A Hall With A 3 In. Radius Has Volume V . A Second Ball Has A 9 In. Radius And Volume V ' Which Equation

Understanding the Relationship Between Radius and Volume of Spheres

A Hall With A 3 In. Radius Has Volume V. A Second Ball Has A 9 In. Radius And Volume V' Which Equation raises an interesting mathematical question involving the properties of spheres. To analyze this scenario, it is essential to understand how the volume of a sphere relates to its radius. This understanding forms the foundation for deriving the equation that links the two given spheres' radii and volumes.

Fundamentals of Sphere Geometry

The Formula for the Volume of a Sphere

The volume V of a sphere with radius r is given by the well-known formula:

- $V = \frac{4}{3} \pi r^3$

This formula indicates that the volume of a sphere is directly proportional to the cube of its radius. As the radius increases, the volume increases cubically, which is a significant point when comparing spheres of different sizes.

Implication of the Cubic Relationship

Given the volume formula, if we have two spheres with radii r_1 and r_2, their volumes V_1 and V_2 satisfy:

- $V_1 = \frac{4}{3} \pi r_1^3$
- $V_2 = \frac{4}{3} \pi r_2^3$

Dividing one volume by the other gives:

$\frac{V_2}{V_1} = \frac{r_2^3}{r_1^3}$

This ratio is crucial for establishing the equation relating the volumes and radii of the two spheres.

Applying the Given Data to Derive the Equation

Known Data

1. First sphere radius: $r_1 = 3$ inches
2. Second sphere radius: $r_2 = 9$ inches
3. Volumes: V and V', corresponding to the first and second spheres respectively

Determine the Volume of the First Sphere

Using the volume formula:

- $V = \frac{4}{3} \pi (3)^3 = \frac{4}{3} \pi \times 27 = 36 \pi$

Determine the Volume of the Second Sphere

Similarly:

- $V' = \frac{4}{3} \pi (9)^3 = \frac{4}{3} \pi \times 729 = 972 \pi$

Establishing the Relationship Between V and V'

From the calculations:

- $V = 36 \pi$
- $V' = 972 \pi$

Dividing V' by V:

$\frac{V'}{V} = \frac{972 \pi}{36 \pi} = \frac{972}{36} = 27$

This confirms that the volume of the larger sphere is 27 times the volume of the smaller sphere, consistent with the ratio of the cubes of their radii, since:

- $\frac{r_2^3}{r_1^3} = \frac{9^3}{3^3} = \frac{729}{27} = 27$

General Equation Connecting Radii and Volumes

Deriving the General Equation

From the proportionality established, the general relationship between the volume and radius of two spheres is:

- $V' = V \times \left(\frac{r_2}{r_1} \right)^3$

Or, equivalently, given the radii and unknown volume:

- $V' = \frac{4}{3} \pi r_2^3$
- $V = \frac{4}{3} \pi r_1^3$

Thus, the key equation relating the volumes and radii of two spheres is:

Final Equation

$$V' = V \times \left(\frac{r_2}{r_1} \right)^3$$

Applications and Further Insights

Using the Equation in Practical Scenarios

- Predicting the volume of a larger sphere when the volume of a smaller sphere and their radii are known.
- Determining the radius of a sphere given its volume and the volume or radius of another sphere.

Example Calculation

Suppose a sphere with radius $r_1 = 4$ inches has volume V. If we want to find the volume V' of a sphere with radius $r_2 = 12$ inches:

- Calculate $V' = V \times \left(\frac{12}{4} \right)^3 = V \times 3^3 = V \times 27$

This illustrates how volume scales with the cube of the radius, emphasizing the importance of the cubic relationship in sphere geometry.

