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6. A Sequence Is Defined By A = 1, An = (an-1 + 6) For N 2. (1) Find A And Ag. (2) Assume Lim An Exists.

Understanding sequences is fundamental in mathematics, especially in the study of limits, series, and calculus. The sequence defined by the recurrence relation $A_1 = 1$ and An = An-1 + 6 for N ≥ 2 represents an arithmetic sequence, a common type of sequence characterized by a constant difference between successive terms. This article delves into the properties of this sequence, explores methods to find its explicit formula, analyzes its limit behavior, and discusses its significance in broader mathematical contexts.

Introduction to Sequences and Recurrence Relations

Sequences are ordered lists of numbers following a specific pattern or rule. They are crucial in various branches of mathematics, including algebra, calculus, and number theory. A key aspect of understanding sequences involves identifying their explicit formulas and analyzing their limits.

Recurrence relations define each term of a sequence based on previous terms. They provide a recursive way to generate sequence elements but often need to be transformed into explicit formulas for easier analysis.

In our case, the sequence is defined as:

- $A_1 = 1$
- $A_n = A_{n-1} + 6$, for N ≥ 2

This is an arithmetic sequence, where each term increases by a fixed amount, 6, from the previous term.

Understanding the Given Sequence

Defining the Sequence

The sequence is given by:

- First term: $A_1 = 1$
- Recurrence relation: $A_n = A_{n-1} + 6$

This indicates each subsequent term is obtained by adding 6 to the previous term. To better understand its behavior and properties, it is essential to derive its explicit formula.

Properties of Arithmetic Sequences

An arithmetic sequence has the general form:

$A_n = A_1 + (n - 1)d$

where:

- A_1 is the first term
- d is the common difference between terms
- n is the term number

Given this, the sequence's explicit formula can be derived directly.

Part 1: Finding the First Term (A) and the Sequence Terms (Ag)

Calculating the First Term

From the problem statement, the first term is explicitly given:

$A_1 = 1$

This serves as the starting point for the sequence.

Explicit Formula for the Sequence

Using the general form of an arithmetic sequence:

$A_n = A_1 + (n - 1)d$

Substituting $A_1 = 1$ and $d = 6$:

$A_n = 1 + (n - 1) 6$

Simplify:

$A_n = 1 + 6n - 6$

$A_n = 6n - 5$

Thus, the explicit formula for the sequence is:

$A_n = 6n - 5$

This formula allows us to compute any term directly without relying on previous terms.

Calculating the Sequence Terms (Ag)

Using the explicit formula, we can find specific terms:

- For n = 1:

$A_1 = 6(1) - 5 = 6 - 5 = 1$

- For n = 2:

$A_2 = 6(2) - 5 = 12 - 5 = 7$

- For n = 3:

$A_3 = 6(3) - 5 = 18 - 5 = 13$

- For n = 4:

$A_4 = 6(4) - 5 = 24 - 5 = 19$

And so on. This sequence progresses as 1, 7, 13, 19, 25, 31, 37, etc., increasing by 6 each time.

Part 2: Analyzing the Limit of the Sequence (Lim An)

Assumption: Limit Exists

The problem states, "Assume Lim An exists." For an arithmetic sequence, the limit as n approaches infinity depends on the sign of the common difference:

- If $d > 0$, the sequence diverges to infinity.
- If $d < 0$, the sequence diverges to negative infinity.
- If $d = 0$, the sequence is constant, and the limit is that constant.

In our sequence, $d = 6$, which is positive. Therefore, intuitively, the sequence increases without bound as n becomes large, and the limit does not exist in the finite real numbers. However, the problem asks us to assume the limit exists, prompting a theoretical exploration.

Calculating the Limit

Given the explicit formula:

$A_n = 6n - 5$

As n approaches infinity:

$\lim_{n\to\infty} A_n = \lim_{n\to\infty} (6n - 5) = \infty$

Since the sequence increases without bound, its limit is infinite. In the context of real analysis, we say the limit does not exist finitely but diverges to infinity.

Implications of the Limit

- The sequence is unbounded.
- The sequence does not converge to a finite number.
- For practical purposes, in calculus and analysis, sequences like this are said to diverge.

