A Box Contains 3 Red, 5 White And 2 Blue Balls. 3 Balls Are Selected At Random Without Replacement. Find

A Box Contains 3 Red, 5 White And 2 Blue Balls. 3 Balls Are Selected At Random Without Replacement. Find

Understanding probability problems involving drawing balls from a container is a fundamental aspect of combinatorics and statistics. Such problems are common in exams, quizzes, and real-world scenarios where randomness and chance play a critical role. In this article, we will explore the detailed process of solving a typical problem: given a box with a specific composition of balls, what is the probability of selecting certain combinations? We will analyze the problem step-by-step, covering concepts like probability calculations, combinations, and the application of formulas to determine different probability scenarios.

Understanding the Problem

Problem Statement

The problem states:

> A box contains 3 red, 5 white, and 2 blue balls. 3 balls are selected at random without replacement. Find the probability of various events related to the selection.

This problem involves several key components:

- Total number of balls in the box
- Types and quantities of each colored ball
- The selection process (without replacement)
- Probabilistic outcomes (e.g., selecting a certain number of balls of a specific color)

Key Concepts Involved

To solve this problem, understanding the following concepts is crucial:

- Total number of objects (balls): Sum of all balls in the box.
- Combinations (nCr): The number of ways to select r objects from n objects without regard to order.
- Probability: The ratio of favorable outcomes to total possible outcomes.

Step-by-Step Solution Approach

1. Determine Total Number of Balls

Calculate the total number of balls in the box:

- Red balls = 3
- White balls = 5
- Blue balls = 2

Total balls = 3 + 5 + 2 = 10

2. Total Number of Ways to Select 3 Balls

Since the selection is without replacement and order does not matter, total possible combinations:

$$
\text{Total ways} = \binom{10}{3} = \frac{10!}{3! \times 7!} = 120
$$

This is the denominator for probability calculations.

Calculating Probabilities of Various Events

Different events can occur when selecting 3 balls. Here, we explore some common probability scenarios:

Event A: All three balls are of the same color

Event B: Exactly two balls are of one color, and the third is of another color

Event C: At least one blue ball is selected

Event D: Selecting exactly 1 red, 1 white, and 1 blue ball

Event A: All three balls are of the same color

Analysis:

- Red: Only 3 red balls, so selecting all 3 red:

$$
\binom{3}{3} = 1
$$

- White: 5 white balls, select all 3:

$$
\binom{5}{3} = 10
$$

- Blue: Only 2 blue balls, cannot select 3:

$$
\binom{2}{3} = 0
$$

Total favorable outcomes:

$$
1 + 10 + 0 = 11
$$

Probability:

$$
P(\text{all same color}) = \frac{11}{120}
$$

Event B: Exactly two balls of one color and one of another

Analysis:

Possible cases:

- Two red and one white
- Two red and one blue
- Two white and one red
- Two white and one blue
- Two blue and one red
- Two blue and one white

Calculate each:

1. Two red, one white:

$$
\binom{3}{2} \times \binom{5}{1} = 3 \times 5 = 15
$$

2. Two red, one blue:

$$
\binom{3}{2} \times \binom{2}{1} = 3 \times 2 = 6
$$

3. Two white, one red:

$$
\binom{5}{2} \times \binom{3}{1} = 10 \times 3 = 30
$$

4. Two white, one blue:

$$
\binom{5}{2} \times \binom{2}{1} = 10 \times 2 = 20
$$

5. Two blue, one red:

$$
\binom{2}{2} \times \binom{3}{1} = 1 \times 3 = 3
$$

6. Two blue, one white:

$$
\binom{2}{2} \times \binom{5}{1} = 1 \times 5 = 5
$$

Total favorable outcomes:

$$
15 + 6 + 30 + 20 + 3 + 5 = 79
$$

Probability:

$$
P(\text{exactly two of one color and one of another}) = \frac{79}{120}
$$

Event C: At least one blue ball is selected

Analysis:

It's often easier to calculate the complement — the probability of selecting no blue balls — and subtract from 1.

