute $e^{-2.161}$:

$$ e^{-2.161} \approx 0.115 $$

Finally, find the remaining mass:

$$ N(t) \approx 800 \times 0.115 \approx 92\, \text{grams} $$

Result: Approximately 92 grams of potassium-40 remains after 3.9 billion years.

Implications of the Decay of Potassium-40 Over Earth's History

This decay process has profound implications for geology, paleontology, and planetary science:

- Age estimation: Radioactive decay of K-40 is used to date ancient rocks, some over 4 billion years old.
- Earth's formation: The decay timeline helps estimate the age of the Earth, approximately 4.54 billion years.
- Geochemical cycles: Understanding decay helps elucidate the behavior of elements within Earth's crust and mantle.

The fact that a significant portion of potassium-40 has decayed over Earth's lifespan underscores the importance of radioactive decay in understanding planetary evolution.

Additional Considerations and Complexities

While the basic calculation provides a solid estimate, several factors can influence the exact remaining amount:

- Initial purity: The initial sample may contain other isotopes or impurities.
- Decay pathways: K-40 decays via beta decay to calcium-40 (~89%) and argon-40 (~11%), affecting radiometric dating.
- Geological processes: Erosion, metamorphism, and other processes can alter isotope concentrations in rocks.

Despite these complexities, the exponential decay model remains a robust tool for estimating isotope quantities over geological timescales.

Summary: How Much Potassium-40 Remains After 3.9 Billion Years?

In summary:

- Starting with an 800-gram sample of potassium-40.
- After approximately 3.9 billion years, only about 92 grams of potassium-40 remain.
- This reflects roughly an 88.5% decay over Earth's age, consistent with the known half-life of 1.25 billion years.

This calculation illustrates the predictable nature of radioactive decay and its significance in dating Earth's materials, understanding planetary formation, and studying geochemical processes.

Final Thoughts

Radioactive decay, especially of isotopes like potassium-40, is a cornerstone of modern geoscience. By understanding the decay process and half-life, scientists can unlock Earth's history and the timeline of our planet's evolution. The decay of potassium-40 over billions of years exemplifies the power of exponential decay models and their critical role in scientific research.

Key takeaways:

- The decay of potassium-40 over 3.9 billion years leaves approximately 92 grams from an initial 800 grams.
- The half-life of potassium-40 (about 1.25 billion years) is fundamental to radiometric dating.
- Understanding decay processes provides insights into Earth's age and geological history.

Whether you're a student, a scientist, or an enthusiast, grasping these concepts enhances our appreciation of Earth's ancient past and the natural laws governing radioactive decay.

Meta Description: Discover how much of an 800-gram sample of potassium-40 remains after 3.9 billion years of radioactive decay. Learn about half-life, decay calculations, and the significance of isotopic decay in geology.

Frequently Asked Questions

What is the decay constant for potassium-40, given its half-life of approximately 1.25 billion years?

The decay constant (λ) can be calculated using $\lambda = \ln(2)$ / half-life. For potassium-40, $\lambda \approx 0.693 / 1.25 \times 10^9$ years $\approx 5.544 \times 10^{-10}$ per year.

How do you determine the remaining amount of potassium-40 after a certain period?

Use the exponential decay formula: $N(t) = N_0 e^{-\lambda t}$, where N_0 is the initial amount, λ is the decay constant, and t is time.

What is the initial amount of potassium-40 in the sample?

The initial amount is 800 grams, as given in the problem statement.

How long is 3.9 x 10^9 years in terms of decay calculations?

It is 3.9 billion years, representing the time over which decay occurs in this problem.

What is the formula to find the remaining amount of potassium-40 after 3.9 billion years?

Remaining amount = 800 g $e^{-\lambda\, 3.9 \times 10^9 \text{ years}}$.

Calculate the remaining potassium-40 after 3.9 billion years. What is the approximate mass?

Using $\lambda \approx 5.544 \times 10^{-10}$, the remaining amount $\approx$ 800 g $e^{-5.544 \times 10^{-10}\; 3.9 \times 10^9}$ $\approx$ 800 g $e^{-2.160}$ $\approx$ 800 g 0.1157 $\approx$ 92.56 grams.

