A Drawer Contains 20 Socks. 10 Are Black. 6 Are Blue. 4 Are Red. Two Socks Are Taken Out At Random. What

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Introduction

Understanding probability and combinatorics is essential in solving many real-world problems, including those involving random selections from collections. Consider a drawer containing 20 socks, categorized by color: 10 black, 6 blue, and 4 red. If two socks are drawn at random without replacement, what is the probability that they meet certain conditions? This problem provides an excellent opportunity to explore concepts such as probability calculation, conditional probability, and combinatorial analysis.

In this article, we will explore the problem in detail, examine various scenarios and their probabilities, and discuss methods for solving similar problems involving random selection from a finite set. We aim to provide a comprehensive understanding suitable for students, educators, and anyone interested in probability theory.

Understanding the Basic Setup

The Composition of the Drawer

The initial composition of the socks in the drawer is as follows:

- Black socks: 10
- Blue socks: 6
- Red socks: 4

Total socks: 10 + 6 + 4 = 20

The Drawing Process

- Two socks are drawn at random.
- The socks are drawn without replacement (i.e., once a sock is drawn, it is not put back into the drawer).

Possible Outcomes

The total number of ways to draw two socks from 20 is:

$$
\text{Total outcomes} = \binom{20}{2} = \frac{20 \times 19}{2} = 190
$$

This total forms the basis for calculating probabilities of various events.

Probabilities of Basic Events

Drawing Two Socks of the Same Color

Let's first determine the probability that both socks drawn are of the same color.

Both Socks are Black

Number of ways to choose 2 black socks:

$$
\binom{10}{2} = \frac{10 \times 9}{2} = 45
$$

Probability:

$$
P(\text{both black}) = \frac{45}{190} = \frac{9}{38} \approx 0.2368
$$

Both Socks are Blue

Number of ways:

$$
\binom{6}{2} = \frac{6 \times 5}{2} = 15
$$

Probability:

$$
P(\text{both blue}) = \frac{15}{190} = \frac{3}{38} \approx 0.0789
$$

Both Socks are Red

Number of ways:

$$
\binom{4}{2} = \frac{4 \times 3}{2} = 6
$$

Probability:

$$
P(\text{both red}) = \frac{6}{190} = \frac{3}{95} \approx 0.0316
$$

Drawing Two Socks of Different Colors

The probability that the two socks are of different colors can be calculated by subtracting the probability of both being the same color from 1 or by directly summing the probabilities of all mixed color combinations.

Total probability:

$$
P(\text{different colors}) = 1 - P(\text{both black}) - P(\text{both blue}) - P(\text{both red})
$$

Calculations:

$$
P(\text{different colors}) = 1 - \left(\frac{9}{38} + \frac{3}{38} + \frac{3}{95}\right)
$$

Express all fractions with a common denominator, 190:

\[

$$
\frac{9}{38} = \frac{45}{190}
$$

$$
\frac{3}{38} = \frac{15}{190}
$$

$$
\frac{3}{95} = \frac{6}{190}
$$

Sum:

$$
\frac{45 + 15 + 6}{190} = \frac{66}{190} = \frac{33}{95}
$$

Therefore,

$$
P(\text{different colors}) = 1 - \frac{33}{95} = \frac{62}{95} \approx 0.6526
$$

Conditional Probabilities and Specific Scenarios

Beyond basic probabilities, we often need to find the likelihood of a specific event given another. For example, what is the probability that both socks are black given that at least one is black? Or, what is the probability that both socks are red if the first sock drawn was red?

Probability Both Socks Are Black Given At Least One Is Black

- Event A: Both socks are black.
- Event B: At least one sock is black.

Solution:

$$
P(A|B) = \frac{P(A \cap B)}{P(B)}
$$

Note that $A \subseteq B$, so:

$$
P(A|B) = \frac{P(\text{both black})}{P(\text{at least one black})}
$$

Calculating $P(\text{at least one black})$:

$$
P(\text{at least one black}) = 1 - P(\text{no black socks})
$$

Number of ways with no black socks:

$$
\binom{10}{0} \times \binom{6 + 4}{2} = 1 \times \binom{10}{2} = 1 \times 45
$$

But this approach is incorrect because we need to consider the total number of ways to draw two socks excluding all black socks.

Total socks excluding black:

$$
6 + 4 = 10
$$

Number of ways to draw 2 socks without black:

$$
\binom{10}{2} = 45
$$

Total ways to draw any 2 socks:

$$
\binom{20}{2} = 190
$$

Hence,

$$
P(\text{no black}) = \frac{45}{190} = \frac{9}{38}
$$

Thus,

$$
P(\text{at least one black}) = 1 - \frac{9}{38} = \frac{29}{38}
$$

Now,

$$

$$
P(A|B) = \frac{\frac{9}{38}}{\frac{29}{38}} = \frac{9}{29} \approx 0.3103
$$

Interpretation: If at least one sock drawn is black, there's approximately a 31% chance that both are black.

