A Cylinder Has A Volume Of 72 Cubic Inches. What Is The Volume Of A Cone With The Same Height And Radius

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Understanding the relationship between different geometric solids can be both fascinating and practically useful. When working with cylinders and cones that share the same height and radius, it's important to recognize how their volumes compare and how to calculate one based on the other. In this article, we will explore how to determine the volume of a cone given the volume of a cylinder, specifically when both have identical dimensions. We will delve into the formulas, step-by-step calculations, and real-world applications to provide a comprehensive understanding.

Understanding the Volumes of Cylinder and Cone

Before calculating the volume of the cone, it's crucial to understand the formulas that define the volume of both shapes and how they relate to each other.

Volume of a Cylinder

The volume $V_{cylinder}$ of a cylinder with radius r and height h is given by:

$$
V_{cylinder} = \pi r^2 h
$$

Where:
- π (pi) is approximately 3.1416
- r is the radius of the circular base
- h is the height of the cylinder

Given that the volume of a particular cylinder is 72 cubic inches, we can express this as:

$$
\pi r^2 h = 72
$$

Volume of a Cone

The volume V_{cone} of a cone with the same radius r and height h is:

$$
V_{cone} = \frac{1}{3} \pi r^2 h
$$

Notice the similarity between the formulas for the cylinder and cone, with the cone's volume being exactly one-third of the cylinder's volume when both share the same base radius and height.

Calculating the Cone's Volume When Given the Cylinder's Volume

Since both the cylinder and cone in question have the same radius and height, and the volume of the cylinder is already known to be 72 cubic inches, calculating the cone's volume becomes straightforward.

Step 1: Recognize the Relationship

Given:

$$
V_{cylinder} = \pi r^2 h = 72
$$

The cone's volume is:

$$
V_{cone} = \frac{1}{3} \pi r^2 h
$$

We can see that:

$$
V_{cone} = \frac{1}{3} V_{cylinder}
$$

Step 2: Compute the Cone's Volume

Using the relationship:

$$
V_{cone} = \frac{1}{3} \times 72 = 24
$$

Therefore, the volume of the cone with the same height and radius as the cylinder is 24 cubic inches.

Additional Insights and Applications

Understanding this relationship between cylinders and cones is not only academically interesting but also practically valuable in various fields such as engineering, manufacturing, and design.

Practical Example: Material Usage

Suppose you're designing a container that needs to be filled with a certain amount of liquid. Knowing that a cylindrical container with specific dimensions holds 72 cubic inches allows you to determine the volume of a conical container with the same dimensions. This can help in material estimation, cost calculation, or optimizing storage space.

Design and Manufacturing

In manufacturing, cones and cylinders are common shapes used in bottles, funnels, and tanks. Recognizing that the cone's volume is one-third of the cylinder's volume for the same dimensions can assist in designing components that fit specific volume requirements efficiently.

Summary of Key Points

- The volume of a cylinder is calculated as $\pi r^2 h$.
- The volume of a cone with the same radius and height is $\frac{1}{3} \pi r^2 h$.
- If a cylinder has a volume of 72 cubic inches, the cone with the same radius and height has a volume of 24 cubic inches.
- The relationship simplifies to the cone's volume being exactly one-third of the cylinder's volume when dimensions are shared.

Conclusion

In summary, when dealing with a cylinder that has a volume of 72 cubic inches and a cone sharing the same height and radius, the cone's volume is simply one-third of the cylinder's volume. This means the cone's volume is 24 cubic inches. Recognizing these relationships makes it easier to solve volume problems involving these shapes and can be applied in various real-world scenarios, from design to manufacturing.

By understanding the fundamental formulas and their relationships, you can quickly determine the volume of one shape based on the other, saving time and enhancing your geometric problem-solving skills.

Frequently Asked Questions

If a cylinder has a volume of 72 cubic inches, and a cone has the same height and radius, what is the volume of the cone?

The volume of the cone is 24 cubic inches, which is one-third of the cylinder's volume.

How do you find the volume of a cone with the same height and radius as a given cylinder?

Use the formula for the cone's volume: $(1/3) \times \pi \times r^2 \times h$. Since the cylinder's volume is known, you can relate the two to find the cone's volume.

Given a cylinder volume of 72 cubic inches, what is the radius if the height is known?

You can find the radius using the cylinder volume formula: $V = \pi r^2 h$. Rearranged as $r = \text{sqrt}(V / (\pi h))$.

Can the volume of the cone be directly calculated from the cylinder's volume without knowing the height or radius?

No, because the cone's volume depends on the same height and radius, so knowing these dimensions is necessary to calculate the cone's volume.

If the height of the cylinder is doubled, how does that affect the volume of the cone with the same radius and height?

Doubling the height doubles the volume of the cone, since volume is proportional to height when radius remains constant.

Is the volume of the cone with the same dimensions as a cylinder always one-third of the cylinder's volume?

Yes, when the cone and cylinder have the same height and radius, the cone's volume is always one-third of the cylinder's volume.

How do you verify that the cone's volume is one-third of the cylinder's volume given the same dimensions?

By calculating both volumes using their formulas, you'll find the cone's volume equals one-third of the cylinder's volume, confirming the relationship.

What is the step-by-step process to find the cone's volume given the cylinder's volume and common dimensions?

Step 1: Confirm the dimensions (radius and height). Step 2: Use the formula for the cone's volume: $(1/3) \pi r^2 h$. Step 3: Substitute the known values to compute the volume. Alternatively, recognize that the cone's volume is one-third of the cylinder's volume if dimensions are the same, so the answer is 24 cubic inches.

