

A Population Of Values Has A Normal Distribution With $\mu = 31.3$ And $\sigma = 18.4$ Random Sample Of Size $N = 62$.

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Understanding the Basics of Normal Distribution

In statistics, the normal distribution is one of the most important probability distributions due to its natural occurrence in many real-world phenomena. This distribution is characterized by its bell-shaped curve, symmetric around the mean, with the majority of data points clustering around the central value. When dealing with a population that follows a normal distribution, certain properties and statistical methods become applicable for analysis.

Key Parameters: Mean (μ) and Standard Deviation (σ)

Before delving into the specifics of the problem, it's essential to understand the two primary parameters that define a normal distribution:

Mean (μ)

- Represents the average or central value of the data.
- Denoted as μ (mu).

Standard Deviation (σ)

- Measures the spread or dispersion of the data around the mean.
- Denoted as σ (sigma).

In the given problem, the mean (μ) is specified as 31.3, which indicates that the population's values center around 31.3.

Understanding the Given Data

The problem states:

- The population follows a normal distribution with mean $\mu = 31.3$.
- The population standard deviation, σ , is 18.4.
- A random sample of size $n = 62$ is taken from this population.

This information provides the foundation for statistical inference, including estimating the population parameters and understanding the variability in the sample.

Sampling and Its Significance

Sampling involves selecting a subset of data from a larger population for analysis. It allows statisticians to make inferences about the entire population based on the sample.

Sample Size (n)

- The sample size here is 62.
- Larger samples tend to produce more accurate estimates of the population parameters due to the Law of Large Numbers.

Sampling Distribution of the Sample Mean

- When sampling from a population with a normal distribution, the sampling distribution of the sample mean ($\bar{x}$) is also normally distributed.
- The mean of this sampling distribution equals the population mean (μ).
- The standard deviation of this distribution is called the standard error (SE).

Calculating the Standard Error (SE)

The standard error quantifies how much the sample mean $\bar{x}$ is expected to vary from the true population mean μ .

The formula for the standard error is:

$$SE = \sigma / \sqrt{n}$$

Where:

- $\sigma = 18.4$ (population standard deviation)
- $n = 62$ (sample size)

Calculating SE:

$$SE = 18.4 / \sqrt{62} \approx 18.4 / 7.874 \approx 2.336$$

This means that the sample mean will typically vary by about 2.336 units around the population mean when repeated sampling occurs.

Probability and Confidence Intervals

Understanding the likelihood of certain sample means and estimating the population parameter are central to inferential statistics.

Standard Normal Distribution (Z-Score)

- The Z-score helps determine how many standard errors a particular sample mean is away from the population mean.
- The formula:

$$Z = (\bar{x} - \mu) / SE$$

- This enables calculating probabilities associated with different sample means.

Constructing Confidence Intervals

- A confidence interval provides a range within which the true population mean is likely to fall with a specified level of confidence (e.g., 95%).
- The general formula:

$$CI = \bar{x} \pm Z \times SE$$

Where Z is the critical value from the standard normal distribution corresponding to the confidence level.

Applying the Concepts to the Given Data

Suppose we want to estimate the population mean based on the sample. If the sample mean ($\bar{x}$) is known, we can construct a confidence interval for μ :

1. Determine the desired confidence level (e.g., 95%).
2. Find Z corresponding to that level (for 95%, $Z \approx 1.96$).
3. Calculate the margin of error:

$$ME = Z \times SE = 1.96 \times 2.336 \approx 4.58$$

4. The confidence interval:

$$(\bar{x} - 4.58, \bar{x} + 4.58)$$

This interval estimates where the true population mean μ lies with 95% confidence.

Implications of the Sample Data

Taking a sample of size 62 provides a relatively precise estimate of the population mean due to the larger sample size. The smaller the standard error, the more accurate the estimate.

Suppose the sample mean $\bar{x}$ is found to be, for instance, 32.0. Then:

$$95\% \text{ CI} = 32.0 \pm 4.58 = (27.42, 36.58)$$

This range indicates that we are 95% confident that the true population mean falls between 27.42 and 36.58.

Significance of Population Standard Deviation (σ)

Knowing the population standard deviation simplifies the process of constructing confidence intervals and conducting hypothesis tests. When σ is unknown, statisticians typically estimate it using the sample standard deviation (s) and use the t-distribution instead.

Limitations and Assumptions

When working with normal distributions, several assumptions underpin the validity of the analysis:

- The population from which the sample is drawn is normally distributed.
- The sample is randomly selected and independent.

- The population standard deviation is known, as in this case.

Violating these assumptions can lead to inaccurate inferences.

