6. For The Function Shown Below, Find All Values Of X In The Interval [0,21t): Y = Cos X Cot(x) To Which

6. For The Function Shown Below, Find All Values Of X In The Interval [0,21π): Y = Cos X Cot(x) To Which

Understanding and solving trigonometric functions is a fundamental aspect of mathematics, especially when dealing with periodic functions and their properties. The problem at hand requires us to analyze the function Y = Cos X Cot(x) and find all values of X within the interval [0, 21π) that satisfy certain conditions. This involves exploring the behavior of the cosine and cotangent functions, understanding their domains, and solving the resulting equations systematically. This comprehensive guide will walk you through the process step-by-step, ensuring clarity and thoroughness.

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1. Introduction to the Functions Involved

Before diving into the solution, it's essential to understand the individual functions involved in the expression: cosine (Cos X) and cotangent (Cot x).

1.1 Cosine Function (Cos X)

- The cosine function, Cos X, is a fundamental trigonometric function that describes the x-coordinate of a point on the unit circle at an angle X measured from the positive x-axis.
- Properties:
- Periodicity: Cos X has a period of 2π.
- Range: [-1, 1].
- Symmetry: Even function, meaning Cos(-X) = Cos X.
- Zeroes: Cos X = 0 at X = (π/2) + nπ, where n is an integer.

1.2 Cotangent Function (Cot x)

- Cotangent, Cot x, is the reciprocal of tangent: Cot x = Cos x / Sin x.
- Properties:
- Domain: x ≠ nπ, where n is an integer, because Sin x = 0 at these points.
- Periodicity: Cot x has a period of π.
- Range: (-∞, ∞).
- Zeroes: Cot x = 0 at x = π/2 + nπ, as Cos x = 0 there.

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2. Analyzing the Function Y = Cos X Cot x

The function Y = Cos X Cot x involves both variables X and x. To interpret and solve the problem, we need to clarify the relationship between X and x in the given function, as well as the intended variable of differentiation or evaluation.

Important clarification:
- Typically, in such problems, the function is considered in terms of a single variable, X.
- The notation suggests that the function involves both Cos X and Cot X, possibly misprinted as Cot x.
- For consistency, we interpret the function as Y = Cos X Cot X, where the variable is X.

Assumption:
- The problem asks to find all X in [0, 21π) such that the function Y = Cos X Cot X satisfies certain conditions (e.g., equals a specific value, or is defined).

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3. Simplifying the Function Y = Cos X Cot X

To analyze the function effectively, we should simplify it:

Y = Cos X Cot X
= Cos X (Cos X / Sin X)
= (Cos^2 X) / Sin X

Key observations:
- The function is undefined where Sin X = 0, i.e., at X = nπ, where n is an integer.
- The numerator, Cos^2 X, ranges between 0 and 1.

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4. Domain Considerations

Given the expression Y = (Cos^2 X) / Sin X, the domain restrictions are critical:

- Sin X ≠ 0 → X ≠ nπ, where n ∈ ℤ.
- Since we're considering X in [0, 21π), the points where Sin X = 0 are:
- X = 0, π, 2π, 3π, ..., 20π (since 21π is the upper bound, but not included if considering open interval).

Therefore:
- X ∈ [0, 21π), excluding X = nπ for n = 0, 1, 2, ..., 20.

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5. Solving for Specific Conditions

Depending on what the problem asks—such as finding where Y equals a particular value or where the function reaches extrema—the approach varies.

Assuming the task is to find X where Y = 1:

$Y = (\cos^2 X) / \sin X = 1$

This leads to the equation:

$(\cos^2 X) / \sin X = 1$

Rearranged:

$\cos^2 X = \sin X$

---

6. Solving the Equation Cos^2 X = Sin X

This is a trigonometric equation involving both cosine and sine. To solve it:

Step 1: Use the Pythagorean identity:

$\cos^2 X = 1 - \sin^2 X$

Substitute into the equation:

$1 - \sin^2 X = \sin X$

Step 2: Rearrange to quadratic form:

$\sin^2 X + \sin X - 1 = 0$

Let $S = \sin X$

The quadratic becomes:

$S^2 + S - 1 = 0$

Step 3: Solve quadratic:

$S = [-1 \pm \sqrt{(1^2 - 4\ 1\ (-1))}] / (2\ 1)$

$= [-1 \pm \sqrt{(1 + 4)}] / 2$

$= [-1 \pm \sqrt{5}] / 2$

Step 4: Determine valid solutions:

Since Sin X ∈ [-1, 1], only the solutions within this range are valid.

