

A 94.0 A Current Circulates Around A 2.40-mm -diameter Superconducting Ring What Is The On Axis Magnetic

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Understanding the magnetic field generated by a current-carrying superconducting ring is a fundamental topic in electromagnetism, especially relevant in applications such as magnetic resonance imaging (MRI), particle accelerators, and quantum computing. When a 94.0 A current circulates through a superconducting ring with a diameter of 2.40 mm, it produces a magnetic field that is primarily oriented along the axis passing through the center of the ring. This article explores the principles governing the magnetic field on the axis of such a superconducting ring, providing insights into how it is calculated, its characteristics, and its practical implications.

Fundamentals of Magnetic Fields in Current Loops

Magnetic Field Produced by a Circular Current Loop

A current-carrying loop generates a magnetic field described by the Biot-Savart Law, which states that the magnetic field $d\mathbf{B}$ at a point in space is proportional to the current I , the length element $d\mathbf{L}$, and inversely proportional to the square of the distance r between the element and the point:

$$d\mathbf{B} = \frac{\mu_0}{4\pi} \frac{I d\mathbf{L} \times \hat{\mathbf{r}}}{r^2}$$

where:

- μ_0 is the permeability of free space ($4\pi \times 10^{-7} \text{ T}\cdot\text{m/A}$)
- $d\mathbf{L}$ is the differential length element of the current element
- $\hat{\mathbf{r}}$ is the unit vector from the current element to the point of observation
- r is the distance between the current element and the point

For a circular loop, the magnetic field along its axis simplifies due to symmetry, leading to a well-known formula that describes the magnetic field at any point along the axis passing through the center.

Key Parameters of the Superconducting Ring

- Current (I): 94.0 A
- Diameter (D): 2.40 mm = 0.0024 m
- Radius (R): $D/2 = 1.20 \text{ mm} = 0.0012 \text{ m}$

Since the ring is superconducting, it can sustain this current indefinitely without resistance, making it ideal for generating stable magnetic fields.

Calculating the On-Axis Magnetic Field

Magnetic Field Formula on the Axis

The magnetic field B_z at a point located a distance z along the axis from the center of a circular current loop is given by:

$$B_z(z) = \frac{\mu_0 I R^2}{2(R^2 + z^2)^{3/2}}$$

Where:

- z is the axial distance from the center of the ring
- R is the radius of the ring
- I is the current in the ring

This formula allows calculating the magnetic field at any point along the axis, including at the center ($z=0$) and far away from the ring ($z \rightarrow \infty$).

Calculating the Magnetic Field at the Center of the Ring

At the center of the ring ($z=0$), the formula simplifies to:

$$B_{\text{center}} = \frac{\mu_0 I}{2 R}$$

Substituting the known values:

$$B_{\text{center}} = \frac{(4\pi \times 10^{-7} \text{ H/m}) \times 94.0 \text{ A}}{2 \times 0.0012 \text{ m}}$$

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Calculations:

1. Numerator: $(4\pi \times 10^{-7} \times 94.0 \approx 1.178 \times 10^{-4})$
2. Denominator: $(2 \times 0.0012 = 0.0024)$

Thus:

$$B_{\text{center}} \approx \frac{1.178 \times 10^{-4}}{0.0024} \approx 0.0491, \text{ T}$$

or approximately 49.1 millitesla (mT).

This indicates a strong magnetic field at the center of the superconducting ring, essential for various applications.

Magnetic Field at Various Axial Positions

The magnetic field decreases as the distance from the center increases, following the inverse cubic relation in the denominator. For points far away ($z \gg R$), the field diminishes rapidly, approximating that of a magnetic dipole.

Implications and Applications of the On-Axis Magnetic Field

Practical Significance in Scientific and Medical Devices

Understanding the magnetic field generated by a superconducting ring is vital in several fields:

- Magnetic Resonance Imaging (MRI): Superconducting coils create uniform magnetic fields necessary for high-resolution imaging.
- Particle Accelerators: Superconducting rings serve as magnets to steer and focus particle beams.
- Quantum Computing: Superconducting loops form the basis of qubits, where precise magnetic control is crucial.
- Magnetic Traps: Used in trapping ultracold atoms or ions, relying on well-characterized magnetic fields.