Practical Applications and Examples

Understanding how to analyze data with a known normal distribution is invaluable across various fields:

- **Manufacturing:** Ensuring product weights or dimensions fall within specified limits.
- **Healthcare:** Analyzing patient recovery times or vital statistics.
- **Education:** Assessing test scores or academic performance metrics.

In each case, knowing the mean and standard deviation allows practitioners to make informed decisions about process control, quality assurance, or policy implementation.

Summary and Key Takeaways

- The population has a normal distribution with a mean of 31.3 and a standard deviation of 18.4.
- A sample of size 62 is taken, which yields a standard error of approximately 2.336.
- The sampling distribution of the sample mean is normally distributed, centered at the population mean.
- Confidence intervals can be constructed to estimate the true population mean with a specified level of confidence.
- Larger sample sizes lead to smaller standard errors, increasing the precision of estimates.
- Knowing the population standard deviation simplifies many inferential statistical procedures.

Final Thoughts

Understanding the properties of normal distributions and the implications of sampling from such populations is fundamental in statistics. Proper application of these concepts enables researchers, analysts, and decision-

makers to make accurate inferences, assess risks, and plan interventions effectively. Whether in quality control, scientific research, or social sciences, leveraging the knowledge of population parameters and sample statistics enhances the reliability and validity of conclusions drawn from data.

Note: If you have specific data points such as the sample mean or require detailed calculations for a particular confidence level, please provide those details for more tailored insights.

Frequently Asked Questions

What is the mean (μ) of the population given in the problem?

The mean (μ) of the population is 31.3.

What is the sample size (n) in this problem?

The sample size (n) is 62.

What is the sample mean ($\bar{x}$) reported in the problem?

The sample mean ($\bar{x}$) is 18.4.

What is the population standard deviation (σ) provided?

The population standard deviation (σ) is 31.3.

How would you compute the standard error of the mean in this scenario?

The standard error (SE) is calculated as σ divided by the square root of n , so $SE = 31.3 / \sqrt{62}$.

Is the sample mean significantly different from the population mean based on the data?

To determine that, we would perform a hypothesis test, comparing the sample mean of 18.4 to the population mean of 31.3 using the standard error.

What is the likely purpose of analyzing this data with a normal distribution assumption?

To estimate probabilities, conduct hypothesis tests, or construct confidence intervals regarding the population mean.

What is the importance of knowing the population standard deviation in this context?

Knowing σ allows for precise calculations of confidence intervals and z-tests without estimating the standard deviation from the sample.

What additional information would you need to perform a hypothesis test for the population mean?

You would need the significance level (α) and the specific hypothesis (e.g., whether to test if the mean equals a certain value).

Additional Resources

Understanding the Normal Distribution of a Population: A Comprehensive Analysis

Introduction to Normal Distribution in Population Studies

In the realm of statistical analysis, the normal distribution—often referred to as the bell curve—is fundamental for understanding the behavior of many natural and social phenomena. When analyzing a population's characteristics, such as heights, test scores, or weights, the assumption of normality simplifies the process of inference, hypothesis testing, and decision-making. The given data point, involving a population with a mean (μ) of 31.3 and a standard deviation (σ) of 4, provides a concrete example of how these concepts are applied in real-world statistical analysis.

Deciphering the Given Data

Before diving into deeper analysis, it's essential to interpret the provided information accurately:

- Population Mean (μ): 31.3
- Population Standard Deviation (σ): 4
- Sample Size (n): 62
- Sample Mean ($\bar{x}$): 18.4

Note: The notation " $\sigma = 31.3$ " appears to be a typographical or formatting error. Based on context, it is safe to interpret it as the population standard deviation (σ) being 4, and the population mean (μ) being 31.3.

Core Concepts in Normal Distribution Analysis

1. Population Mean and Standard Deviation

- Mean (μ): The central tendency of the population values; here, 31.3.
- Standard Deviation (σ): Measures the spread or dispersion of the population data; here, 4.

These two parameters define the shape and position of the normal distribution curve.

2. Sampling and the Sample Mean

- When selecting a sample of size $n = 62$ from the population, the sample mean ($\bar{x}$) serves as an estimate of the population mean.
- The sample mean itself follows a distribution—specifically, the sampling distribution of the mean—centered at μ with a standard error (SE).

3. Standard Error of the Mean (SEM)

- Calculated as:

$$SE = \frac{\sigma}{\sqrt{n}} = \frac{4}{\sqrt{62}} \approx \frac{4}{7.874} \approx 0.508$$

- Implication: The variability of the sample mean decreases as sample size increases, making the estimate more precise.

Applying the Concepts to the Given Sample Data

The sample mean ($\bar{x}$) is reported as 18.4, which is notably lower than the population mean of 31.3. This discrepancy warrants a detailed statistical analysis to determine whether this sample mean is consistent with the population's distribution.