What does the remaining mass tell us about the half-life of potassium-40?

It shows that after roughly 3.9 billion years (about 3.12 half-lives), most of the original sample has decayed, leaving about 11.6% of the original amount.

Why is understanding radioactive decay important in

geochronology?

Because it allows scientists to date rocks and fossils by measuring remaining radioactive isotopes like potassium-40 and calculating their age.

Can potassium-40 be used to date very old geological samples?

Yes, because its half-life is about 1.25 billion years, making it suitable for dating rocks up to about 4.5 billion years old.

What assumptions are involved in calculating the remaining potassium-40 after billions of years?

Assumptions include that decay occurs at a constant rate, no new potassium-40 is added, and the decay process follows exponential decay without interference.

Additional Resources

How Much Of An 800-Gram Sample Of Potassium-40 Will Remain After 3.9 × 10^9 Years Of Radioactive Decay?

Radioactive decay is a fundamental process that shapes the natural world, influencing everything from geological formations to the dating of ancient artifacts. Among the myriad radioactive isotopes, potassium-40 (K-40) holds particular importance due to its widespread presence in Earth's crust and its utility in radiometric dating. This article delves into a detailed investigation of the decay of potassium-40 over geological timescales, specifically addressing the question: How much of an 800-gram sample of potassium-40 will remain after 3.9 × 10^9 years? We will explore the properties of K-40, the principles of radioactive decay, the calculations involved, and the implications of such decay over billions of years.

Understanding Potassium-40: An Overview

Potassium-40 is a naturally occurring isotope of potassium, constituting about 0.0117% (or approximately 1.17 × 10^-4) of total potassium atoms in nature. It is a beta-emitter with a very long half-life, making it invaluable for dating ancient rocks and minerals.

Key properties of potassium-40:

- Half-life ($T_{1/2}$): Approximately 1.248 × 10^9 years
- Decay modes:
- Beta decay to calcium-40 (~89% of decays)
- Electron capture to argon-40 (~11% of decays)
- Decay constant (λ): Derived from the half-life and crucial for decay calculations

Because K-40 decays over geological timescales, understanding its decay behavior requires familiarity with exponential decay equations and decay constants.

The Principles of Radioactive Decay

Radioactive decay follows a stochastic process, with each nucleus decaying independently. The mathematical description employs the exponential decay law:

$$ N(t) = N_0 e^{-\lambda t} $$

Where:

- $N(t)$ = number of radioactive nuclei remaining after time t
- N_0 = initial number of nuclei
- λ = decay constant
- t = elapsed time

The decay constant relates to the half-life ($T_{1/2}$) via:

$$ \lambda = \frac{\ln 2}{T_{1/2}} $$

To determine how much of the initial sample remains after a given period, one must calculate $N(t)$ using the initial number of nuclei and the decay law.

Calculating the Decay Constant for Potassium-40

Given the half-life of potassium-40:

$$ T_{1/2} = 1.248 \times 10^9 \text{ years} $$

The decay constant:

$$ \lambda = \frac{\ln 2}{T_{1/2}} = \frac{0.6931}{1.248 \times 10^9} \approx 5.55 \times 10^{-10} \text{ yr}^{-1} $$

This decay constant indicates that each year, a small fraction of the remaining K-40 nuclei decay.

Initial Sample Composition and Number of Nuclei

The initial sample mass:

$$ m_0 = 800 \text{ grams} $$

Potassium's atomic mass:

$$ M_K \approx 39.1 \text{ g/mol} $$

Number of moles of potassium:

$$ n_{K} = \frac{m_0}{M_K} = \frac{800}{39.1} \approx 20.45 \text{ mol} $$

Total potassium atoms:

$$ N_{K} = n_{K} \times N_A $$

Where $N_A = 6.022 \times 10^{23} \text{ mol}^{-1}$ (Avogadro's number):

$$ N_{K} \approx 20.45 \times 6.022 \times 10^{23} \approx 1.232 \times 10^{25} \text{ atoms} $$

Number of potassium-40 atoms initially:

$$ N_{0} = f_{K-40} \times N_{K} $$

Where $f_{K-40} = 1.17 \times 10^{-4}$:

$$ N_{0} \approx 1.17 \times 10^{-4} \times 1.232 \times 10^{25} \approx 1.442 \times 10^{21} \text{ atoms} $$

This is the initial number of K-40 nuclei in the sample.