- Total ways to select 3 balls with no blue:

Number of balls without blue: 3 red + 5 white = 8

Number of ways:

$$
\binom{8}{3} = 56
$$

- Probability of no blue:

$$
P(\text{no blue}) = \frac{56}{120} = \frac{14}{30} = \frac{7}{15}
$$

- Therefore, probability of at least one blue:

$$
P(\text{at least one blue}) = 1 - P(\text{no blue}) = 1 - \frac{7}{15} = \frac{8}{15}
$$

Event D: Exactly one red, one white, and one blue ball

Analysis:

Number of ways:

$$
\binom{3}{1} \times \binom{5}{1} \times \binom{2}{1} = 3 \times 5 \times 2 = 30
$$

Probability:

$$
P(\text{1 red, 1 white, 1 blue}) = \frac{30}{120} = \frac{1}{4}
$$

Summary of Probabilities

Event	Favorable Outcomes	Total Outcomes	Probability
All same color	11	120	11/120
Exactly two of one color + one of another	79	120	79/120
At least one blue	—	120	8/15
Exactly 1 red, 1 white, 1 blue	30	120	1/4

Key Takeaways for Probability Calculations

- **Total combinations:** Always start by calculating the total possible combinations using $\binom{n}{r}$.
- **Favorable outcomes:** Count the number of ways specific conditions are satisfied, considering the composition of the box.
- **Complement rule:** Use this to simplify calculations for events like "at least" or "none."
- **Symmetry in cases:** Recognize when different scenarios are symmetrical to reduce computation effort.

Applications of This Problem in Real Life

Understanding the principles behind this problem extends beyond theoretical exercises. Here are some practical applications:

1. **Quality control:** Sampling items from a batch and calculating probabilities of defects.
2. **Games and gambling:** Calculating chances of specific outcomes in card games, lotteries, or roulette.
3. **Medical testing:** Estimating probabilities of detecting diseases based on sample testing.
4. **Supply chain management:** Assessing risks associated with randomly selecting products or components.

Conclusion

Probability problems like selecting balls from a box involve a combination of combinatorial mathematics and logical reasoning. By understanding how to compute total outcomes and favorable outcomes using combinations, one can accurately determine the likelihood of various events. The problem with a box containing 3 red, 5 white, and 2 blue balls exemplifies fundamental concepts such as total arrangements, complement probability, and specific event calculation. Mastery of these techniques not only enhances problem-solving skills but also prepares individuals for more complex statistical analysis and real-world decision-making scenarios.

Additional Tips for Solving Similar Problems

- Always start by clearly identifying the total number of items and the specific conditions.
- Break complex events into simpler, mutually exclusive cases.
- Use the complement rule to simplify calculations when appropriate.
- Verify your calculations by considering extreme cases or symmetry.
- Practice

Frequently Asked Questions

What is the total number of balls in the box?

The total number of balls is 3 Red + 5 White + 2 Blue = 10 balls.

How many ways can 3 balls be selected from the box without replacement?

The total number of ways is C(10, 3) = 120.

What is the probability of selecting 3 Red balls?

Since there are only 3 Red balls, the probability of selecting all 3 Red balls is 1/120.

What is the probability of selecting exactly 2 White and 1 Blue ball?

Number of ways to select 2 White from 5 = C(5, 2) = 10; Number of ways to select 1 Blue from 2 = C(2, 1) = 2; Total ways = 10 2 = 20. Therefore, probability = 20/120 = 1/6.

What is the probability of selecting at least one Blue ball?

First, find the probability of selecting no Blue balls (all from Red and White): C(8, 3) = 56. Total ways = 120. So, probability of no Blue = 56/120 = 7/15. Therefore, probability of at least one Blue = 1 - 7/15 = 8/15.

What is the probability of selecting exactly one Red, one White, and one Blue ball?

Number of ways: Red = C(3,1)=3, White = C(5,1)=5, Blue = C(2,1)=2. Total ways = 352=30. Probability = 30/120=1/4.

Is it possible to select three balls all of the same color? If so, what are the probabilities?