- $S_1 = (-1 + \sqrt{5}) / 2$
- $S_2 = (-1 - \sqrt{5}) / 2$

Calculate approximate values:

- $S_1 \approx (-1 + 2.236) / 2 \approx 1.236 / 2 \approx 0.618$
- $S_2 \approx (-1 - 2.236) / 2 \approx -3.236 / 2 \approx -1.618$

Since Sin X cannot be less than -1 or greater than 1:

- $S_1 \approx 0.618$ (valid)
- $S_2 \approx -1.618$ (invalid, outside [-1, 1])

Therefore:

$|S| = (-1 + \sqrt{5})/2 \approx 0.618$

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7. Finding X Corresponding to Valid Solution

Now, find all X in $[0, 21\pi)$ such that Sin X ≈ 0.618.

Step 1: Find the general solutions:

$X = \arcsin(0.618) + 2\pi n$

or

$X = \pi - \arcsin(0.618) + 2\pi n$

Where n is an integer, and X is within the interval $[0, 21\pi)$.

Step 2: Calculate the principal value:

$\arcsin(0.618) \approx 0.666$ radians (approximate)

Step 3: Determine all solutions within the interval:

- First set:

$X = 0.666 + 2\pi n$

- Second set:

$X = \pi - 0.666 \approx 3.142 - 0.666 \approx 2.476 + 2\pi n$

Now, find all n such that $X \in [0, 21\pi)$:

Since $21\pi \approx 65.973$.

Step 4: Find possible n values:

For the first set:

$n \geq 0$

- n=0: $X \approx 0.666$
- n=1: $X \approx 0.666 + 6.283 \approx 6.949$
- n=2: $X \approx 0.666 + 12.566 \approx 13.232$
- n=3: $X \approx 0.666 + 18.849 \approx 19.515$
- n=4: $X \approx 0.666 + 25.132 \approx 25.798$ (> 21π, so exclude)

For the second set:

- n=0: $X \approx 2.476$
- n=1: $X \approx 2.476 + 6.283 \approx 8.759$
- n=2: $X \approx 2.476 + 12.566 \approx 15.042$
- n=3: $X \approx 2.476 + 18.849 \approx 21.325$
- n=4: $X \approx 2.476 + 25.132 \approx 27.608$ (> 21π), exclude.

Final list of solutions:

- From first set: $X \approx 0.666, 6.949, 13.232, 19.515$
- From second set: $X \approx 2.476, 8.759, 15.042, 21.325$

All these X values satisfy the original equation Y = 1, within the interval $[0, 21\pi)$.

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8. Further Considerations and Variations

Depending on the actual question, you might need to:

- Find where Y = 0
- Find maximum or minimum values of Y
- Analyze the behavior of Y over the interval
- Find

Frequently Asked Questions

What is the primary goal when solving for values of X in the interval [0, 21π) for the function Y = cos X · cot X?

The goal is to find all X in [0, 21π) where the function Y is defined and satisfies specific conditions, such as equal to a particular value or within a certain range.

How can the function Y = cos X · cot X be simplified using trigonometric identities?

Recall that cot X = cos X / sin X, so Y = cos X · (cos X / sin X) = (cos^2 X) / sin X.

Are there any restrictions on X when solving Y = cos X · cot X in the interval [0, 21π)?

Yes, since cot X = cos X / sin X, X cannot be values where sin X = 0, i.e., X ≠ nπ for integers n, within the interval.

How do you find the zeros of Y = (cos^2 X) / sin X?

Set numerator equal to zero: cos^2 X = 0, which implies cos X = 0. The solutions are X = π/2 + nπ. Also, check for points where the entire expression equals a specific value if needed.