Additional Resources

A Cylinder Has A Volume Of 72 Cubic Inches. What Is The Volume Of A Cone With The Same Height And Radius

In the realm of geometry, understanding the relationships between different three-dimensional shapes is fundamental. Among these shapes, cylinders and cones share similar attributes—most notably, their dependence on radius and height for volume calculation. A common question that arises in educational settings and practical applications alike is: If a cylinder has a volume of 72 cubic inches, what is the volume of a cone with the same height and radius?

This question not only tests comprehension of volume formulas but also offers insight into the proportional relationships between these shapes. In this article, we will explore the mathematical principles underlying this problem, break down the formulas involved, and demonstrate how to arrive at the solution with clarity and confidence.

Understanding the Geometric Shapes: Cylinder and Cone

Before delving into calculations, it is essential to understand the basic properties of the shapes involved.

What Is a Cylinder?

A cylinder is a three-dimensional object characterized by:

- Two parallel circular bases
- A curved surface connecting these bases
- Equal radius for both bases
- Height (h): the perpendicular distance between the bases

The volume of a cylinder is given by the formula:

$$
V_{cylinder} = \pi r^2 h
$$

where:

- r is the radius of the base
- h is the height of the cylinder
- π (pi) is approximately 3.14159

What Is a Cone?

A cone is a three-dimensional object with:

- A circular base
- A vertex (tip) not in the plane of the base
- The same radius r as the cylinder's base
- Height h: the perpendicular distance from the base to the vertex

The volume of a cone is given by:

$$
V_{cone} = \frac{1}{3} \pi r^2 h
$$

Notice that the volume formula for the cone is similar to that of the cylinder but scaled down by a factor of 1/3.

The Relationship Between Cylinder and Cone Volumes

Given the formulas:

$$
V_{cylinder} = \pi r^2 h
$$

$$
V_{cone} = \frac{1}{3} \pi r^2 h
$$

It becomes clear that, for the same radius and height, the volume of a cone is exactly one-third the volume of a cylinder.

This proportionality is fundamental and applies universally under the same height and radius conditions.

Applying the Given Data: Calculating the Cone's Volume

Let's analyze the problem step by step.

Given:

- Volume of the cylinder, $V_{cylinder} = 72 \text{ cubic inches}$
- Same height h and radius r for the cone

Goal:

- Find the volume of the cone, V_{cone}

Based on the relationship:

$$
V_{cone} = \frac{1}{3} V_{cylinder}
$$

Substituting the known value:

$$
V_{cone} = \frac{1}{3} \times 72
$$

Calculating:

$$
V_{cone} = 24
$$

Therefore, the volume of the cone is 24 cubic inches.

Deep Dive: Confirming the Calculation with Actual Dimensions

While the calculation above relies on the proportionality, it’s instructive to see how the dimensions relate to the volume, especially if you want to verify or understand the underlying factors.

Suppose we want to find the actual radius and height, assuming the given volume of the cylinder. Let's break down the process:

Step 1: Express the volume of the cylinder in terms of r and h:

$$
72 = \pi r^2 h
$$

Step 2: Express the cone's volume in terms of the same r and h:

$$
V_{cone} = \frac{1}{3} \pi r^2 h
$$

Given the cylinder's volume, the cone's volume becomes:

$$
V_{cone} = \frac{1}{3} \times 72 = 24
$$

This confirms the earlier conclusion.

Step 3: Finding specific dimensions (optional)

If, for example, the height and radius are known or assumed, you can verify the calculation:

Suppose:

- Radius, $r = 3$ inches
- Height, h can be found from the cylinder volume:

$$
72 = \pi \times 3^2 \times h \Rightarrow 72 = \pi \times 9 \times h
$$
$$
h = \frac{72}{9\pi} = \frac{8}{\pi} \approx 2.546 \text{ inches}
$$

Using these dimensions, the cone volume:

$$
V_{cone} = \frac{1}{3} \pi r^2 h = \frac{1}{3} \pi \times 9 \times \frac{8}{\pi} = \frac{1}{3} \times 9 \times 8 = 3 \times 8 = 24
$$

which matches the earlier result.

Practical Applications and Significance

Understanding the relationship between the volumes of cylinders and cones isn’t just a theoretical exercise; it has real-world applications in various fields:

- Manufacturing: Designing containers, tanks, or funnels often involves understanding how different shapes relate volumetrically.
- Architecture: Calculations for structural components, such as columns (cylinders) and conical roofs, require precise volume considerations.
- Education: Grasping these relationships enhances spatial reasoning and prepares students for advanced geometry or physics topics.
- Engineering: Volume calculations impact material estimations, cost analysis, and efficiency assessments.

Summary and Key Takeaways

- The volume of a cylinder is calculated using $V_{cylinder} = \pi r^2 h$.
- The volume of a cone with the same radius and height is $V_{cone} = \frac{1}{3} \pi r^2 h$.
- For a cylinder with a volume of 72 cubic inches, the cone sharing the same dimensions has a volume of 24 cubic inches.
- The proportional relationship is consistent: the cone's volume is always one-third that of the cylinder with identical radius and height.
- Knowing this relationship allows for quick calculations and better understanding of three-dimensional shapes.

Final Thoughts

The problem of determining the volume of a cone given the volume of a cylinder with the same height and radius highlights the elegant proportionality inherent in geometric figures. Recognizing these relationships simplifies complex calculations and enhances spatial intuition. Whether designing practical objects or exploring mathematical principles, understanding the interplay between different shapes is a valuable skill that bridges theory and application.

By mastering these foundational concepts, students, engineers, architects, and enthusiasts can confidently navigate the world of three-dimensional geometry, making informed decisions and fostering innovation across various disciplines.

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