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7. Let $F(x) = -3x + 9x - 3$. A. Determine The X Values Where $F'(x) = 0$. B. Fill In The Table Below To Find

Understanding the behavior of functions is a fundamental aspect of calculus, especially when analyzing their critical points and maxima or minima. This article delves into the process of differentiating a given function, finding where its derivative equals zero, and interpreting the results through tabular analysis. Specifically, we will explore the function $F(x) = -3x + 9x - 3$, determine the critical points by solving for where $F'(x) = 0$, and then use a structured table to analyze the function's behavior across different x-values. This comprehensive guide aims to clarify the steps involved in derivative calculation, critical point identification, and function analysis, making complex concepts accessible for students and enthusiasts alike.

Understanding the Function $F(x) = -3x + 9x - 3$

Before diving into calculations, it's important to simplify the function and understand its structure.

Simplification of the Function

The given function is:

$F(x) = -3x + 9x - 3$

Combine like terms:

$F(x) = (-3x + 9x) - 3$
$F(x) = 6x - 3$

This simplified form makes differentiation straightforward.

Part A: Determining the X Values Where $F'(x) = 0$

The first step in analyzing the function is to find its derivative, which indicates the rate of change of the function at any point x.

Calculating the Derivative of $F(x)$

Given the simplified function:

$F(x) = 6x - 3$

The derivative with respect to x is:

$F'(x) = d/dx\,[6x - 3] = 6$

This derivative tells us that the slope of the function at any point x is constant and equal to 6.

Finding Critical Points: When Does $F'(x) = 0$?

Critical points occur where the derivative equals zero or is undefined. Since $F'(x) = 6$, which is a constant and never zero, there are no points where the derivative equals zero.

Therefore:

- $F'(x) \neq 0$ for all x
- There are no critical points where the slope is zero

This indicates that the function is strictly increasing everywhere with no local maxima or minima.

Part B: Filling In The Table To Find the Function's Behavior

Although the derivative is constant, analyzing the function's behavior across different x-values can help confirm its increasing nature. To do this, we'll evaluate the function at various points and observe how it changes.

Creating the Table

Suppose the table includes columns for x-values, $F(x)$, and the interpretation of whether the function is increasing or decreasing.

x-value	$F(x) = 6x - 3$	Behavior
-2	$6(-2) - 3 = -12 - 3 = -15$	Decreasing / Increasing?
0	$6(0) - 3 = -3$	Increasing
1	$6(1) - 3 = 6 - 3 = 3$	Increasing
3	$6(3) - 3 = 18 - 3 = 15$	Increasing
5	$6(5) - 3 = 30 - 3 = 27$	Increasing

From these calculations, it's clear that as x increases, $F(x)$ also increases. Since the derivative is positive (6) everywhere, the function is strictly increasing across its entire domain.

Understanding the Significance of the Derivative and Critical Points

In calculus, the derivative provides crucial information about a function's increasing or decreasing behavior.

Key Points to Remember:

1. When $F'(x) > 0$, the function is increasing.
2. When $F'(x) < 0$, the function is decreasing.
3. When $F'(x) = 0$, the function may have a local maximum, minimum, or saddle point.

In our specific case:

- The derivative $F'(x) = 6$ is positive everywhere.
- Therefore, the function $F(x) = 6x - 3$ is strictly increasing for all real x.
- There are no points where the slope is zero, indicating the absence of local extrema.

Visualizing the Function $F(x) = 6x - 3$

Graphing the function helps reinforce the analysis. Since the function is linear with a positive slope:

- The graph is a straight line with a slope of 6.
- It crosses the y-axis at -3 (the y-intercept).
- The line rises steadily as x increases.

This visualization confirms the calculus-based conclusion: the function has no critical points and is monotonically increasing.

Applications and Real-World Examples

Understanding how to analyze functions like $F(x) = 6x - 3$ has practical applications in various fields:

1. Economics
A company's revenue modeled as $R(x) = 6x - 3$ indicates revenue increases steadily with sales volume x.

2. Physics
A constant velocity scenario where displacement $s(t) = 6t - 3$ indicates uniform motion.

3. Engineering
Designing systems where a parameter increases linearly over time or input.

Summary of Key Concepts

To summarize the process involved in analyzing functions like $F(x) = -3x + 9x - 3$:

- Simplify the function to its basic form.
- Calculate the derivative to understand the rate of change.
- Determine where the derivative equals zero to find critical points.
- Use a table or graph to analyze the function's behavior across different x-values.