What is the significance of the interval [0, 21π) in solving the problem?

The interval specifies the domain within which all solutions for X are to be found, considering the periodicity of the trigonometric functions involved.

How many solutions are expected within [0, 21π) for the zeros of cos X?

Since cos X = 0 at X = π/2 + nπ, for n = 0, 1, 2, ..., 20, the solutions are at X = π/2 + nπ, up to but not including 21π, resulting in 21 solutions.

What methods can be used to verify the solutions obtained for X?

Use substitution back into the original function to ensure the solutions satisfy the equation, and check for any points where the function is undefined due to division by zero.

Are there any points where Y is undefined in the given function?

Yes, Y is undefined where sin X = 0, i.e., at X = nπ, which must be excluded from the solution set.

How does the periodicity of cosine and cotangent functions influence the solution set?

Both functions are periodic with period 2π, which means solutions repeat every 2π, allowing for systematic counting of solutions within [0, 21π).

What is the final step to present all solutions X in [0, 21π) for the function Y = cos X · cot X?

Express all solutions as X = π/2 + nπ for n = 0, 1, 2, ..., 20, excluding values where sin X = 0, and verify they lie within the interval [0, 21π).

Additional Resources

6. For The Function Shown Below, Find All Values Of X In The Interval [0,21π): Y = Cos X Cot(x) To Which

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Introduction

Mathematics often presents intriguing challenges that deepen our understanding of the relationships between functions and their behaviors over specific intervals. One such problem involves analyzing a trigonometric function to identify all solutions within a given domain. The problem statement reads: "6. For the function shown below, find all values of x in the interval [0, 21π): Y = Cos x Cot x to which...". While the original prompt appears incomplete, it essentially invites us to investigate the solutions to the equation involving the function \(Y = \cos x \cot x \) within the interval from 0 to \(21\pi \).

This task combines fundamental trigonometric identities and analytical techniques to determine the precise values of \(x \) satisfying the equation. Whether you're a student brushing up on trigonometric functions or a mathematics enthusiast exploring the depths of periodic functions, this exploration offers a comprehensive insight into how to handle such problems systematically.

Understanding the Function \(Y = \cos x \cot x \)

What is \(\cot x \)?

Before delving into the solutions, it’s essential to understand the components of the function:

- \(\cos x \) is the cosine function, which oscillates between -1 and 1 with a period of \(2\pi \).
- \(\cot x \) is the cotangent function, defined as:

$$
\cot x = \frac{\cos x}{\sin x}
$$

with the domain excluding points where $\sin x = 0$, i.e., $x = n\pi$, where n is an integer, because division by zero is undefined.

Expressing Y in a more manageable form

Given:

$$
Y = \cos x \cot x
$$

Substitute $\cot x = \frac{\cos x}{\sin x}$:

$$
Y = \cos x \times \frac{\cos x}{\sin x} = \frac{\cos^2 x}{\sin x}
$$

This expression is valid everywhere $\sin x \neq 0$, which is consistent with the domain of $\cot x$.

Clarifying the Goal: Solving for x

The original problem suggests finding all x within the interval $[0, 21\pi)$ that satisfy a particular condition involving Y. While the prompt is incomplete, typical problems of this nature often involve setting the function equal to a constant or analyzing where the function attains certain values.

Assumption:

Suppose the problem asks to find all solutions to:

$$
Y = \cos x \cot x = k
$$

for some constant k, or perhaps to find all x for which the function equals a specific value, such as zero.

Given the structure, a common case is:

$$
\cos x \cot x = 0
$$

which simplifies the analysis and is a typical scenario in trigonometric equations.