Design Considerations for Superconducting Magnets

When designing superconducting rings for specific magnetic field strengths:

- Current magnitude: Higher currents produce stronger fields but require robust superconducting materials.
- Ring dimensions: Larger rings produce more extensive fields, but size constraints may apply.
- Material properties: Superconductors with high critical current densities enable higher current flow without resistance.
- Thermal management: Maintaining superconductivity requires cryogenic cooling to prevent quenching.

Safety and Limitations

While superconducting rings can generate intense magnetic fields, safety considerations include:

- Magnetic field exposure: Strong magnetic fields can affect electronic devices and pose health risks.
- Quenching risk: Sudden loss of superconductivity can cause rapid changes in the magnetic field and local heating.
- Structural integrity: Mechanical stresses due to magnetic forces must be managed in the design.

Advanced Topics and Related Concepts

Magnetic Field Uniformity and Shielding

In practical applications, achieving a uniform magnetic field is often necessary. Techniques involve:

- Using multiple coils in specific arrangements (e.g., Helmholtz coils)
- Incorporating magnetic shields to contain stray fields
- Employing active feedback systems for field stabilization

Superconducting Material Selection

Common superconducting materials used in rings include:

- Niobium-titanium (NbTi)
- Niobium-tin (Nb₃Sn)
- High-temperature superconductors like YBCO (Yttrium Barium Copper Oxide)

Material choice impacts:

- Critical current density
- Operating temperature
- Mechanical properties

Quantum Effects in Superconducting Rings

Superconducting rings exhibit quantum phenomena such as:

- Flux quantization: Magnetic flux through the ring occurs in discrete units ($\Phi_0 = h/2e \approx 2.07 \times 10^{-15} \text{ Wb}$)
- Persistent currents: Currents may persist indefinitely without decay
- Quantum interference: Used in SQUIDs (Superconducting Quantum Interference Devices) for ultra-sensitive magnetic measurements

Conclusion

The magnetic field on the axis of a superconducting ring with a 94.0 A current and a diameter of 2.40 mm is a critical parameter in many advanced technological applications. At the center of the ring, the magnetic field reaches approximately 49.1 mT, a substantial value for such a small structure. Understanding the calculation of this field through the Biot-Savart Law and the simplified axial formula is essential for designing and optimizing devices that rely on superconducting magnetic fields. As technology advances, the ability to precisely control and utilize these magnetic fields opens new frontiers in medicine, physics, and quantum computing.

Keywords: superconducting ring, magnetic field calculation, on-axis magnetic field, Biot-Savart Law, magnetic resonance imaging, quantum computing, superconducting magnets, flux quantization, persistent currents, high-temperature superconductors

Frequently Asked Questions

What is the significance of a 94.0 A current circulating

around a superconducting ring?

The 94.0 A current indicates a substantial persistent current flowing through the superconducting ring, which can generate a significant magnetic field due to the ring's superconducting properties and zero resistance, enabling stable magnetic flux trapping.

How does the diameter of 2.40 mm affect the magnetic field produced on the axis of the superconducting ring?

The small diameter of 2.40 mm results in a localized magnetic field on the axis, with the magnetic field strength depending on the current and the size of the ring, producing a more intense and concentrated magnetic field near the ring's center.

How can the magnetic field on the axis of the superconducting ring be calculated?

The magnetic field on the axis can be calculated using the Biot-Savart law or the formula for a current loop, which relates the field to the current, the radius of the ring, and the position along the axis, typically involving the expression $B = (\mu_0 I R^2) / (2(R^2 + x^2)^{3/2})$.

What role does superconductivity play in maintaining the 94.0 A current in the ring?

Superconductivity allows the current to persist indefinitely without resistance, enabling the 94.0 A current to circulate continuously and generate stable magnetic fields without energy loss.

What practical applications can arise from such a superconducting ring with a circulating current?

Applications include magnetic resonance imaging (MRI), superconducting magnets for particle accelerators, magnetic energy storage systems, and quantum computing components that rely on stable magnetic fields generated by persistent currents.

How does the on-axis magnetic field vary with distance from the center of the superconducting ring?

