A Certain Ideal Gas Has A Molar Specific Heat Of CV = 7/2 R . A 2.00-mol Sample Of The Gas Always Starts

A Certain Ideal Gas Has A Molar Specific Heat Of CV = 7/2 R . A 2.00-mol Sample Of The Gas Always Starts in a specified initial state and undergoes various processes. Understanding the thermodynamic properties of this gas, including its behavior during heating, cooling, and work interactions, requires a detailed analysis rooted in the principles of thermodynamics. This comprehensive guide explores the fundamental concepts, calculations, and applications relevant to this ideal gas, emphasizing the significance of its molar specific heat capacity and how it influences the gas's response during different thermodynamic processes.

Understanding the Molar Specific Heat at Constant Volume (CV)

Definition and Significance

The molar specific heat at constant volume, denoted as CV, describes the amount of heat required to raise the temperature of one mole of a substance by one degree Celsius (or Kelvin) at constant volume. For an ideal gas, CV is a key parameter because it relates directly to the internal energy changes during temperature variations.

Given Value and Its Implication

In this case, the ideal gas has:

- CV = 7/2 R

where R is the universal gas constant (~8.314 J/(mol·K)). The value of CV indicates that the gas is monatomic, as classical thermodynamics tells us that for monatomic ideal gases:

- CV = (3/2) R
- CP = (5/2) R

However, the given CV of 7/2 R suggests a different scenario, perhaps considering additional degrees of freedom or a specific model.

Initial Conditions and Thermodynamic State

Initial State of the Gas

The problem states that a 2.00-mol sample of this gas "always starts" at a particular initial state. While specific initial conditions (temperature, pressure, volume) are not explicitly provided, we can analyze general scenarios assuming standard initial parameters:

1. Initial Temperature (T_1): Assume an arbitrary initial temperature, say $T_1 = T_0$.
2. Initial Pressure (P_1): Can be related via the ideal gas law, $P_1V_1 = nRT_1$.
3. Initial Volume (V_1): Chosen based on the initial state or given conditions.

Understanding the initial state is crucial for subsequent calculations involving heat transfer, work done, and changes in internal energy.

Thermodynamic Processes Involving the Gas

Types of Processes

The ideal gas can undergo various thermodynamic processes, each characterized by different paths and energy interactions:

1. Isothermal Process (constant T)
2. Adiabatic Process (no heat exchange)
3. Isobaric Process (constant pressure)
4. Isochoric Process (constant volume)

Each process involves specific calculations of heat transfer (Q), work done (W), and change in internal energy (ΔU).

General Expressions for Thermodynamic Quantities

For an ideal gas:

- Change in internal energy:

- $\Delta U = nCV\Delta T$

- Heat transfer:

- $Q = \Delta U + W$

- Work done (for expansion or compression):

- $W = P\Delta V$

    - For processes at constant volume:

        - $Q = \Delta U$

    - For processes at constant pressure:

        - $Q = nCP\Delta T$

Calculations and Examples

Example 1: Heating the Gas at Constant Volume

Suppose the gas is heated from an initial temperature T_1 to a final temperature T_2 at constant volume.

1. Calculate the heat added:
    - $Q = nCV(T_2 - T_1)$

2. Calculate the change in internal energy:
    - $\Delta U = nCV(T_2 - T_1)$

3. Determine the work done:
    - $W = 0$ (since volume is constant)

Example 2: Adiabatic Expansion or Compression

In an adiabatic process ($Q = 0$), the relationship between initial and final states:

1. Use the adiabatic relation:
    - $TV^{\gamma-1} = \text{constant}$

2. Determine the final temperature or volume given initial conditions.

where $\gamma = C_P / C_V$.

Implications of the Specific Heat Value

Calculating CP

Given $C_V = 7/2\ R$, and knowing the relation for ideal gases:

- For monatomic gases:

  - $\gamma = 5/3$
  - $C_P = C_V + R = (7/2) R + R = (9/2) R$

However, the value $C_V = 7/2\ R$ suggests the gas may have more degrees of freedom or specific internal structures influencing its heat capacities.

Impact on Thermodynamic Behavior

The molar specific heats influence:

1. Rate of temperature change during heating or cooling

2. Work done during expansion or compression
3. Energy transfer during different processes

The higher CV indicates that more energy is needed to change the temperature, reflecting the energy stored in additional degrees of freedom.

Applications and Real-World Relevance

Engine Cycles and Power Generation

Understanding the thermodynamics of gases with specific heat capacities is fundamental in designing engines and power plants, where gases undergo various processes:

1. Internal combustion engines
2. Gas turbines
3. Refrigeration cycles

Knowledge of heat capacities aids in optimizing efficiency and predicting performance.

Industrial and Laboratory Uses

Precise control over heating and cooling processes in laboratories depends on understanding the specific heat properties of gases involved.

Summary and Key Takeaways

- The molar specific heat at constant volume, CV, is a critical property influencing how an ideal gas responds to thermal processes. In this case, CV = 7/2 R indicates a particular set of degrees of freedom or internal energy structure.
- Initial conditions define the starting point for thermodynamic calculations involving heat transfer, work, and energy changes.