Yes, only Red balls can be selected all of the same color (since there are 3 Red). Probability = C(3,3)/C(10,3) = 1/120. For White and Blue, it's not possible as there are 5 White and 2 Blue, but selecting 3 Blue is possible, so for Blue: C(2,3)=0. So, only Red is possible, with probability 1/120.

What is the probability of selecting 3 balls with at least two of the same color?

Possible combinations include 2 Red + 1 White, 2 White + 1 Blue, etc., plus all three of the same color. Calculating all these would involve summing the probabilities of these events, but generally, the probability of at least two of the same color is 1 minus the probability of all three being of different colors, which is negligible here because with only 2 Blue balls, all three cannot be Blue. The detailed calculation yields approximately 11/15.

Additional Resources

A Box Contains 3 Red, 5 White, And 2 Blue Balls. 3 Balls Are Selected At Random Without Replacement. Find

In the realm of probability and statistics, problems involving the random selection of objects from a finite set are fundamental. They serve as essential building blocks for understanding uncertainty, risk assessment, and decision-making processes. One classic problem involves a box containing balls of different colors, and the task is to analyze the likelihood of specific outcomes when a subset of these balls is drawn at random without replacement. This article delves into a comprehensive exploration of such a problem, focusing on a box containing 3 red, 5 white, and 2 blue balls, from which three balls are selected without replacement. We will systematically analyze the problem, derive probabilistic expressions, and interpret the results in a detailed, structured manner.

Understanding the Problem: The Basic Setup

Details of the Scenario

The problem describes a box with a total of 10 balls, categorized by color:

- Red balls: 3
- White balls: 5
- Blue balls: 2

The total number of balls is:
$$ 3 + 5 + 2 = 10 $$

The selection process involves choosing 3 balls at random without replacement. This means once a ball is drawn, it is not returned to the box, affecting the probabilities of subsequent draws.

Key Concepts Involved

- Sample Space: The set of all possible outcomes when selecting 3 balls from 10.
- Event: A specific outcome or a set of outcomes, such as "all three balls are of the same color" or "exactly two are white."
- Probability: The likelihood of an event occurring, calculated as the ratio of the favorable outcomes to total possible outcomes.

Fundamental Probabilistic Principles

Calculating Total Number of Outcomes

Since the balls are distinguishable only by color, and we are selecting 3 balls without replacement, the total number of possible combinations is given by the combination formula:

$$ \binom{10}{3} $$

Calculating:
$$ \binom{10}{3} = \frac{10!}{3! \times 7!} = \frac{10 \times 9 \times 8}{3 \times 2 \times 1} = 120 $$

Thus, there are 120 equally likely ways to select 3 balls from the box.

Understanding the Concept of Favorable Outcomes

Depending on the specific event under consideration (e.g., all balls are red, exactly two are white, etc.), we need to count the number of combinations that satisfy the condition.

Common Events and Their Probabilities

In analyzing such problems, it is customary to explore various events of interest. Here, we examine some common scenarios:

1. Probability that all three selected balls are of the same color

- Red: Since there are only 3 red balls, selecting all three red balls is the only possibility:
$$ \text{Number of ways} = \binom{3}{3} = 1 $$
- White: Selecting all three white balls (out of 5):
$$ \binom{5}{3} = 10 $$
- Blue: Only 2 blue balls exist, so selecting all three is impossible:
$$ \binom{2}{3} = 0 $$

Total favorable outcomes:
$$ 1 + 10 + 0 = 11 $$

Probability:
$$ P(\text{all same color}) = \frac{11}{120} $$

---

2. Probability that exactly two balls are of one color and the third is of another

This scenario involves multiple subcases. For each color, we determine how many combinations involve exactly two balls of that color and one of a different color.

a) Two red and one non-red:
- Ways to choose 2 red balls: $\binom{3}{2} = 3$
- Ways to choose 1 non-red ball: total non-red balls = 5 white + 2 blue = 7
- Ways to select 1 non-red ball: $\binom{7}{1} = 7$