The magnetic field on the axis decreases with increasing distance from the ring's center, following a specific mathematical relationship that depends on the current and the ring's radius, reaching maximum at the center and diminishing further away.

What safety considerations are important when working with superconducting rings carrying high currents?

Safety considerations include managing cryogenic temperatures required for superconductivity, preventing magnetic field hazards, ensuring proper insulation, and avoiding quenching events that can lead to rapid energy release and potential damage.

Additional Resources

A 94.0 A Current Circulates Around A 2.40-mm Diameter Superconducting Ring: An In-Depth Analysis of On-Axis Magnetic Fields

Introduction

Superconducting rings are fundamental components in various advanced technological applications, including magnetic resonance imaging (MRI), particle accelerators, quantum computing, and magnetic levitation systems. The behavior of magnetic fields generated by currents in such rings is a pivotal aspect of their functionality. In this review, we delve into the scenario where a substantial current of 94.0 A circulates through a superconducting ring with a diameter of 2.40 mm, focusing specifically on the on-axis magnetic field produced.

Understanding the magnetic field distribution generated by a superconducting ring not only provides insights into fundamental electromagnetic principles but also informs practical design considerations in high-precision magnetic systems. This comprehensive analysis covers the theoretical framework, calculations, physical implications, and potential applications of such a system.

Fundamental Concepts

Superconductivity and Its Significance

Superconductivity is a quantum mechanical phenomenon characterized by zero electrical resistance and the expulsion of magnetic fields (Meissner effect) below a critical temperature. Superconducting materials can sustain persistent currents with negligible energy loss, making them ideal for generating stable magnetic fields over extended periods.

Magnetic Fields Around Current-Carrying Loops

A current-carrying loop produces a magnetic field that can be modeled using classical electromagnetic principles. The geometry and current magnitude significantly influence the field's strength and spatial distribution. For small loops, the magnetic field at points along the axis can be approximated using simplified formulas derived from the Biot-Savart law.

Geometry and Parameters of the Superconducting Ring

- Ring Diameter (D): 2.40 mm
- Ring Radius (r): Half of diameter = 1.20 mm = 1.20×10^{-3} m
- Current (I): 94.0 A

The ring is assumed to be a perfect, thin, superconducting loop with negligible thickness compared to its radius, simplifying the magnetic field calculations.

Theoretical Framework for Magnetic Field Calculation

Biot-Savart Law

The Biot-Savart law relates the magnetic field $\mathbf{d}$ at a point P to a current element $d\ell$:

$$\mathbf{B} = \frac{\mu_0}{4\pi} \int \frac{I \, d\boldsymbol{\ell} \times \mathbf{\hat{r}}}{r^2}$$

where:

- $\mu_0 = 4\pi \times 10^{-7} \, \text{H/m}$ (permeability of free space)
- $d\boldsymbol{\ell}$: differential length element of the current path
- $\mathbf{\hat{r}}$: unit vector from the current element to the point P
- r : distance from the current element to P

For a circular loop, the calculation simplifies when considering points along the axis of the ring.

On-Axis Magnetic Field for a Circular Loop

The magnetic field at a point z along the axis of a circular current loop is given by:

$$B_z = \frac{\mu_0 I r^2}{2 (r^2 + z^2)^{3/2}}$$

where:

- r : radius of the loop
- z : axial distance from the center of the loop to the point where the field is measured

This formula assumes an idealized thin, circular loop with uniform current distribution.

Calculating the On-Axis Magnetic Field

Scenario: Field at the Center of the Ring

The most straightforward case is on the axis at the center of the superconducting ring, i.e., $z = 0$.

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$$B_{\text{center}} = \frac{\mu_0 I}{2r}$$

Plugging in the values:

$$B_{\text{center}} = \frac{(4\pi \times 10^{-7}) \times 94.0}{2 \times 1.20 \times 10^{-3}}$$

Calculations:

1. Calculate numerator:

$$4\pi \times 10^{-7} \times 94.0 \approx (12.566 \times 10^{-7}) \times 94.0 \approx 1.180 \times 10^{-4}$$

2. Denominator:

$$2 \times 1.20 \times 10^{-3} = 2.40 \times 10^{-3}$$

3. Magnetic field:

$$B_{\text{center}} \approx \frac{1.180 \times 10^{-4}}{2.40 \times 10^{-3}} \approx 0.0492 \text{ T}$$

Final Result:

$$\boxed{\textbf{Magnetic field at the center} \approx 49.2 \text{ mT}}$$

This is a substantial magnetic field, showcasing the potential of superconducting rings to generate strong, stable magnetic fields.