- Different thermodynamic processes—such as isothermal, adiabatic, isobaric, and isochoric—have unique equations and implications for the energy interactions of the gas.
- Calculations of heat transfer, work, and internal energy changes rely heavily on understanding the specific heat capacities and their relation to temperature and volume changes.
- These principles are vital in practical applications spanning energy production, industrial processes, and scientific research.

Conclusion

Analyzing an ideal gas with a molar specific heat of CV = 7/2 R involves understanding its thermodynamic properties and behavior under various processes. While the provided information offers a foundation, detailed calculations depend on the initial conditions and specific process parameters. Mastery of these concepts enables engineers, scientists, and students to predict and optimize the behavior of gases in real-world applications, ensuring efficiency and safety in technological advancements and industrial operations.

Frequently Asked Questions

What does the molar specific heat at constant volume (CV) = 7/2 R imply about the degrees of freedom of the gas molecules?

It indicates that the gas molecules have 3 translational degrees of freedom and 2 rotational degrees of freedom, typical of diatomic or linear molecules, resulting in a total of 5 degrees of freedom.

How can you calculate the internal energy change of the gas when its temperature changes?

The change in internal energy (ΔU) is given by $\Delta U = n\ CV\ \Delta T$. Since CV = 7/2 R, for a 2.00 mol sample, $\Delta U = 2.00\ \text{mol}\ (7/2\ R)\ \Delta T$.

What is the significance of the gas always starting in a specific initial state in thermodynamic processes?

It allows for consistent calculations of heat, work, and energy transfer during processes, making it easier to analyze changes relative to a known baseline.

If the gas undergoes an isochoric process, how much heat is added or removed per degree of temperature change?

Heat added or removed per degree (Q per ΔT) is $Q = n \, CV \, \Delta T$. For 2 mol, $Q = 2 \, (7/2 \, R) \, \Delta T = 7 \, R \, \Delta T$.

How does the specific heat at constant volume (CV) relate to the total energy stored in the gas?

CV represents the amount of heat required to raise the temperature of the gas by one Kelvin per mole at constant volume, directly related to the internal energy stored by the molecules.

What additional information is needed to determine the work done during a thermodynamic process involving this gas?

The process path (e.g., isothermal, isobaric, adiabatic) and the change in volume are needed to calculate the work done on or by the gas.

Additional Resources

A Certain Ideal Gas Has A Molar Specific Heat Of CV = 7/2 R

Understanding the thermodynamic properties of gases is fundamental to both theoretical physics and practical applications in engineering and chemistry. When examining an ideal gas with a given molar specific heat at constant volume (CV), such as CV = (7/2) R, one gains insight into its microscopic behavior, energy storage capacity, and response to thermal processes. This article delves into the significance of this specific heat value, exploring its implications, underlying physics, and practical considerations. By analyzing a 2.00-mol sample that always starts in a specified initial state, we will explore various thermodynamic processes, calculations, and conceptual insights relevant to this particular gas.

Understanding Molar Specific Heat at Constant Volume

Definition and Significance

The molar specific heat at constant volume, CV, measures how much heat energy is needed to raise the temperature of one mole of a substance by one Kelvin (or degree Celsius) while keeping the volume fixed. For an ideal gas, CV is directly related to the degrees of freedom of the molecules, reflecting how energy is partitioned among translational, rotational, and vibrational modes.

Mathematically,
$$ C_V = \left(\frac{\partial Q}{\partial T}\right)_V $$
where Q is the heat added, and T is the temperature.

In the case of the gas under discussion, CV = (7/2) R, where R is the universal gas constant (~8.314 J/mol·K). This specific heat value indicates that each molecule has a certain number of degrees of freedom contributing to energy storage.

Relation to Degrees of Freedom

According to the equipartition theorem, each quadratic degree of freedom contributes $\frac{1}{2} R$ to the molar heat capacity at constant volume. Therefore, the total CV relates to the number of degrees of freedom (f):

$$ C_V = \frac{f}{2} R $$

Given CV = (7/2) R, it implies:

$$ \frac{f}{2} R = \frac{7}{2} R \Rightarrow f = 7 $$

This indicates that each molecule of this gas possesses seven degrees of freedom.

Features and implications:

- Translational motion: 3 degrees (x, y, z axes)
- Rotational motion: Typically 2 degrees for nonlinear molecules, but for linear molecules, rotational degrees are fewer; however, in this context, the total degrees of freedom sum to 7, suggesting additional internal modes.
- Vibrational modes: Vibrational degrees of freedom contribute in molecules with more complex structures; each vibrational mode adds 2 degrees (kinetic + potential energy).

Since the total is 7, the molecule likely has:

- 3 translational
- 2 rotational
- 2 vibrational modes

This detailed understanding helps predict how the gas behaves under various conditions, including heating, cooling, and compression.

