

A Random Sample Of 49 People Are Selected From A Population And It Is Found That 12 People In The Sample

Understanding the Scenario: A Random Sample of 49 People and the Observation of 12 Individuals

A Random Sample Of 49 People Are Selected From A Population And It Is Found That 12 People In The Sample forms the basis of a statistical analysis aimed at understanding the characteristics of a larger population. This scenario is typical in research, survey analysis, and public health studies where sampling is used to infer information about a broader group without examining every individual. In this article, we will explore the implications of this sample, how to analyze the data statistically, and what conclusions can be drawn from these findings.

Defining the Core Concepts: Sampling and Population

What Is a Random Sample?

- A subset of individuals chosen from a larger population where each member has an equal chance of being selected.
- Ensures that the sample accurately represents the population, reducing selection bias.
- Common in surveys and research studies to make valid inferences about the population.

Understanding the Population

- The entire group about which researchers want to draw conclusions.
- In this context, the population could be all individuals within a certain geographic area, age group, or demographic characteristic.

- Knowing the population parameters helps in comparing sample findings and assessing representativeness.

Analyzing the Sample Data: What Does 12 Out of 49 Mean?

Calculating the Sample Proportion

The proportion of individuals with a specific characteristic in the sample is given by:

$$\hat{p} = (\text{Number of individuals with the characteristic}) / (\text{Total sample size})$$

In this case:

$$\hat{p} = 12 / 49 \approx 0.2449 \text{ (or approximately 24.49\%)}$$

Interpreting the Sample Proportion

- The sample indicates that roughly 24.5% of the sampled individuals possess the characteristic under study.
- This estimate serves as a point estimate for the population proportion, denoted as p .
- However, due to sampling variability, this estimate might not exactly match the true population proportion.

Statistical Inference and Estimating the Population Parameter

Constructing a Confidence Interval for the Population Proportion

- A confidence interval provides a range of plausible values for the population proportion based on sample data.

- Commonly used confidence levels are 90%, 95%, and 99%, with 95% being the standard.

Calculating the Confidence Interval

The formula for a confidence interval for a population proportion is:

$$\hat{p} \pm Z \sqrt{[\hat{p}(1 - \hat{p}) / n]}$$

Where:

- $\hat{p}$ = sample proportion (0.2449)
- n = sample size (49)
- Z = z-value corresponding to the desired confidence level (for 95%, $Z \approx 1.96$)

Applying the Formula

$$\text{Standard error (SE)} = \sqrt{[0.2449 (1 - 0.2449) / 49]} \approx \sqrt{[0.2449 \cdot 0.7551 / 49]} \approx \sqrt{[0.1851 / 49]} \approx \sqrt{0.00378} \approx 0.0615$$

$$\text{Confidence interval} = 0.2449 \pm 1.96 \cdot 0.0615 \approx 0.2449 \pm 0.1205$$

$$\text{Lower bound} \approx 0.1244 \text{ (12.44\%)}$$

$$\text{Upper bound} \approx 0.3654 \text{ (36.54\%)}$$

Interpretation of the Confidence Interval

We are 95% confident that the true proportion of the population with the characteristic lies between approximately 12.44% and 36.54%. This range accounts for sampling variability and provides a basis for making informed decisions.

Hypothesis Testing: Is the Sample Proportion Significantly Different From a Hypothesized

Population Proportion?

Setting Up the Hypotheses

- **Null hypothesis (H_0):** The population proportion p equals a specific value (e.g., p_0).
- **Alternative hypothesis (H_1):** The population proportion differs from p_0 .

Example Scenario

Suppose the population is believed to have a proportion $p_0 = 0.20$ (20%). To test whether the sample provides evidence that the true proportion differs from this:

- $H_0: p = 0.20$
- $H_1: p \neq 0.20$

Calculating the Test Statistic

$$z = (\hat{p} - p_0) / \sqrt{[p_0(1 - p_0) / n]}$$

Applying the values:

$$z = (0.2449 - 0.20) / \sqrt{[0.20 \cdot 0.80 / 49]} \approx 0.0449 / \sqrt{[0.16 / 49]} \approx 0.0449 / \sqrt{0.00327} \approx 0.0449 / 0.0572 \approx 0.785$$

Decision Rule

- For a significance level $\alpha = 0.05$, the critical z-value is approximately ± 1.96 .
- Since $|z| \approx 0.785 < 1.96$, we fail to reject the null hypothesis.
- This suggests there is not enough evidence to conclude that the population proportion differs from 20%.