Exploring Advanced Probability Scenarios

Scenario 1: Probability Both Socks Are Red

As previously calculated, the probability that both socks are red:

$$
P(\text{both red}) = \frac{6}{190} = \frac{3}{95} \approx 0.0316
$$

Scenario 2: Probability First Sock Is Red, Second Is Blue

Since the drawing is without replacement, the probability that the first sock is red and the second is blue:

$$
P(\text{first red}) = \frac{4}{20} = \frac{1}{5}
$$

After drawing one red sock, remaining socks:

- Total remaining: 19
- Blue socks: 6 (unchanged)

- Red socks: 3

Probability that second sock is blue:

$$
P(\text{second blue} | \text{first red}) = \frac{6}{19}
$$

Therefore, the combined probability:

$$
P(\text{red then blue}) = P(\text{first red}) \times P(\text{second blue} | \text{first red}) = \frac{1}{5} \times \frac{6}{19} = \frac{6}{95} \approx 0.0632
$$

Scenario 3: Probability First Sock Is Blue, Second Is Red

Similarly:

$$
P(\text{first blue}) = \frac{6}{20} = \frac{3}{10}
$$

Remaining socks after drawing a blue sock:

- Total: 19
- Red socks: 4
- Blue socks: 5

Probability second sock is red:

$$
P(\text{second red} | \text{first blue}) = \frac{4}{19}
$$

Combined probability:

$$
P(\text{blue then red}) = \frac{3}{10} \times \frac{4}{19} = \frac{12}{190} = \frac{6}{95} \approx 0.0632
$$

Calculating Probabilities for Multiple Events

Probability Both Socks Are of the Same Color (General Formula)

For each color:

$$
P(\text{both of color } c) = \frac{\binom{\text{number of socks of color } c}{2}}{\binom{20}{2}}
$$

Total probability both socks are the same color:

$$
P(\text{same color}) = P(\text{both black}) + P(\text{both blue}) + P(\text{both red})
$$

which is:

\[

\frac{\binom{10}{2} + \binom{6}{2} + \binom{4}{2}}{190} = \frac{45 + 15 + 6}{190} = \frac{66}{190} = \frac{33}{95} \

Frequently Asked Questions

What is the probability that both socks drawn are black?

The probability that both socks are black is (10/20) (9/19) = 90/380 = 9/38 ≈ 0.2368 or 23.68%.

What is the probability that both socks are of the same color?

Calculating the probabilities for each color and summing: (10/20)(9/19) + (6/20)(5/19) + (4/20)(3/19) = (90/380) + (30/380) + (12/380) = 132/380 ≈ 0.3474 or 34.74%.

What is the chance that one sock is red and the other is blue?

Probability that one sock is red and the other blue: (4/20)(6/19) + (6/20)(4/19) = (24/380) + (24/380) = 48/380 ≈ 0.1263 or 12.63%.

If you draw two socks at random, what is the probability that both are blue?

The probability both are blue is (6/20) (5/19) = 30/380 = 3/38 ≈ 0.0789 or 7.89%.

What is the probability that at least one sock drawn is red?

Probability that none are red: (16/20) (15/19) = 240/380 = 12/19 ≈ 0.6316. Therefore, probability that at least one is red = 1 - 12/19 = 7/19 ≈ 0.3684 or 36.84%.

What is the probability that the two socks are of different colors?

Total probability of different colors: 1 minus probability of same color = 1 - (9/38 + 15/38 + 6/38) = 1 -

(30/38) = 8/38 ≈ 0.2105 or 21.05%.

If one sock is black, what is the probability that the other sock is red?

Given one sock is black, the probability the other is red is (4/19) because there are 19 remaining socks, of which 4 are red.

Additional Resources

Socks Probability Puzzle: An In-Depth Analysis of Drawing Socks from a Drawer

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Introduction

Probability puzzles have long captivated mathematicians, students, and enthusiasts alike. They blend everyday scenarios with mathematical rigor, offering insights into how chance operates in real life. One such classic problem involves understanding the likelihood of drawing certain socks from a drawer containing a mixture of colors. This problem is not just an academic exercise; it showcases foundational principles of probability, combinatorics, and decision-making under uncertainty. In this review, we will explore the scenario of a drawer containing 20 socks of different colors, analyze the probabilities involved when drawing two socks at random, and interpret the implications of these calculations.

Understanding the Scenario

The Composition of the Drawer

The problem states:

- Total socks: 20
- Black socks: 10
- Blue socks: 6
- Red socks: 4

This distribution indicates a diverse assortment, with black socks being the most prevalent, followed by blue, then red. The composition offers an excellent basis for probability calculations due to the clear counts.

Key Assumptions

Before diving into calculations, several assumptions are typically made:

- Socks are well mixed and randomly accessible.
- The socks are indistinguishable aside from their color; no pattern or other distinguishing features.
- When drawing socks, they are removed one after the other without replacement, meaning the total decreases after each draw.
- The order of drawing matters; the first sock drawn is distinct from the second in the analysis.