Total combinations for this case:
$$ 3 \times 7 = 21 $$

b) Two white and one non-white:
- Ways to select 2 white balls: $\binom{5}{2} = 10$
- Non-white balls: 3 red + 2 blue = 5

- Ways to select 1 non-white: $\binom{5}{1} = 5$

Total combinations:
$$ 10 \times 5 = 50 $$

c) Two blue and one non-blue:
- Ways to select 2 blue balls: $\binom{2}{2} = 1$
- Non-blue balls: 3 red + 5 white = 8
- Ways to select 1 non-blue: $\binom{8}{1} = 8$

Total combinations:
$$ 1 \times 8 = 8 $$

Total favorable outcomes for exactly two of one color:
$$ 21 + 50 + 8 = 79 $$

Probability:
$$ P(\text{exactly two of same color}) = \frac{79}{120} $$

---

3. Probability of selecting a mix of different colors, such as one of each color

- One Red, one White, one Blue:
Number of ways:
$$ \binom{3}{1} \times \binom{5}{1} \times \binom{2}{1} = 3 \times 5 \times 2 = 30 $$

Probability:
$$ P(\text{one of each color}) = \frac{30}{120} = \frac{1}{4} $$

Advanced Analytical Approaches

Beyond basic probability calculations, more sophisticated methods include calculating expected values, variances, and conditional probabilities.

Expected Number of Balls of Each Color in the Draw

- The expected value of red balls in 3 draws:
$$ E(\text{Red}) = 3 \times \frac{3}{10} = 0.9 $$
because the probability of drawing a red ball in any one draw, without replacement, is approximately $\frac{3}{10}$.

- Similarly, for white balls:
$$E(\text{White}) = 3 \times \frac{5}{10} = 1.5$$

- For blue balls:
$$E(\text{Blue}) = 3 \times \frac{2}{10} = 0.6$$

These expected values give us insight into the average composition of the selected subset.

Conditional Probabilities and Sequential Analysis

Suppose we are interested in the probability that the second ball drawn is blue, given the first ball was white. This involves conditional probability and is computed considering the reduced total and remaining balls after the first draw.

Real-World Applications and Significance

The problem's structure is analogous to various real-world scenarios where understanding probabilities of complex outcomes is critical. Examples include:

- Quality Control: Selecting items from a batch for inspection and assessing the likelihood of defects based on color-coded indicators.
- Card and Board Games: Drawing specific combinations or sequences, which is essential for game strategies.
- Medical Testing: Probabilistic models of test outcomes when selecting samples from a population with different health statuses.
- Risk Management: Estimating the likelihood of certain combinations of events in logistical or supply chain contexts.

Understanding the probabilistic framework for such problems enables better decision-making, risk assessment, and optimization.

Conclusion: Integrating the Analyses

This detailed exploration of the problem involving a box with 3 red, 5 white, and 2 blue balls demonstrates the power and versatility of combinatorial probability. By systematically calculating total outcomes, favorable cases, and associated probabilities, we gain a comprehensive understanding of the likelihood of various events in random sampling without replacement.

The key takeaways include:

- The total number of ways to select 3 balls from 10 is 120.

- Probabilities can be computed for events like all balls being of the same color, exactly two of a color, or one of each color.
- The problem illustrates fundamental concepts in probability, including combinations, conditional probability, and expectations.
- Such analyses are not merely academic; they have practical applications across various fields requiring probabilistic reasoning.

By mastering these techniques, students, researchers, and practitioners can better analyze complex random processes, make informed predictions, and develop robust strategies in uncertain environments.

A Box Contains 3 Red 5 White And 2 Blue Balls 3 Balls Are Selected At Random Without Replacement Find

A Box Contains 3 Red 5 White And 2 Blue Balls 3 Balls Are Selected At Random Without Replacement Find

Related Articles

- _________ Occur(s) In Information Systems When The Output Of One System Feeds Another System.a) Context
- 15. Which Excerpt From Virginia Woolf's A Room Of One's Own Contains An Allusion? (1 Point) O"She Looked
- 2. Suppose You Are Working For A Marketing Agency, And You Have To Present To Your Team The Distribution

Back to Home