Summary and Key Takeaways

- The sequence defined by $A_1 = 1$ and $An = An-1 + 6$ is an arithmetic sequence with a common difference of 6.
- Its explicit formula is $A_n = 6n - 5$, which allows direct calculation of any term.
- The sequence progresses as 1, 7, 13, 19, 25, 31, ..., increasing by 6 each time.
- As n approaches infinity, the sequence diverges to infinity, and the limit does not exist in finite terms.
- Understanding the behavior of such sequences is essential in calculus, especially in the study of limits and divergence.

Applications of Arithmetic Sequences in Mathematics and Real Life

Arithmetic sequences are fundamental in various fields, including:

- Mathematics Education: Teaching the concepts of sequences, series, and limits.
- Finance: Calculating fixed-rate savings or loan payments over time.
- Computer Science: Iterative algorithms with linear progression.
- Engineering: Signal processing involving linear signals.

Practical Examples

- Savings Account: Saving a fixed amount each month, where the total savings over time form an arithmetic sequence.
- Construction Planning: Number of bricks added each week to a project.
- Population Modeling: Growth with a constant rate added per period.

Conclusion

The sequence defined by $A_1 = 1$ and $An = An-1 + 6$ exemplifies an arithmetic sequence characterized by a constant difference. Its explicit formula, $A_n = 6n - 5$, simplifies the process of finding any term directly. While the sequence increases indefinitely, diverging to infinity, understanding its structure offers valuable insights into the behavior of linear sequences. Recognizing whether a sequence converges or diverges is crucial when analyzing limits, especially in calculus and mathematical analysis. Mastery of these concepts enables mathematicians and students to apply them across numerous real-world scenarios, from financial calculations to engineering designs.

Keywords: arithmetic sequence, recurrence relation, explicit formula, sequence limit, divergence, mathematical sequences, calculus, limits, sequence analysis

Frequently Asked Questions

What is the explicit formula for the sequence defined by $A_1 = 1$ and $A_n = A_{n-1} + 6$?

The explicit formula is $A_n = 1 + 6(n - 1)$.

How do you find the value of A_n in the sequence $A_1 = 1$, $A_n = A_{n-1} + 6$?

Using the explicit formula, $A_n = 6n - 5$.

What is the value of A_n when n = 2 in this sequence?

$A_2 = 6(2) - 5 = 12 - 5 = 7$.

If the limit of A_n exists as n approaches infinity, what is its value?

Since the sequence increases without bound, the limit as n approaches infinity is infinity.

Does the sequence $A_n = 1 + 6(n - 1)$ have a finite limit, and why?

No, because as n increases, A_n increases without bound, so the limit does not exist finitely; it diverges to infinity.

Additional Resources

Understanding the Sequence Defined by $A_1 = 1$ and $A_n = A_{n-1} + 6$ for $n \geq 2$: A Comprehensive Guide

Sequences are foundational concepts in mathematics, especially in algebra and calculus, serving as the building blocks for understanding limits, series, and functions. One particular sequence, defined by a simple recursive relation, offers insight into how patterns develop and how limits can be explored. In this article, we delve into the sequence where $A_1 = 1$ and for $n \geq 2$, $A_n = A_{n-1} + 6$, providing a detailed analysis, solutions to key problems, and an understanding of the sequence's behavior.

Introduction to the Sequence

The sequence in question is an arithmetic sequence, characterized by a fixed difference between consecutive terms. The recursive definition:

- $A_1 = 1$
- $A_n = A_{n-1} + 6$ for $n \geq 2$

implies that each term is obtained by adding 6 to the previous term, starting from 1. This simple rule creates a sequence of numbers that increase steadily, following a predictable pattern.

Part 1: Finding the General Terms A_n and $A_{n(g)}$

Understanding Recursive and Explicit Definitions

When dealing with sequences, two main approaches are often used:

- Recursive definition: Describes each term based on the previous one.
- Explicit (closed-form) formula: Provides a direct way to compute the nth term without referencing earlier terms.

The recursive definition here:

$$A_n = A_{n-1} + 6 \quad \text{with} \quad A_1 = 1$$

can be transformed into an explicit formula, allowing easier calculation and analysis.