Thermodynamic Behavior of the Gas

Initial Conditions and Sample Size

The problem states that a 2.00-mol sample of this gas always starts in a particular initial state. This initial state serves as a baseline for thermodynamic processes—such as heating, cooling, or expansion—and allows for the calculation of energy changes, work done, and heat transfer.

Knowing the starting point is crucial for:

- Calculating changes in internal energy ΔU
- Applying the first law of thermodynamics
- Analyzing isochoric, isobaric, isothermal, and adiabatic processes

Internal Energy of the Gas

For an ideal gas, the internal energy U depends solely on temperature and the molar heat capacity at constant volume:

$$U = n C_V T$$

where $n = 2.00\, \text{mol}$.

Given CV = (7/2) R, the total internal energy at temperature T is:

$$U = 2 \times \frac{7}{2} R T = 7 R T$$

This linear relationship indicates that every degree of freedom contributes equally to the internal energy, confirming the molecular interpretation.

Features:

- Linearity: Internal energy scales directly with temperature.

- Predictability: Changes in energy are straightforward to compute for temperature variations.
- Energy Storage: The energy stored per mole is high due to the large degrees of freedom.

Energy Exchange in Thermodynamic Processes

Heating and Cooling

When heat Q is added or removed at constant volume:

$$\Delta U = Q$$

Since the internal energy depends on temperature:

$$\Delta U = n C_V \Delta T$$

For a 2.00-mol sample:

$$\Delta U = 2 \times \frac{7}{2} R \times \Delta T = 7 R \Delta T$$

Thus, to increase the temperature by 1 K:

$$\Delta U = 7 R \approx 7 \times 8.314 \approx 58.2 \text{ J}$$

This quantifies the energy requirement for a temperature change and emphasizes the relatively high energy capacity of this gas.

Pros:

- Predictable energy change for temperature shifts.
- Facilitates energy budgeting in thermodynamic cycles.

Cons:

- Large amounts of heat may be necessary for significant temperature changes, especially for large samples.

Work Done During Expansion or Compression

When the gas expands or compresses at constant temperature (isothermal process), the work done W is:

$$W = n R T \ln \left(\frac{V_f}{V_i}\right)$$

- The energy change is zero at constant temperature, but work is performed.
- In adiabatic processes, both U and W vary, and the relationship between temperature and volume is governed by adiabatic equations involving CV and the adiabatic index γ.

Adiabatic Processes and the Adiabatic Index

Calculating γ for the Gas

The ratio of specific heats:

$$\gamma = \frac{C_P}{C_V}$$

where C_P is the molar heat capacity at constant pressure. Using:

$$C_P = C_V + R = \frac{7}{2} R + R = \frac{9}{2} R$$

then:

$$\gamma = \frac{\frac{9}{2} R}{\frac{7}{2} R} = \frac{9/2}{7/2} = \frac{9}{7} \approx 1.2857$$

This value influences the behavior of adiabatic processes, such as the relation between temperature and volume:

$$TV^{\gamma - 1} = \text{constant}$$

Features:

- Moderate γ: Slightly above 1, indicating the gas compresses and expands with moderate temperature changes during adiabatic processes.

Real-World Applications and Practical Considerations

Potential Uses of Such Gases

Understanding gases with CV = 7/2 R has implications in:

- High-temperature environments: Where vibrational modes are excited.
- Complex molecules: Such as polyatomic gases with multiple internal degrees of freedom.
- Thermodynamic cycle design: Including engines and refrigerators, where specific heat ratios impact efficiency.

Pros and Cons of Using Such Gases

Pros:

- High energy capacity: Due to multiple degrees of freedom.
- Predictable thermodynamic behavior: Simplifies modeling and design.
- Relevance in complex molecular gases: Provides insights into real gases beyond simple monatomic models.

Cons:

- Complex internal structure: Can complicate modeling when vibrational modes become excited at high temperatures.
- Potential for non-ideal behavior: At high pressures or low temperatures, deviations from ideality may occur.
- Energy input required: Significant heat energy needed for temperature changes due to high CV.

Conclusion

The ideal gas characterized by a molar specific heat at constant volume CV = (7/2) R offers a fascinating window into molecular physics and thermodynamics. Its high degrees of freedom reflect a complex internal structure, likely involving vibrational modes, making it suitable for modeling gases with rich molecular behavior. The predictable relationships between internal energy, heat, and temperature facilitate detailed analysis of thermodynamic processes, from heating and cooling to adiabatic expansions. While this gas's properties make it valuable in theoretical and applied contexts, practical considerations such as energy input and potential deviations from ideality must be acknowledged. Overall, understanding such gases enhances our ability to design efficient thermal systems and deepens our grasp of molecular energy dynamics.

Features Summary:

- High degrees of freedom (~7): Indicates complex molecular structure.
- Linear energy-temperature relation: Simplifies thermodynamic calculations.
- Moderate adiabatic index (~1.2857): Slightly above monatomic gases, influencing process behavior.
- Energy-intensive for temperature changes: Due to large CV.

Pros:

- Facilitates detailed thermodynamic analysis.
- Applicable to complex molecules.
- Useful in high-temperature

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