A New Sample Of 325 Employed Adults Is Chosen. Find The Probability That Less Than 6% Of The Individuals

A New Sample Of 325 Employed Adults Is Chosen. Find The Probability That Less Than 6% Of The Individuals

When conducting statistical analysis, understanding the likelihood of certain outcomes within a sample is crucial. In this context, suppose a surveyor has randomly selected a new sample of 325 employed adults and wants to determine the probability that less than 6% of these individuals exhibit a particular characteristic or behavior. This type of problem involves applying probability concepts, particularly the binomial distribution and its approximation using the normal distribution under certain conditions. This article explores how to calculate such probabilities, the underlying assumptions, and practical applications, providing a comprehensive guide for students and professionals alike.

Understanding the Problem: Key Concepts and Terminology

Sample Size and Population Proportion

In this scenario, the sample size is $n = 325$ employed adults. The population proportion, p, refers to the true proportion of all employed adults who share a specific characteristic of interest (e.g., those who prefer remote work). The goal is to find the probability that less than 6% ($p < 0.06$) of the sampled individuals possess this trait.

What Does "Less Than 6%" Mean in Probability Terms?

The phrase indicates a focus on the probability that the observed sample proportion, $\hat{p}$, is less than 0.06. Since $\hat{p}$ is a random variable, we want $P(\hat{p} < 0.06)$, which depends on the underlying population proportion p and the sampling distribution of $\hat{p}$.

Modeling the Sampling Distribution of the Sample Proportion

The Binomial Distribution Framework

The number of individuals with the characteristic in the sample, X, can be modeled as a binomial random variable:

- Number of trials: $n = 325$
- Probability of success (having the characteristic): p
- Probability of failure: $1 - p$

The sample proportion $\hat{p} = X / n$.

The Distribution of $\hat{p}$

The sampling distribution of $\hat{p}$ is binomially distributed with:
- Mean: $\mu = p$
- Variance: $\sigma^2 = p(1 - p) / n$

For large n, the binomial distribution can be approximated by a normal distribution, thanks to the Central Limit Theorem.

Applying the Normal Approximation to the Binomial Distribution

Conditions for Approximation

The normal approximation is appropriate when both np and $n(1 - p)$ are at least 5:

- $np \geq 5$
- $n(1 - p) \geq 5$

If these conditions are met, the distribution of $\hat{p}$ can be approximated by a normal distribution with:
- Mean: $\mu = p$
- Standard deviation: $\sigma_{\hat{p}} = \sqrt{[p(1 - p) / n]}$

Calculating the Probability

To find $P(\hat{p} < 0.06)$, we:
1. Assume a value for p (if unknown, consider the worst-case scenario or use an estimated p).
2. Calculate $\sigma_{\hat{p}}$.
3. Use the z-score formula:

$$z = (\hat{p} - p) / \sigma_{\hat{p}}$$

4. Find the probability corresponding to the z-score using standard normal distribution tables or software.

Step-by-Step Example Calculation

Suppose, based on prior data, the estimated population proportion $p = 0.05$ (5%). Let's calculate the probability that less than 6% of the sample have the characteristic.

Step 1: Check Conditions for Approximation

- $np = 325 \ 0.05 = 16.25 \geq 5$ ✓
- $n(1 - p) = 325 \ 0.95 = 308.75 \geq 5$ ✓

Since both conditions are satisfied, normal approximation is valid.

Step 2: Calculate Standard Error

$\sigma_{\hat{p}} = \sqrt{[p(1 - p) / n]} = \sqrt{[0.05 \ 0.95 / 325]} \approx \sqrt{[0.0475 / 325]} \approx \sqrt{[0.00014615]} \approx 0.01209$

Step 3: Compute Z-Score

$\hat{p} = 0.06$

$z = (0.06 - 0.05) / 0.01209 \approx 0.01 / 0.01209 \approx 0.827$

Step 4: Find Corresponding Probability

Using standard normal tables or calculators, $P(z < 0.827) \approx 0.7967$

Therefore, the probability that less than 6% of the sample have the characteristic, assuming $p = 0.05$, is approximately 79.67%.

Exploring Different Population Proportions

When the True p Is Unknown

If the exact value of p is unknown, analysts often:
- Use the sample proportion from previous studies as an estimate.
- Consider a range of p values to understand how the probability varies.

Impact of p on the Probability

As p approaches 0.06, the probability of $\hat{p}$ being less than 0.06 increases, and vice versa. For extreme values of p (near 0 or 1), the probability calculations change significantly.

Practical Applications and Significance

Policy and Decision Making

Understanding the probability that less than 6% of employed adults have a certain trait can influence:

- Workplace policies
- Resource allocation
- Targeted interventions

Market Research and Survey Analysis

Companies can use these probability calculations to:

- Estimate market penetration
- Assess the effectiveness of campaigns
- Predict future trends based on sample data

Limitations and Considerations

Sample Size and Variability

Small samples may not satisfy the conditions for the normal approximation, leading to inaccurate probability estimates.