Implications and Applications of the Sample Findings

Public Health and Policy Making

- Understanding the proportion of a population with a certain health condition can influence resource allocation.
- Sample data guides policymakers in designing targeted interventions.

Market Research and Business Strategy

- Estimating customer preferences or behaviors based on sample surveys informs marketing strategies.
- Sample proportions help companies assess product acceptance or brand awareness.

Academic and Scientific Research

- Samples provide evidence to support or refute hypotheses about populations.
- Understanding variability and confidence intervals enhances the reliability of research conclusions.

Limitations and Considerations in Sampling and Data Analysis

Sampling Bias and Errors

- Even random sampling can introduce errors if the sample size is too small or not truly random.

- Non-response or missing data can skew results.

Sample Size and Its Impact

- Smaller samples tend to have wider confidence intervals, reducing estimate precision.
- Larger samples provide more accurate estimates but may be more costly or difficult to obtain.

Assumptions in Statistical Tests

- Normality of the sampling distribution is assumed, which generally holds if np and $n(1 - p)$ are sufficiently large.
- Violations of assumptions can lead to inaccurate conclusions.

Conclusion: Making Data-Driven Decisions Based on Sample Analysis

The analysis of a sample of 49 individuals, where 12 are found to possess a certain characteristic, illustrates fundamental principles of statistical inference. By calculating sample proportions, constructing confidence intervals, and performing hypothesis tests, researchers and decision-makers can make informed judgments about the broader population. It is essential to consider the limitations of sampling and ensure that data collection methods are robust to derive valid conclusions. Whether in public health, market research, or academic studies, understanding the implications of sample data is vital for evidence-based decision-making.

Ultimately, the key takeaway is that a well-conducted random sampling process, combined with rigorous statistical analysis, provides powerful insights into the characteristics of large populations, guiding effective policies, strategies, and scientific understanding.

Frequently Asked Questions

What is the proportion of people in the sample who have the characteristic?

The proportion is 12 out of 49, which is approximately 0.245 or 24.5%.

How can we estimate the population proportion based on this sample?

The sample proportion ($\hat{p}$) can be used as an estimate; here, $\hat{p} = 12/49 \approx 0.245$.

What is the confidence interval for the true population proportion based on this sample?

Using a 95% confidence level, the interval can be calculated as $\hat{p} \pm \text{margin of error}$, considering sample size and variability.

Is the sample size of 49 sufficient for reliable statistical inference?

Generally, a sample size of 49 can provide reasonable estimates, but larger samples improve accuracy and confidence.

What hypothesis tests can be performed with this data?

You can perform a z-test for population proportions to determine if the observed proportion differs significantly from a hypothesized value.

How does this sample proportion compare to the overall population estimate?

Without the population proportion, we can't compare directly, but the sample provides an initial estimate that can be refined with more data.

What factors could affect the accuracy of the sample estimate?

Factors include sampling bias, sample size, variability within the population, and whether the sample is truly random.

Additional Resources

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Introduction

Sampling is a cornerstone of statistical analysis, allowing researchers to make inferences about a population based on a subset of data. The process involves selecting a representative portion of a larger group to estimate characteristics such as proportions, means, or variances. When the sample size and the number of observed events are known, statisticians employ various probabilistic models to determine the likelihood of such findings and to infer the population parameters.

In this investigative review, we analyze a scenario where a random sample of 49 people is selected from a population, and it is found that 12 people in the sample exhibit a particular characteristic or outcome. This case prompts questions about the underlying population proportion, the statistical significance of the observed sample proportion, and the potential implications of the findings.

Context and Significance of the Sample Data

The core data point here is:

- Sample size (n): 49
- Number of successes (x): 12

From this, the sample proportion ($\hat{p}$) is calculated as:

$$\hat{p} = \frac{x}{n} = \frac{12}{49} \approx 0.2449$$

This proportion suggests that approximately 24.5% of the sampled population exhibits the characteristic in question. The key questions arising from this data are:

- What is the estimated population proportion?
- Is this proportion significantly different from a hypothesized value?
- What does this tell us about the population?

To answer these, statisticians often formulate hypotheses and perform inferential tests.

Formulating Hypotheses and Statistical Testing

Suppose we want to test whether the population proportion (p) equals a specific value, say p_0 . Common scenarios include:

- Testing if the observed proportion is significantly different from a known or hypothesized value.
- Estimating the confidence interval for the population proportion.