Conclusion

The initial question about two spheres with different radii and volumes is elegantly answered through the fundamental relationship between a sphere's radius and its volume. The key takeaway is that the volume of a sphere scales with the cube of its radius, leading to the important equation:

$$V' = V \times \left(\frac{r_2}{r_1} \right)^3$$

Understanding this relationship enables mathematicians, engineers, and scientists to make precise calculations involving spheres of various sizes. Whether in designing physical objects, modeling natural phenomena, or solving geometric problems, this principle remains a cornerstone of three-dimensional geometry.

Frequently Asked Questions

How does the volume of a sphere relate to its radius in the given problem?

The volume of a sphere is proportional to the cube of its radius, as given by the formula $V = (4/3)\pi r^3$. Therefore, increasing the radius significantly increases the volume.

What is the general equation connecting the radii of the two balls with the same volume V?

Since both balls have the same volume V, and volume $V = (4/3)\pi r^3$, the equation relating their radii is $(4/3)\pi(3)^3 = (4/3)\pi(9)^3$, which simplifies to $r_1^3 = r_2^3$ for equal volumes, but with different radii, their volumes are proportional to r^3.

If a smaller ball has a radius of 3 inches and volume V, what is the radius of the larger ball with the same volume?

Since the volumes are equal, and volume $V = (4/3)\pi r^3$, the larger ball's radius is 9 inches, as given, confirming that $r_2 = 3\ 3 = 9$ inches.

What is the equation to find the radius of a sphere given its volume V?

The radius r can be found using the formula r = [(3V) / (4π)]^(1/3).

How can we verify that two spheres with radii 3 in. and 9 in. have the same volume V?

Calculate their volumes: $V_1 = (4/3)\pi(3)^3$ and $V_2 = (4/3)\pi(9)^3$. Since $(3)^3 = 27$ and $(9)^3 = 729$, the volumes are $V_1 = (4/3)\pi 27$ and $V_2 = (4/3)\pi 729$. V_2 is 27 times larger than V_1, so unless the volumes are scaled or adjusted, they are not equal unless specified otherwise. To have equal volume V, the equation is $(4/3)\pi r_1^3 = (4/3)\pi r_2^3$, leading to $r_1^3 = r_2^3$.

Additional Resources

A Hall With A 3 In. Radius Has Volume V. A Second Ball Has A 9 In. Radius And Volume V' Which Equation?

In the realm of geometry and spatial analysis, understanding the relationships between the dimensions of objects and their volumes is fundamental. This understanding not only deepens our comprehension of three-dimensional shapes but also finds practical applications across engineering, architecture, manufacturing, and even computer graphics. Today, we explore a compelling mathematical problem involving two spherical objects—specifically, a hall and a ball—each with different radii and volumes. The question that arises is: "Which equation relates the volumes of two spheres given their radii?" This seemingly straightforward query unravels into a rich discussion about the geometry of spheres, the formulas governing their volumes, and how these principles are applied in real-world scenarios.

Understanding the Geometry of a Sphere

The Sphere: Basic Definitions

A sphere is a perfectly symmetrical three-dimensional shape where every point on its surface is equidistant from its center. The key measurements associated with a sphere include:

- Radius (r): The distance from the center to any point on the surface.
- Diameter (d): Twice the radius (d = 2r).
- Circumference: The perimeter of the circle formed by a cross-section through the center.
- Surface Area: The total area covering the sphere's surface.
- Volume (V): The three-dimensional space enclosed by the sphere.

Understanding these parameters is essential because they are interconnected through mathematical formulas, especially when calculating volume.

The Volume Formula for a Sphere

The volume of a sphere is given by the well-established mathematical formula:

$$
V = \frac{4}{3} \pi r^3
$$

where:

- V is the volume,
- r is the radius,
- π (pi) is a constant approximately equal to 3.14159.

This formula states that the volume is directly proportional to the cube of the radius, scaled by the factor $\frac{4}{3} \pi$.

Applying the Volume Formula to Our Scenario

Given the problem, we have two objects:

1. A hall with a radius of 3 inches and volume V.
2. A second ball with a radius of 9 inches and volume V'.

Our goal is to find the relation between V and V', which involves understanding how their volumes relate based on their radii.