Assumptions of Random Sampling

The calculations assume the sample is randomly selected, representing the population accurately. Biases in sampling can distort results.

Unknown or Changing Population Proportions

If p varies over time or across different groups, static calculations may not reflect current realities.

Conclusion: How to Use These Concepts Effectively

Calculating the probability that less than 6% of a sample of 325 employed adults possess a certain characteristic involves understanding the binomial distribution, applying the normal approximation, and interpreting the results in context. Whether for academic research, business analysis, or policy development, mastering these statistical tools enables informed decision-making based on sample data. Remember to verify the conditions for approximation, consider the accuracy of your estimated p, and interpret the probability within the broader context of your study or application.

By following these steps and principles, researchers and analysts can confidently assess the likelihood of various outcomes, ensuring their conclusions are statistically sound and practically relevant.

Frequently Asked Questions

What is the probability that less than 6% of the 325 employed adults are selected in the sample?

To determine this probability, you need to model the number of individuals less than 6% of 325 (which is approximately 19.5) using a binomial or normal approximation, depending on the context. If the probability of success in the population is known, you can calculate the probability that fewer than 19 or 20 individuals are selected.

How can I approximate the probability that less than 6% of the sample are employed adults using normal distribution?

Since the sample size is large, you can use the normal approximation to the binomial distribution. Calculate the mean and standard deviation for the binomial variable, then find the z-score for 19.5 (less than 6%), and use standard normal tables to estimate the probability.

What information do I need to accurately compute this probability?

You need to know the true proportion of employed adults in the population (the probability p). Without this, you cannot accurately compute the probability that less than 6% are selected. If p is unknown, you may need to estimate it from prior data.

If the true proportion of employed adults in the population is 5%, what is the probability that fewer than 6% are in the sample?

Assuming p=0.05, the expected number of successes in the sample is 325 0.05 = 16.25. You would calculate the probability that the number of successes is less than 19.5 using the binomial or normal approximation. This probability would be quite high, indicating it's likely fewer than 6% in the sample.

Why is it important to consider the sample size when calculating this probability?

A larger sample size justifies using the normal approximation to the binomial distribution, making calculations easier and more accurate. For smaller samples, exact binomial probabilities should be used to avoid approximation errors.

Additional Resources

Understanding the Probability of Less Than 6% of Employed Adults in a Sample of 325

When analyzing statistical data, especially in the context of workforce studies, understanding the probability that a certain proportion of individuals exhibit a particular characteristic is essential. In this case, we analyze a sample of 325 employed adults and explore the probability that less than 6% of these individuals fall into a specific category—perhaps those who possess a certain skill, are part of a particular industry, or meet another criterion.

This comprehensive review delves into the statistical concepts involved, the calculations necessary, and the implications of such probabilities, providing a thorough understanding suitable for students, researchers, or professionals engaged in data analysis.

Context and Significance of the Problem

Before diving into calculations, it’s crucial to interpret the problem properly:

- Sample Size (n): 325 employed adults.
- Proportion of Interest (p): The true proportion of the population with a certain characteristic. The problem asks about the probability that less than 6% ($p < 0.06$) of this sample exhibit that characteristic.
- Objective: Find $P(\hat{p} < 0.06)$, where $\hat{p}$ (p-hat) is the sample proportion.

This problem is rooted in probability theory, particularly in the use of sampling distributions, binomial probabilities, and the normal approximation to the binomial distribution.

Understanding Key Concepts

Sample Proportion ($\hat{p}$)

- The sample proportion $\hat{p}$ is calculated as:

$\hat{p}$ = (Number of individuals with characteristic) / n

- It's a random variable that varies from sample to sample.

- Its distribution reflects the underlying population proportion p and the sample size n.

Sampling Distribution of $\hat{p}$

- When the sample size is sufficiently large, the distribution of $\hat{p}$ approximates a normal distribution due to the Central Limit Theorem.

- The mean of this distribution is p.

- The standard deviation (standard error) is:

SE = sqrt[p(1 - p) / n]

- This approximation is valid when both np and n(1 - p) are greater than 5 or 10.

Binomial Distribution and Normal Approximation

- The number of successes in the sample follows a binomial distribution: Bin(n, p).

- When n is large, Bin(n, p) can be approximated by a normal distribution with mean np and standard deviation sqrt[np(1 - p)].

- The probability that $\hat{p}$ is less than a certain value can then be computed using the normal distribution.

Step-by-Step Approach to Find the Probability

To determine $P(\hat{p} < 0.06)$, we proceed systematically:

1. Identify the assumed population proportion (p):
Since the problem doesn't specify p directly, it's common to interpret it as the true population proportion, or to analyze a range of p values. Often, the question concerns the probability that $\hat{p}$ is less than 6% given some assumed p.

2. Determine the standard error (SE):
Using the assumed p, calculate the standard deviation of the sampling distribution:

SE = sqrt[p(1 - p) / n]

3. Convert the problem into a z-score:

z = (p̂ - p) / SE

For a specific p̂ (here, 0.06), the z-score becomes:

z = (0.06 - p) / SE

4. Calculate the probability:

Use standard normal distribution tables or software to find P(z < computed value), which gives the probability that p̂ is less than 0.06.