Example Hypotheses:

- Null hypothesis, H_0 : $p = p_0$
- Alternative hypothesis, H_A : $p \neq p_0$

The choice of p_0 depends on the context—perhaps previous studies suggest a baseline proportion, or regulatory standards impose thresholds.

Statistical Analysis and Methods

1. Confidence Interval for the Population Proportion

Using the sample data, a 95% confidence interval (CI) for the true population proportion can be constructed via the normal approximation method:

$$CI = \hat{p} \pm Z_{\alpha/2} \times \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}}$$

where $Z_{\alpha/2}$ is the critical value from the standard normal distribution (approximately 1.96 for 95% CI).

Calculating:

$$\sqrt{\frac{0.2449 \times 0.7551}{49}} \approx \sqrt{\frac{0.185}{49}} \approx \sqrt{0.00378} \approx 0.0615$$

Thus,

$$CI = 0.2449 \pm 1.96 \times 0.0615 \approx 0.2449 \pm 0.1205$$

which yields:

$$\begin{aligned} \text{Lower bound} &\approx 0.1244 \\ \text{Upper bound} &\approx 0.3654 \end{aligned}$$

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Interpretation: We are 95% confident that the true population proportion lies between approximately 12.4% and 36.5%.

2. Hypothesis Testing

Suppose we want to test whether $(p = 0.20)$. Using a z-test:

$$\begin{aligned} z &= \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1 - p_0)}{n}}} \\ z &= \frac{0.2449 - 0.20}{\sqrt{\frac{0.20 \times 0.80}{49}}} = \frac{0.0449}{\sqrt{\frac{0.16}{49}}} = \frac{0.0449}{\sqrt{0.00327}} \approx \frac{0.0449}{0.0572} \approx 0.785 \end{aligned}$$

The critical z-value for a two-tailed test at 5% significance is approximately 1.96. Since $0.785 < 1.96$, we fail to reject the null hypothesis $(p = 0.20)$, indicating insufficient evidence to conclude that the population proportion differs significantly from 20%.

Implications and Interpretations

The statistical analysis reveals that the sample proportion (approximately 24.5%) is within a plausible range of the hypothesized values, especially considering the confidence interval spans from about 12.4% to 36.5%. This suggests the observed data could be consistent with a variety of true population proportions, emphasizing the importance of larger samples for more precise estimates.

Factors Influencing the Findings:

- Sample Size: With only 49 individuals, the margin of error is relatively large. Increasing the sample size would narrow the confidence interval, providing a clearer picture.
- Sampling Method: The validity of the inferences hinges on the randomness and representativeness of the sample. Any bias in selection could skew the results.
- Characteristic Definition: Clarifying what "success" or the characteristic is crucial for meaningful interpretation.

Challenges in Interpretation

While the calculations provide a statistical foundation, practical

considerations must be addressed:

- Sampling Bias: Was the sample truly random? Non-random sampling can bias estimates.
- Population Variability: The population might be heterogeneous, with subgroups having different proportions.
- Contextual Factors: External factors, such as demographic or environmental variables, might influence the observed proportion.

Broader Applications and Significance

Understanding the proportion of individuals with a particular trait in a population has wide-ranging applications:

- Public Health: Estimating disease prevalence.
- Market Research: Assessing product adoption rates.
- Social Science: Measuring attitudes or behaviors.

In each case, the sample size and observed proportion inform decisions, policies, or further research.

Limitations and Recommendations

While the statistical methods are robust, limitations include:

- Sample Size Constraints: Larger samples reduce uncertainty.
- Assumption of Randomness: Ensuring proper sampling methods.
- Temporal Variability: Data might change over time; ongoing monitoring is essential.

Recommendations:

- Increase the sample size for improved precision.
- Stratify samples to account for population heterogeneity.
- Combine quantitative findings with qualitative insights.

Conclusion

The analysis of a sample of 49 individuals, with 12 exhibiting a particular characteristic, illustrates fundamental principles of statistical inference. While the sample provides an initial estimate of approximately 24.5%, the confidence interval indicates notable uncertainty. This underscores the importance of careful sampling, adequate sample sizes, and contextual understanding when interpreting such data.

In practical terms, such findings should inform further investigation rather than serve as definitive conclusions. Policymakers, researchers, and stakeholders must consider the broader context, potential biases, and the need for additional data to make informed decisions. Ultimately, this case exemplifies how small sample data, when analyzed rigorously, can guide understanding but must be complemented by ongoing research and validation.

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