

A Rectangle Measures 10 Feet By 6 Feet. The Length And Width Are Each Increased By 50%. What Is The Area

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Understanding how to calculate the area of a rectangle after its dimensions are increased by a certain percentage is a fundamental concept in geometry and real-world applications such as construction, interior design, and land surveying. This article provides a comprehensive explanation of this problem, including step-by-step calculations, relevant formulas, and practical insights. Whether you're a student preparing for exams or a professional dealing with area calculations regularly, this guide will help clarify the process and improve your understanding of proportional increases in dimensions and their impact on area.

Introduction to Rectangle Area Calculation

Rectangles are one of the most basic geometric shapes, characterized by four sides with opposite sides equal in length and four right angles. Calculating the area of a rectangle involves multiplying its length by its width:

$$\text{Area} = \text{Length} \times \text{Width}$$

This simple formula allows us to determine the space enclosed within the rectangle. For example, a rectangle measuring 10 feet by 6 feet has an initial area of:

$$10 \text{ ft} \times 6 \text{ ft} = 60 \text{ square feet}$$

However, real-world scenarios often involve changes to the dimensions, such as increasing the length and width by a certain percentage. These modifications can significantly impact the total area, which is crucial in planning and resource estimation.

Understanding Percentage Increase in Dimensions

When the length and width of a rectangle are increased by a percentage, each dimension expands proportionally. In our case, both the length and the width are increased by 50%. Here's what that entails:

- Original dimensions:
 - Length = 10 feet
 - Width = 6 feet
- Percentage increase:

- 50%

- Resulting dimensions:

- New length = Original length + (50% of original length)

- New width = Original width + (50% of original width)

Calculating these new dimensions involves multiplying the original measurements by 1 plus the percentage increase expressed as a decimal:

$$\text{New dimension} = \text{Original dimension} \times (1 + \frac{\text{Percentage increase}}{100})$$

For a 50% increase:

$$1 + \frac{50}{100} = 1 + 0.5 = 1.5$$

Thus,

- New length:

$$10 \text{ ft} \times 1.5 = 15 \text{ ft}$$

- New width:

$$6 \text{ ft} \times 1.5 = 9 \text{ ft}$$

Notice how each original dimension is multiplied by 1.5, reflecting a 50% increase.

Calculating the New Dimensions

Having established that the increased dimensions are 15 feet in length and 9 feet in width, the next step is to compute the new area.

- Original dimensions:

- Length = 10 feet

- Width = 6 feet

- Increased dimensions:

- Length = 15 feet

- Width = 9 feet

This change illustrates a proportional increase, which affects the area non-linearly. Specifically, increasing both dimensions by 50% results in the area increasing by a factor of:

$$(1 + \frac{\text{Percentage increase}}{100})^2 = (1.5)^2 = 2.25$$

This is because when both length and width are scaled by the same factor, the area scales by the square of that factor.

Calculation:

$$\text{New area} = \text{Original area} \times 2.25$$

Using the original area of 60 square feet:

$$60 \text{ sq ft} \times 2.25 = 135 \text{ sq ft}$$

Alternatively, directly calculating using the new dimensions:

$$15 \text{ ft} \times 9 \text{ ft} = 135 \text{ sq ft}$$

Both methods yield the same result, confirming the accuracy of the calculations.

Step-by-Step Calculation Process

To ensure clarity, here's a concise step-by-step process:

1. Identify original dimensions:

- Length = 10 ft
- Width = 6 ft

2. Calculate the scale factor for increase:

- Percentage increase = 50%
- Convert percentage to decimal: 0.5
- Scale factor = $1 + 0.5 = 1.5$

3. Compute new dimensions:

- New length = $10 \text{ ft} \times 1.5 = 15 \text{ ft}$
- New width = $6 \text{ ft} \times 1.5 = 9 \text{ ft}$

4. Calculate new area:

- New area = $15 \text{ ft} \times 9 \text{ ft} = 135 \text{ sq ft}$

5. Verify the area increase:

- Original area = $10 \text{ ft} \times 6 \text{ ft} = 60 \text{ sq ft}$
- Area increase factor = $(1.5)^2 = 2.25$
- New area = $60 \text{ sq ft} \times 2.25 = 135 \text{ sq ft}$

This process demonstrates how proportional increases in dimensions translate into a squared increase in the area.

Implications of Increasing Dimensions by a Percentage

Understanding the effect of proportional increases is crucial in various fields:

- Construction: When designing larger structures, knowing how increases in length and width affect the total area helps in material estimation.
- Land Development: Expanding land plots proportionally impacts the total area, influencing planning and resource allocation.
- Interior Design: Increasing room dimensions affects the total space, which impacts furniture placement and aesthetic considerations.
- Manufacturing: Scaling product dimensions proportionally alters the surface area and volume, affecting production costs and material usage.

In our specific example, increasing each dimension by 50% results in more than double the original area, emphasizing the nonlinear relationship between linear dimensions and area.

Additional Considerations and Practical Applications

While the calculation provided is straightforward, practical scenarios may involve additional factors:

- Material Constraints: When enlarging a space, the amount of material needed increases quadratically, impacting budgets.
- Design Limitations: Structural integrity and design standards may limit how much a space can be expanded.
- Cost Estimation: Accurate area calculations are essential for estimating costs in construction projects.
- Scaling Models: Engineers and architects often use scale models, where proportional increases help visualize larger plans.

Furthermore, understanding the mathematical principles behind these calculations allows professionals to make informed decisions, optimize designs, and avoid costly errors.

Summary of Key Points

- The original rectangle measures 10 feet by 6 feet, with an initial area of 60 square feet.
- Increasing each dimension by 50% involves multiplying each by 1.5.
- The new dimensions are 15 feet (length) and 9 feet (width).
- The new area is calculated as $15 \text{ ft} \times 9 \text{ ft} = 135 \text{ square feet}$.
- The area increases by a factor of 2.25, which is the square of the scale factor (1.5).