Field at a Point Along the Axis ($z \neq 0$)

For points away from the center, the field diminishes according to:

$$B_z = \frac{\mu_0 I r^2}{2 (r^2 + z^2)^{3/2}}$$

This formula enables mapping the magnetic field along the axis, which is critical in

applications like MRI, where field homogeneity is essential.

Physical Implications and Physical Limitations

Superconducting State and Persistent Currents

Superconductors can sustain the 94.0 A current indefinitely without resistive losses, provided they remain below their critical temperature, critical magnetic field, and critical current density. This ensures a stable and persistent magnetic field, advantageous for applications requiring long-term field stability.

Magnetic Field Strength and Safety Considerations

The calculated magnetic field (~ 49.2 mT at the center) is significant but well within safety limits for many applications. However, in practice, higher currents or larger loops could produce stronger fields, necessitating safety protocols, magnetic shielding, and structural considerations.

Superconductor Material and Critical Parameters

The actual maximum current and magnetic field a superconductor can carry depend on its material properties, including:

- Critical temperature (T_c)
- Critical magnetic field (H_c)
- Critical current density (J_c)

High-temperature superconductors like YBCO or BSCCO are often used for such applications due to their higher (T_c) and (J_c) . Ensuring the superconductor remains below these critical parameters is essential for maintaining superconductivity.

Practical Considerations in Experimental and Applied Contexts

Manufacturing and Material Constraints

- Thinness of the ring: The assumption of a thin, uniform superconductor simplifies calculations but in practice, the thickness must be sufficient to handle the current without flux penetration or heating.
- Material purity and defect control: Imperfections can cause localized flux entry, reducing the maximum sustainable current.
- Cooling requirements: Superconductivity necessitates cryogenic cooling, often using liquid nitrogen or helium, influencing design and operational costs.

Magnetic Field Homogeneity and Design Optimization

In applications like MRI or particle accelerators, the uniformity of the magnetic field is critical. To achieve this:

- Multiple rings or coils may be arranged in specific geometries.
- Shimming techniques are employed to correct field inhomogeneities.
- The size, shape, and current distribution are optimized through computational models.

Limitations and Challenges

- Flux trapping: Superconducting rings can trap magnetic flux during cooling, affecting the field.
- Quenching: Sudden loss of superconductivity (quench) can lead to rapid heating and potential damage.
- Mechanical stability: The magnetic forces exerted by high currents can cause mechanical stress, requiring robust structural support.

Advanced Topics and Emerging Research

Quantum Effects in Superconducting Rings

At the quantum level, superconducting rings exhibit phenomena such as flux quantization, where magnetic flux threading the ring takes discrete values, and persistent current states, which are foundational in quantum computing technologies like superconducting qubits.

High-Field Superconducting Magnets

Research continues into increasing the maximum achievable magnetic fields with superconducting coils, exploring high- J_c materials and innovative configurations.

Integration into Quantum Devices

Superconducting rings are integral in devices such as SQUIDs (Superconducting Quantum Interference Devices), which are ultra-sensitive magnetometers used in geophysics, medicine, and fundamental physics research.

Conclusion

The analysis of a 94.0 A current circulating around a 2.40-mm diameter superconducting ring reveals a highly potent magnetic field, approximately 49.2 millitesla at the ring's center. This demonstrates the remarkable ability of superconducting loops to generate strong, stable magnetic fields with minimal energy loss, underpinning their importance in cutting-edge technological applications.

Understanding the theoretical underpinnings, precise calculations, and practical constraints allows engineers and physicists to optimize superconducting magnetic systems for diverse uses. As research advances, the capabilities of such systems will continue to expand, pushing the boundaries of magnetic field strength, stability, and application scope, ultimately contributing to innovations across medicine, industry, and fundamental science.

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