Solving $Y = 0$: The Case $\cos x \cot x = 0$

Let’s analyze the scenario where:

$$
\cos x \cot x = 0
$$

This equation holds when either:

1. $\cos x = 0$, or
2. $\cot x = 0$

Analyzing each case:

Case 1: $\cos x = 0$

- $\cos x = 0$ at:

$$
x = \frac{\pi}{2} + n\pi, \quad n \in \mathbb{Z}
$$

- Within the interval $[0, 21\pi)$, determine all such x:

Since $x \geq 0$, and n is an integer, the solutions are:

$$
x = \frac{\pi}{2} + n\pi, \quad n \geq 0
$$

Find the maximum n such that:

$$
\frac{\pi}{2} + n\pi < 21\pi
$$

Divide all parts by π:

$$
\frac{1}{2} + n < 21
$$

$$
n < 20.5
$$

Since n is an integer, the largest n satisfying this is $n = 20$.

- So, the solutions are:

$$
x = \frac{\pi}{2} + n\pi, \quad n = 0, 1, 2, \ldots, 20
$$

Explicitly, solutions:

$$
x = \frac{\pi}{2} + n\pi, \quad n=0,1,2,...,20
$$

Total solutions in this case: 21 solutions.

Case 2: $\cot x = 0$

Recall:

$$
\cot x = \frac{\cos x}{\sin x} = 0
$$

This occurs when numerator $\cos x = 0$, but only when $\sin x \neq 0$, which is already accounted for in the previous case.

But, more precisely, $\cot x = 0$ when:

$$
\cos x = 0, \quad \text{and } \sin x \neq 0
$$

As established, $\cos x = 0$ at $x = \frac{\pi}{2} + n\pi$, and at these points, $\sin x = \pm 1 \neq 0$. Therefore, the solutions to $\cot x = 0$ are:

$$
x = \frac{\pi}{2} + n\pi, \quad n \in \mathbb{Z}
$$

which coincides with the solutions for $\cos x=0$.

Conclusion: Both cases lead to the same set of solutions.

Summary of Solutions for $Y = 0$

All solutions in $[0, 21\pi)$ are:

$$
\boxed{
x = \frac{\pi}{2} + n\pi, \quad n = 0, 1, 2, \ldots, 20
}
$$

with total solutions being 21 points.

Extending the Analysis: Other Possible Values of Y

While the above analysis covers the case where $Y = 0$, the original problem may involve more general conditions where Y equals some constant or a function of x.

Suppose the problem asks for solutions where:

$$
Y = \cos x \cot x = c
$$

for some constant c. The methodology would involve:

1. Expressing Y as:

$$
Y = \frac{\cos^2 x}{\sin x}
$$

2. Setting up the equation:

$$
\frac{\cos^2 x}{\sin x} = c
$$

3. Rearranging:

$$
\cos^2 x = c \sin x
$$

4. Using the identity $\cos^2 x = 1 - \sin^2 x$:

$$
1 - \sin^2 x = c \sin x
$$

5. Forming a quadratic in $\sin x$:

$$
\sin^2 x + c \sin x - 1 = 0
$$

\]

6. Solving for \(\sin x \):

\[
\sin x = \frac{-c \pm \sqrt{c^2 + 4}}{2}
\]

Solutions are valid only if:

\[
|\sin x| \leq 1
\]

and then, for each valid \(\sin x \), find all \(x \) in \([0, 21\pi) \) satisfying:

\[
\sin x = \text{that value}
\]

which occurs at:

\[
x = \arcsin (\text{value}) + 2k\pi, \quad \text{or} \quad x = \pi - \arcsin (\text{value}) + 2k\pi
\]

for integers \(k \), within the interval constraints.

Considerations for Domain and Discontinuities

In the analysis above, some key points require attention:

- The domain excludes points where \(\sin x = 0 \), i.e., \(x = n\pi \), because \(\cot x \) is undefined there.
- When solving for \(\sin x \), ensure the solutions do not violate the domain restrictions.
- The periodicity of sine and cosine functions implies solutions repeat every \(2\pi \). To find all solutions within \([0, 21\pi) \), count how many periods fit into the interval.

Visualizing the Function and Its Solutions

Graphical analysis significantly aids understanding. Plotting \(Y = \cos x \cot x \):

- Reveals the behavior near discontinuities at \(x = n\pi \).
- Shows the oscillations and where the function crosses specific values.
- Highlights asymptotes at points where \(\sin x = 0 \), i.e., \(

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