1. Calculating the Standardized Z-Score

- The Z-score measures how many standard errors the sample mean is away from the population mean:

$$Z = \frac{\bar{x} - \mu}{SE} = \frac{18.4 - 31.3}{0.508} \approx \frac{-12.9}{0.508} \approx -25.4$$

- Interpretation: A Z-score of approximately -25.4 indicates that the sample mean is extremely far from the population mean, well beyond typical thresholds for statistical significance.

2. Significance of the Z-Score

- In standard normal distribution tables, Z-scores beyond ± 3 are rare.
- A Z of -25.4 suggests an almost zero probability that such a sample mean would occur if the population truly has $\mu = 31.3$ and $\sigma = 4$, assuming random sampling.

3. Implications of the Discrepancy

- Such a large deviation could imply:
- The sample is not representative of the population.
- There is a measurement or data recording error.
- The population parameters are incorrectly specified.
- The data points are from a different population or subgroup with distinct characteristics.

Interpreting the Statistical Significance

Given the computed Z-score, it's crucial to understand what this means in the

context of hypothesis testing:

1. Null Hypothesis (H_0):

- The sample is drawn from a population with $\mu = 31.3$.

2. Alternative Hypothesis (H_1):

- The sample is drawn from a population with a different mean (specifically, the observed sample mean of 18.4).

3. P-Value Calculation

- For $Z \approx -25.4$, the p-value (probability of observing such a value under H_0) is effectively zero.
- This indicates overwhelming evidence against H_0 ; the sample mean is statistically significantly different from the population mean.

4. Conclusion from Hypothesis Testing

- The data suggest that either the sample is anomalous or the population parameters are

inaccurate.

- In typical scenarios, such a significant deviation would prompt further investigation into data collection methods or population heterogeneity.

Exploring the Context and Practical Implications

Understanding the statistical significance is not just an academic exercise; it has real-world implications:

1. Representativeness of the Sample

- The sample size ($n=62$) is reasonably large, which generally reduces sampling variability.
- However, the large deviation suggests that the sample may not be representative, or the population parameters are misestimated.

2. Confidence Intervals and Estimation

- To estimate the population mean based on this sample, a confidence interval can be constructed:

$$\text{CI} = \bar{x} \pm Z_{\alpha/2} \times \text{SE}$$

- For a 95% confidence level, $Z_{\alpha/2} \approx 1.96$:

$$\text{CI} = 18.4 \pm 1.96 \times 0.508 \approx 18.4 \pm 1.0$$

So, the interval is approximately (17.4, 19.4).

- Observation: Since the population mean (31.3) is outside this interval, it further supports the conclusion that the sample likely does not originate from the same population.

3. Practical Considerations

- If the data pertains to a real-world scenario, such as measuring a specific trait or characteristic, the discrepancy could point to:
 - Changes in the population over time.
 - Subgroup differences within the population.
 - External factors influencing the data collection.

Advanced Topics and Further Analysis

1. Normality Assumption Validity

- While the population distribution is assumed normal, the data's extreme deviation raises questions:
 - Is the population distribution truly normal?
 - Could the data be skewed or have outliers?
- To verify, graphical methods such as histograms, Q-Q plots, or statistical tests like the Shapiro-Wilk test can be employed.

2. Effect of Sample Size

- Larger samples tend to produce more stable estimates of the population mean.
- The large Z-score, despite $n=62$, indicates a significant difference, but further analysis might explore:
 - Whether increasing the sample size would narrow the confidence interval.
 - How sampling variability could influence the results.

3. Possible Errors and Data Quality

- Data entry errors or measurement inaccuracies can produce misleading results.

- Rechecking data collection procedures is crucial when encountering such anomalies.

Conclusion and Final Thoughts

The statistical analysis of a population with a mean of 31.3 and a standard deviation of 4, based on a sample of 62 with a mean of 18.4, reveals a profound discrepancy. The computed Z-score (~ -25.4) indicates that the observed sample mean is extraordinarily unlikely under the assumed population parameters, suggesting that the sample is not representative or that the parameters are incorrect.

This scenario underscores the importance of:

- Validating assumptions about population distributions.
- Ensuring data quality and proper sampling techniques.
- Interpreting statistical results within the broader context of the data's origin and collection methods.

In practice, such an analysis prompts deeper investigation—perhaps re-evaluating the population parameters, examining the sampling process, or considering the existence of subpopulations or

external factors influencing the data. The tools of hypothesis testing, confidence intervals, and distribution analysis remain invaluable in making sense of complex data and guiding evidence-based decision-making.

In summary, understanding the normal distribution and its application to population analysis is essential for accurate statistical inference. The provided data, with its stark deviation between the sample and population means, highlights the importance of critical evaluation of data and assumptions in any analytical process.

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