Calculating the Volume of the First Sphere (the Hall)

Using the volume formula:

$$
V = \frac{4}{3} \pi (3)^3
$$

Calculating:

$$
V = \frac{4}{3} \pi \times 27 = 36 \pi
$$

Thus,

$$
V = 36 \pi \quad \text{cubic inches}
$$

Calculating the Volume of the Second Sphere (the Ball)

Similarly, for the second ball with radius 9 inches:

$$
V' = \frac{4}{3} \pi (9)^3
$$

Calculating:

$$
V' = \frac{4}{3} \pi \times 729 = 972 \pi
$$

Therefore,

$$
V' = 972 \pi \quad \text{cubic inches}
$$

Deriving the Relation Between V and V'

Now, observe that:

$$
V = 36 \pi
$$

$$
V' = 972 \pi
$$

Dividing V' by V:

$$
\frac{V'}{V} = \frac{972 \pi}{36 \pi} = \frac{972}{36} = 27
$$

This indicates:

$$
V' = 27 V
$$

Key insight: The volume of the second sphere (radius 9 inches) is 27 times the volume of the first sphere (radius 3 inches).

General Relationship: Volumes and Radii

From the calculations, we see that:

$$
V \propto r^3

\]

$$
V' \propto (3r)^3 = 27 r^3
$$

In general, for two spheres with radii r_1 and r_2, the volumes relate as:

$$
\frac{V_2}{V_1} = \left(\frac{r_2}{r_1}\right)^3
$$

Applying this to our specific case:

$$
\frac{V'}{V} = \left(\frac{9}{3}\right)^3 = 3^3 = 27
$$

which confirms our earlier calculation.

Formulating the Equation Relating V and V'

Based on the proportionality, the precise relation between the volumes and radii can be expressed as:

$$
V' = \left(\frac{r_2}{r_1}\right)^3 V
$$

or, explicitly,

$$
V' = \left(\frac{9}{3}\right)^3 V = 27 V
$$

Rearranged, the general equation connecting the two is:

$$
V' = \left(\frac{r_2}{r_1}\right)^3 V
$$

This formula encapsulates the core principle: the volume of a sphere scales with the cube of its radius.

Implications and Applications

Understanding this relationship is particularly useful in various fields:

- Engineering: Designing spherical tanks or containers, where knowing how volume scales with size is critical.
- Architecture: Calculating the capacity of spherical structures based on their dimensions.
- Manufacturing: Estimating material requirements for spherical objects.
- Physics and Astronomy: Understanding celestial bodies' sizes and volumes based on radius measurements.

Summary and Key Takeaways

- The volume of a sphere is given by $V = \frac{4}{3} \pi r^3$.
- When comparing two spheres, their volumes are proportional to the cubes of their radii.
- The relation between the volumes of two spheres with radii r_1 and r_2 is:

$$
V_2 = \left(\frac{r_2}{r_1}\right)^3 V_1
$$

- In the specific case of a hall with a radius of 3 inches and a second ball with a radius of 9 inches, the volume of the second ball is 27 times the volume of the hall.

This exploration underscores the elegance of geometric relationships and how a simple cubic law governs the scaling of volumes in spheres. Whether designing physical structures or analyzing natural phenomena, these principles serve as foundational tools for precise calculations and insightful understanding.

Final Thoughts: The Power of Mathematical Relationships

The problem exemplifies how fundamental mathematical formulas provide clarity and precision in understanding the physical world. Recognizing that volume scales with the cube of the radius allows us to quickly determine how changing the size of an object impacts its capacity or volume. This knowledge is invaluable across disciplines, from engineering to astronomy, guiding both theoretical understanding and practical application.

By mastering these relationships, scientists, engineers, and designers can make informed decisions, optimize designs, and deepen their appreciation for the geometric harmony that underpins our universe.

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