The Core Question

"Two Socks Are Taken Out At Random. What is the probability that both socks are of the same color?"

This question allows us to explore multiple probability pathways and is foundational for understanding combinatorial probability.

Breakdown of the Problem

1. Types of Possible Outcomes

When two socks are drawn, there are several possible scenarios:

- Both socks are black.
- Both socks are blue.
- Both socks are red.
- Socks are of different colors.

Our goal is to find the probability that both socks are the same color, which involves summing the probabilities of the first three scenarios.

Probability Calculations

2. Total Number of Ways to Draw Two Socks

To understand probabilities, we need to know the total number of possible outcomes when drawing two socks from 20:

$$
\text{Total ways} = \binom{20}{2} = \frac{20 \times 19}{2} = 190
$$

This is the total number of unordered pairs of socks that can be selected.

3. Calculating Same-Color Pair Probabilities

Now, for each color, compute how many pairs can be formed:

- Black socks: $\binom{10}{2} = \frac{10 \times 9}{2} = 45$
- Blue socks: $\binom{6}{2} = \frac{6 \times 5}{2} = 15$
- Red socks: $\binom{4}{2} = \frac{4 \times 3}{2} = 6$

Sum of same-color pairs:

$$
45 + 15 + 6 = 66
$$

Therefore, the probability that both socks are of the same color is:

$$
P(\text{same color}) = \frac{66}{190} \approx 0.347
$$

or roughly 34.7%.

Analyzing the Complement: Different Colors

To deepen our understanding, consider the alternative—what's the probability that the two socks are of different colors?

Total pairs with different colors:

$$
\text{Different color pairs} = \text{Total pairs} - \text{Same color pairs} = 190 - 66 = 124
$$

Probability:

$$
P(\text{different colors}) = \frac{124}{190} \approx 0.653
$$

which is about 65.3%.

This complementary approach simplifies calculations and illustrates the balance between same-color and mixed-color pairs.

Conditional Probabilities and Practical Implications

4. Probability of Drawing a Specific Color First and Second

Suppose you are interested in the probability that both socks are red, but you want to consider the sequential process.

- First sock red: $\frac{4}{20} = 0.2$
- Second sock red (given first was red): $\frac{3}{19} \approx 0.1579$

Probability both are red:

$$
0.2 \times 0.1579 \approx 0.0316
$$

which aligns with the combinatorial calculation:

$$
\frac{\binom{4}{2}}{\binom{20}{2}} = \frac{6}{190} \approx 0.0316
$$

This confirms the consistency of different probability calculation methods.

Extending the Analysis: Specific Cases & Strategy

5. Drawing Socks of Different Colors

If you want the probability that the socks are specifically of different colors, you could analyze particular pairs:

- Black & Blue
- Black & Red
- Blue & Red

For example, the probability of drawing a black sock first and a blue sock second:

$$
\frac{10}{20} \times \frac{6}{19} = \frac{10}{20} \times \frac{6}{19} = 0.5 \times 0.3158 \approx 0.1579
$$

Similarly, for all mixed pairs, the total is:

$$
P(\text{black & blue}) + P(\text{black & red}) + P(\text{blue & red}) + \text{their reverse orders}
$$

which, when summed, confirms the total probability of drawing two different-colored socks (~65.3%).

Practical Applications & Insights

This probability analysis extends beyond theoretical curiosity and has practical implications:

- Clothing Industry: Understanding the likelihood of matching socks when randomly selecting pairs for packaging or customer orders.
- Quality Control: Estimating the chances of mismatched socks in manufacturing batches.
- Game Design: Creating probability-based challenges involving random draws, such as in puzzles or educational tools.
- Decision-Making: Making informed choices based on probabilistic outcomes, especially in scenarios of uncertainty.

Variations and Extensions

The basic problem can be expanded or modified in numerous ways:

- Replacing socks after each draw: Changing the probability calculations for independent events.
- Drawing more than two socks: Calculating probabilities for triplets or larger sets.
- Different distributions: Varying the number of socks per color to model different real-world scenarios.
- Introducing additional colors: Expanding the problem to include more categories and complex calculations.

Each variation enhances understanding of combinatorics and probability theory, serving as valuable educational tools.

Conclusion

The seemingly simple task of drawing socks from a drawer reveals a rich tapestry of mathematical principles. Through detailed combinatorial analysis, we have determined that the probability of drawing two socks of the same color from a drawer containing 10 black, 6 blue, and 4 red socks is approximately 34.7%. Conversely, there is a roughly 65.3% chance of drawing socks of different colors. These calculations not only demonstrate fundamental probability concepts but also highlight the importance of strategic thinking when dealing with randomness in everyday scenarios.

Whether applied to quality control, game design, or simply understanding the odds, this problem exemplifies how a straightforward question can unlock complex insights. Embracing such puzzles fosters critical thinking and enhances our appreciation of the mathematics woven into daily life.

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