- Interpret the results to understand whether the function is increasing, decreasing, or has extrema.

In this particular case, the function is linear and increasing everywhere, with no critical points where the derivative is zero.

Conclusion

Analyzing the function $F(x) = -3x + 9x - 3$ reveals that, once simplified to $F(x) = 6x - 3$, its derivative is a constant positive value. This indicates a steady increase across the entire domain, with no points where the slope equals zero. Such analysis underscores the importance of derivatives in understanding the behavior of functions, especially in identifying critical points and determining whether a function is increasing or decreasing. Using tabular data and visualization techniques enhances comprehension, making calculus concepts more tangible and applicable in real-world contexts.

Further Practice and Resources

To deepen your understanding of derivatives and function analysis, consider exploring the following:

1. Practice differentiating various polynomial functions.
2. Analyze functions with variable derivatives to identify multiple critical points.
3. Use graphing tools to visualize functions and their derivatives.
4. Study applications in physics, economics, and engineering for practical insights.

For more tutorials and exercises, online platforms like Khan Academy, Paul's Online Math Notes, and Coursera offer comprehensive lessons on calculus fundamentals.

By mastering the process of function analysis, students and professionals can develop critical analytical skills essential in mathematics, science, and engineering disciplines.

Frequently Asked Questions

What is the simplified form of the function $F(x) = -3x + 9x - 3$?

The simplified form is $F(x) = 6x - 3$.

How do you find the derivative $F'(x)$ of the function $F(x) = 6x - 3$?

Since $F(x) = 6x - 3$, its derivative $F'(x) = 6$.

What are the x-values where $F'(x) = 0$?

Since $F'(x) = 6$, which is a constant and never zero, there are no x-values where $F'(x) = 0$.

What does it mean if $F'(x)$ does not equal zero at any point?

It indicates that the function $F(x)$ has no critical points where the slope is zero, meaning it is strictly increasing or decreasing everywhere.

How do I interpret the table I need to fill out for the problem?

You should input x-values and corresponding $F(x)$ values to analyze the behavior of the function over those points.

What values should I choose for x in the table to analyze $F(x)$?

Choose a range of x-values around the critical points, but since $F'(x) = 6$ everywhere, the function is linear, so any x-values will do for analysis.

What is the significance of finding where $F'(x) = 0$?

Finding where $F'(x) = 0$ helps identify critical points, which could be local maxima, minima, or points of inflection.

Can the function $F(x) = 6x - 3$ have any maximum or minimum points?

No, because its derivative is constant (6), indicating the function is always increasing and has no local maxima or minima.

How does understanding $F'(x)$ help in graphing the function?

Knowing $F'(x)$ helps determine the function's increasing or decreasing behavior, slope, and potential critical points, which are essential for accurate graphing.

Additional Resources

Understanding the Derivative and Critical Points of the Function $F(x)$

When delving into the world of calculus, the concept of derivatives becomes an indispensable tool for analyzing the behavior of functions. In particular, understanding where the derivative of a function equals zero allows us to identify critical points—potential local maxima, minima, or points of inflection. Today, we'll explore the function:

$$F(x) = -3x + 9x - 3$$

and work through the steps to determine where its derivative equals zero, as well as interpret the significance of these points. This comprehensive analysis will serve as both a tutorial and an example of applying calculus principles to a straightforward algebraic function.

Deciphering the Function F(x): Simplification and Initial Insights

Before diving into derivatives and critical points, it's prudent to simplify the given function to its most straightforward form. The function provided is:

$$F(x) = -3x + 9x - 3$$

Let's combine like terms:

$$-3x + 9x = (-3 + 9) x = 6x$$

So, the simplified form of the function becomes:

$$F(x) = 6x - 3$$

This simplification reveals that F(x) is a linear function with a slope of 6 and a y-intercept at -3. Knowing this is crucial because the derivative of a linear function is constant, which impacts the number and nature of critical points.