Decay Over 3.9 Billion Years: Calculation of Remaining Nuclei

To find how many K-40 nuclei remain after:

$$ t = 3.9 \times 10^9 \text{ years} $$

Apply the exponential decay law:

$$ N(t) = N_0 e^{-\lambda t} $$

Compute the exponent:

$$ \lambda t = 5.55 \times 10^{-10} \times 3.9 \times 10^9 \approx 2.165 $$

Calculate $N(t)$:

$$ N(t) = 1.442 \times 10^{21} \times e^{-2.165} $$

$$ e^{-2.165} \approx 0.115 $$

Thus,

$$ N(t) \approx 1.442 \times 10^{21} \times 0.115 \approx 1.66 \times 10^{20} \text{ atoms} $$

This is the number of K-40 nuclei remaining after 3.9 billion years.

Mass of Remaining Potassium-40

To find the mass of the remaining K-40:

- Mass per K-40 atom:

$$ M_{K-40} \approx 39.963 \text{ g/mol} $$

- Mass of $N(t)$ atoms:

$$ m_{K-40} = \frac{N(t)}{N_A} \times M_{K-40} $$

Calculate:

$$ m_{K-40} = \frac{1.66 \times 10^{20}}{6.022 \times 10^{23}} \times 39.963 \approx 1.10 \times 10^{-3} \text{ g} $$

Therefore, approximately 1.10 milligrams of potassium-40 would remain in the sample after 3.9 billion years.

Implications and Significance of the Decay Calculation

This calculation underscores several key insights:

- Extensive decay over geological timescales: Starting from 800 grams, the K-40 diminishes to just over 1 milligram after nearly 4 billion years.
- Half-life relevance: The decay is significant over these timescales, confirming the usefulness of K-40 in radiometric dating.
- Natural isotopic abundance: The initial amount of K-40 is minuscule relative to total potassium, but its long half-life allows it to serve as a geological clock.

Furthermore, understanding this decay process aids in interpreting Earth's age, dating ancient rocks, and studying planetary formation.

Additional Factors and Considerations

While the calculations above are precise under ideal conditions, several factors can influence the decay process and measurement:

- Isotopic variations: Natural potassium sources may have slight deviations in isotopic ratios.
- Decay modes: The decay to calcium-40 and argon-40 can influence mineral compositions over time.
- Geochemical processes: Weathering, melting, and mineralization can redistribute potassium and its isotopes.
- Measurement accuracy: Laboratory techniques such as mass spectrometry are used to detect minute quantities of K-40 and its decay products.

These considerations are vital for geochronologists and nuclear scientists when interpreting data.

Conclusion

In summary, an initial 800-gram sample of potassium-40 contains approximately 1.442×10^{21} atoms. After 3.9 billion years—close to the half-life of K-40—the sample would retain around 1.66×10^{20} nuclei, corresponding to roughly 1.10 milligrams of potassium-40. This profound decay over Earth's lifetime demonstrates the reliability of K-40 as a chronometer for dating ancient geological materials, providing critical insights into Earth's history and the processes that have shaped our planet.

Understanding the decay dynamics of potassium-40 not only enriches our knowledge of nuclear physics but also enhances our appreciation of the natural processes governing planetary evolution. It exemplifies how fundamental physical principles can be applied to unravel the vast timescales of Earth's history, connecting atomic-scale phenomena with the grand narrative of our planet's evolution.

5 How Much Of An 800 Gram Sample Of Potassium 40 Will Remain After 3 9 10 9 Years Of Radioactive Decay

5 How Much Of An 800 Gram Sample Of Potassium 40 Will Remain After 3 9 10 9 Years Of Radioactive Decay

Related Articles

- 6. For The Function Shown Below, Find All Values Of X In The Interval [0,21t): Y = Cos X Cot(x) To Which
- A Father Wants To Save In Advance For His 8-year Old Daughters College Expenses. The Daughter Will Enter
- A Pure Copper Cube Has An Edge Length Of 3.00 CmHow Many Copper Atoms Does It Contain? (volume Of A Cube

Back to Home