Deriving the Explicit Formula for A_n

Given the recursive relation:

$$A_n = A_{n-1} + 6$$

and the initial term:

$$A_1 = 1$$

This sequence is an arithmetic sequence with:

- First term, $a_1 = 1$
- Common difference, $d = 6$

General formula for the nth term of an arithmetic sequence:

$$A_n = a_1 + (n - 1)d$$

Plugging in the known values:

$$A_n = 1 + (n - 1) \times 6$$

$$\boxed{A_n = 6n - 5}$$

This formula allows us to compute the sequence's terms directly.

Computing Specific Terms

Using the explicit formula:

n	$A_n = 6n - 5$	Calculation
1	$6(1) - 5$	$6 - 5 = 1$
2	$6(2) - 5$	$12 - 5 = 7$
3	$6(3) - 5$	$18 - 5 = 13$
4	$6(4) - 5$	$24 - 5 = 19$
5	$6(5) - 5$	$30 - 5 = 25$

This confirms the sequence:

$$ 1, 7, 13, 19, 25, \dots $$

which increases by 6 each time.

Summary for Part 1:

- The explicit formula for the sequence is:

$$ \boxed{A_n = 6n - 5} $$

- The sequence is an arithmetic sequence with:

- First term: $A_1 = 1$
- Common difference: 6

Part 2: Assuming the Limit of A_n Exists

Understanding Limits of Sequences

In calculus, the limit of a sequence as $n \to \infty$ describes its long-term behavior. For convergent sequences, the terms approach a finite value. For divergent sequences, the terms grow without bound or oscillate.

Given the explicit form:

$$ A_n = 6n - 5 $$

we can analyze whether the sequence converges or diverges.

Does the Limit of A_n Exist?

Since:

$$A_n = 6n - 5$$

as n increases indefinitely, A_n becomes arbitrarily large:

$$\lim_{n \to \infty} A_n = \lim_{n \to \infty} (6n - 5) = \infty$$

which indicates the sequence diverges to infinity.

Therefore:

- The sequence does not have a finite limit.
- The limit does not exist in the finite real numbers.

Implication of Divergence

The divergence of this sequence aligns with the nature of arithmetic sequences with a non-zero common difference. They tend to infinity (or negative infinity if the common difference is negative).

Addressing the Assumption: "Assume Lim A_n Exists"

If we assume the limit exists, then by the properties of limits, the sequence must approach a finite value. But, since the sequence grows without bound, this assumption leads to a contradiction, confirming that the limit does not exist.

Summary and Key Takeaways

Final remarks on the sequence:

- The sequence is arithmetic with initial term $A_1 = 1$ and common difference 6.
- The explicit formula for the sequence:

$$\boxed{A_n = 6n - 5}$$

- The sequence diverges to infinity as $n \to \infty$, so:

$$\boxed{\lim_{n \to \infty} A_n \text{ does not exist in } \mathbb{R}}$$

Practical applications and relevance:

- Recognizing the pattern allows for quick computation of terms.
- Understanding divergence is crucial in calculus and analysis, particularly when studying series and limits.
- The explicit formula provides a foundation for solving related problems, such as summing sequences or analyzing growth.

Additional Notes

Related Problems and Extensions

- Sum of the first n terms:

$$ S_n = \frac{n}{2} (A_1 + A_n) = \frac{n}{2} [1 + (6n - 5)] = \frac{n}{2} (6n - 4) = 3n^2 - 2n $$

- Behavior of the sequence in other contexts: For example, plotting the sequence shows a straight-line increase, emphasizing its linear growth.

Final Thoughts

Understanding how to derive explicit formulas from recursive definitions, analyze limits, and interpret the behavior of sequences is fundamental in mathematics. This sequence, simple yet illustrative, offers a clear example of the principles governing arithmetic progressions and the concept of divergence.

In conclusion, the sequence defined by $A_1=1$ and $A_n = A_{n-1} + 6$ is an arithmetic progression with explicit form $A_n=6n-5$, which diverges to infinity as n approaches infinity, meaning its limit does not exist in the finite realm. Recognizing these properties equips students and professionals with essential tools for mathematical analysis and problem-solving.

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