Considering Different Values of p

Since the problem involves the probability that less than 6% of individuals have a particular trait, it's often framed in terms of:

- The true population proportion p being a certain value.
- The probability that the sample proportion p̂ falls below 6%, given p.

This leads to the concept of conditional probability, where p is fixed, and p̂ varies.

Example 1: Suppose the true population proportion p = 0.10 (10%).

- Calculate the standard error:

SE = sqrt[0.10 (1 - 0.10) / 325] = sqrt[0.10 0.90 / 325] = sqrt[0.09 / 325] ≈ sqrt[0.0002769] ≈ 0.0166

- Compute the z-score for p̂ = 0.06:

z = (0.06 - 0.10) / 0.0166 ≈ (-0.04) / 0.0166 ≈ -2.41

- Find P(z < -2.41):

From standard normal tables, P(z < -2.41) ≈ 0.0080, or 0.8%.

Interpretation: If the true proportion is 10%, there's about a 0.8% chance that the sample proportion will be less than 6%.

Implications and Real-World Applications

Understanding such probabilities is crucial in various contexts:

- Quality Control: Manufacturers may want to know the chance that defect rates are below a certain threshold.
- Workforce Analysis: HR professionals might estimate the likelihood that less than 6% of employees possess a skill.
- Public Health: Epidemiologists could use this approach to estimate the probability that less than 6% of a population has a certain condition.

In each case, the key is to interpret the probability correctly, considering the assumed population proportion and the sample size.

Limitations and Assumptions

While the normal approximation simplifies calculations, it relies on certain assumptions:

- Large sample size: Usually, both np and n(1 - p) should be greater than 5 or 10 to justify the approximation.

- Random sampling: The sample should be representative of the population.

- Independence: Each individual's trait status should be independent of others.

If these assumptions are violated, the calculated probabilities may be inaccurate, and alternative methods like exact binomial probabilities should be used.

Using Software and Calculators

In practice, calculating these probabilities can be efficiently done using statistical software or online calculators:

- Excel: Use the NORM.DIST function.
- R: Use pnorm() function.
- Python: Use scipy.stats.norm.cdf().

Example using R:

```r
Assume p = 0.10
```

```
p <- 0.10
n <- 325
p_hat <- 0.06

SE <- sqrt(p (1 - p) / n)
z <- (p_hat - p) / SE
probability <- pnorm(z)
print(probability)
```

This code computes the probability that $\hat{p} < 0.06$ when the true p is 0.10.

Alternative Scenarios and Further Considerations

- Unknown p: If p is unknown, and the goal is to estimate the probability across different p values, then techniques like confidence intervals or Bayesian methods may be appropriate.
- Two-sided questions: If the question involves the probability that $\hat{p}$ is less than 6% or greater than some value, the calculations would adjust accordingly.
- Small sample sizes: For small n, the normal approximation may not be valid, and exact binomial probabilities should be used instead.

Summary and Final Thoughts

Calculating the probability that less than 6% of 325 employed adults possess a certain characteristic involves understanding the sampling distribution of the sample proportion, the application of the normal approximation to the binomial distribution, and precise calculation of the z-score.

Key takeaways include:

- The importance of verifying assumptions for the normal approximation.
- Recognizing the influence of the true population proportion (p) on the probability.
- Using computational tools to streamline calculations.
- Appreciating the real-world applications of these probability calculations.

In conclusion, such probability assessments are fundamental in statistical inference, helping researchers and analysts make informed decisions, assess risks, and understand the variability inherent in sample data. Whether in workforce studies, quality control, or health sciences, mastering these concepts enhances the ability to interpret data accurately and confidently.

References & Resources:

- Statistics for Business and Economics by Anderson, Sweeney, Williams
- Online calculators: [Khan Academy Probability Calculator](https://www.khanacademy.org/math/statistics-probability)
- R documentation: [pnorm() function](https://stat.ethz.ch/R-manual/R-devel/library/stats/html/pnorm.html)
- Python's SciPy library: [scipy.stats.norm.cdf()](https://docs.scipy.org/doc/scipy/reference/generated/scipy.stats.norm.html)

If further analysis or specific scenarios are required, consulting statistical textbooks or software documentation can provide additional depth and clarity.

A New Sample Of 325 Employed Adults Is Chosen Find The Probability That Less Than 6 Of The Individuals

A New Sample Of 325 Employed Adults Is Chosen Find The Probability That Less Than 6 Of The Individuals

Related Articles

- Which Answer Correctly Distinguishes Among Limestone, Dolostone, And Chert?Limestone, Which Is Composed
- A Very Long Thin Solenoid Carries A Time Dependent Current Isol(t). A Conducting Ring Of A Larger Radius
- A Monopolist Who Price Discriminates Perfectly Will Produce The Socially Efficient Level Of Output. True

Back to Home