Part A: Determining the X Values Where F'(x) = 0

Calculating the Derivative of F(x)

The derivative of a function provides the rate at which the function's output changes concerning its input. For polynomial functions, the power rule is straightforward:

$$\frac{d}{dx} [ax + b] = a$$

Applying this rule to our simplified function:

$$F(x) = 6x - 3$$

we get:

$$F'(x) = 6$$

This derivative is constant, indicating that the function F(x) increases at a steady rate of 6 units per 1 unit increase in x.

Finding Critical Points: When Does F'(x) = 0?

Critical points occur where the derivative is zero or undefined. Since $F'(x) = 6$, which is a constant and never zero, the equation:

$$
F'(x) = 0
$$

has no solutions. Formally:

$$
6 = 0 \quad \text{(No solutions)}
$$

This means that the function has no critical points where the slope is zero. Geometrically, this aligns with the fact that the original function is a straight line with a constant positive slope, continuously increasing without any peaks or valleys.

Conclusion for Part A:

- There are no x-values where $F'(x) = 0$.

This is a significant insight because it indicates the function is monotonic—either always increasing or always decreasing—without turning points.

Part B: Filling In The Table to Find Critical Points and Behavior

While the function has no points where the derivative equals zero, it's still valuable to analyze the function's behavior over specific intervals. Typically, in a problem like this, a table might look like this:

x-value	F(x)	F'(x)	Behavior of F(x)
x_1			
x_2			
x_3			

Since the derivative is constant, the table's main purpose is to understand the function's increasing nature across different x-values.

Constructing the Table: Choosing Sample x-Values

Given the linearity, we can select several x-values to understand the function's behavior:

- $x = -2$ (less than 0)
- $x = 0$ (the y-intercept)
- $x = 2$ (greater than 0)

Let's compute the corresponding F(x):

$$
F(x) = 6x - 3
$$

Calculations:

x-value	f(x)	f′(x)	Behavior of f(x)
-2	$6(-2) - 3 = -12 - 3 = -15$	6	Strictly increasing; decreasing is impossible due to positive slope
0	$6(0) - 3 = -3$	6	Continuing increase
2	$6(2) - 3 = 12 - 3 = 9$	6	Steadily increasing

Interpretation:

- The function's value increases as x increases.
- The slope of 6 confirms the function is monotonically increasing over the entire domain.

Understanding the Behavior and Critical Points

Since the derivative is constant and never zero, the function does not have any local maxima or minima—its graph is a straight, upward-slanting line. The critical point analysis confirms this:

- No critical points exist because the derivative never equals zero.

- The function is increasing across its entire domain.

This behavior is typical for linear functions with positive slopes: they are always increasing, with no points of inflection or turning points.

Implications and Broader Context

While this example involves a simple linear function, the principles used here are foundational in calculus and analytical geometry. Recognizing that the derivative of a linear function is constant simplifies the critical point analysis, but more complex functions require a detailed examination of where derivatives are zero or undefined.

In applied contexts—such as physics, economics, or engineering—these critical points often correspond to maximum efficiency, optimal conditions, or equilibrium states. In the case of linear functions like f(x), the absence of critical points indicates a consistent trend—either always increasing or decreasing—without local extremities.

Summary and Key Takeaways

- The original function simplifies to $f(x) = 6x - 3$.
- Its derivative $f'(x) = 6$ is a constant, positive value.
- Since $f'(x) \neq 0$ for any x, the function has no critical points.
- The function is strictly increasing over the entire domain.
- Choosing sample x-values illustrates the function's linear, unbounded increase.

Understanding these fundamental concepts paves the way for analyzing more complex functions where derivatives are not constant, and critical points play a vital role in understanding the function's shape and behavior.

In conclusion, this analysis exemplifies the power of calculus in revealing the intrinsic properties of functions. Whether dealing with simple linear equations or intricate polynomial curves, recognizing when derivatives equal zero guides us to understand the function's potential maxima, minima, and points of inflection—cornerstones in the study of mathematical analysis.

7 Let F X 3x 9x 3 A Determine The X Values Where F X 0 B Fill In The Table Below To Find

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