Understanding these fundamental concepts is vital for accurate area calculations when dimensions are scaled proportionally.

Conclusion

Calculating the area of a rectangle after increasing its dimensions by a certain percentage is a key skill in geometry with numerous practical applications. In the example of a rectangle measuring 10 feet by 6 feet, increasing both dimensions by 50% results in a new area of 135 square feet, more

than doubling the original space. Mastery of this concept involves understanding the relationship between linear scale factors and their squared effect on area. By applying these principles, professionals and students alike can make precise calculations, plan effectively, and make informed decisions in various fields involving space and resource management.

Whether you're working on a construction project, designing a landscape, or solving academic problems, recognizing the proportional relationship between dimensions and area ensures accuracy and efficiency. Keep practicing these calculations with different percentages and shapes to strengthen your understanding and application of geometric principles.

Frequently Asked Questions

What is the original area of the rectangle that measures 10 feet by 6 feet?

The original area is $10 \text{ ft} \times 6 \text{ ft} = 60$ square feet.

How much is each dimension increased when increased by 50%?

Each dimension increases by 50%; so, the length becomes $10 + (0.5 \times 10) = 15$ feet, and the width becomes $6 + (0.5 \times 6) = 9$ feet.

What are the new dimensions of the rectangle after the increase?

The new length is 15 feet and the new width is 9 feet.

What is the area of the rectangle after each dimension is increased by 50%?

The new area is $15 \text{ ft} \times 9 \text{ ft} = 135$ square feet.

By what percentage does the area of the rectangle increase after the dimensions are increased by 50%?

The area increases from 60 sq ft to 135 sq ft, which is a 125% increase.

How do you calculate the new area after increasing both dimensions by a percentage?

Multiply the original dimensions by $(1 + \text{percentage increase})$, then multiply the new length and width to find the new area.

If the original rectangle measures 10 ft by 6 ft, what is the new area after a 50% increase in both dimensions?

The new area is $15 \text{ ft} \times 9 \text{ ft} = 135$ square feet.

What is the significance of increasing each dimension by 50% in terms of area growth?

Increasing each dimension by 50% results in the area increasing by 125%, since area scales with the square of linear dimensions.

Can you verify the percentage increase in area with calculations?

Yes, original area is 60 sq ft, new area is 135 sq ft. The increase is $135 - 60 = 75$ sq ft, which is 125% of the original area.

What formula is used to find the new area after increasing dimensions by a certain percentage?

New area = (original length $\times$ (1 + percentage)) $\times$ (original width $\times$ (1 + percentage)).

Additional Resources

Understanding the Area of a Rectangle When Dimensions Are Increased by 50%

When exploring geometric figures, particularly rectangles, a fundamental understanding of how their dimensions influence their area is essential. This knowledge is not only crucial for academic purposes but also has practical applications in fields such as architecture, engineering, interior design, and everyday problem-solving. In this detailed review, we will focus on a specific problem involving a rectangle measuring 10 feet by 6 feet, then analyze how increasing its length and width by 50% affects its area.

Introduction to Rectangles and Their Properties

A rectangle is a quadrilateral with four right angles. Its defining features include:

- Two pairs of parallel sides
- Opposite sides are equal in length

The dimensions of a rectangle are typically described by:

- Length (L): The longer side

- Width (W): The shorter side

The area of a rectangle is calculated by multiplying length and width:

$$A = L \times W$$

Understanding how changes in dimensions impact the area provides insight into geometric scaling and proportional reasoning.

The Given Dimensions and Initial Area

Let's start with the original rectangle:

- Length (L): 10 feet

- Width (W): 6 feet

The initial area (A_0) can be calculated as:

$$A_0 = 10 \text{ ft} \times 6 \text{ ft} = 60 \text{ square feet}$$

This straightforward calculation serves as the baseline for understanding how increasing dimensions affect the area.

Increasing Dimensions by 50%: Concept and Calculation

The problem states that both the length and width are increased by 50%. To analyze this, we need to:

1. Convert the percentage increase into a decimal:

$$50\% = 0.50$$

2. Calculate the new dimensions:

- New Length:

$$L_{\text{new}} = L + (0.50 \times L) = L \times (1 + 0.50) = L \times 1.50$$

- New Width:

$$W_{\text{new}} = W + (0.50 \times W) = W \times (1 + 0.50) = W \times 1.50$$

Applying these to our initial dimensions:

- New Length:

$$10 \text{ ft} \times 1.50 = 15 \text{ ft}$$

- New Width:

$$6 \text{ ft} \times 1.50 = 9 \text{ ft}$$

This proportional increase ensures that the rectangle is scaled uniformly, maintaining its shape but enlarging its size.

Calculating the New Area After Dimension Increase

With the new dimensions established, the next step is to calculate the new area (A_{new}):

$$A_{\text{new}} = L_{\text{new}} \times W_{\text{new}}$$

Substituting the values:

$$A_{\text{new}} = 15 \text{ ft} \times 9 \text{ ft} = 135 \text{ square feet}$$

This result demonstrates that the area has increased from 60 square feet to 135 square feet.

Understanding the Relationship Between Dimension Changes and Area

A key insight into this problem is recognizing how proportional increases in dimensions affect the area. Specifically, when both dimensions are increased by a factor (k) , the area scales by (k^2) :

- If:

$$L_{\text{new}} = L \times k$$

$$W_{\text{new}} = W \times k$$

- Then:

$$A_{\text{new}} = (L \times k) \times (W \times k) = L \times W \times k^2 = A_0 \times k^2$$

Applying this to our scenario:

- $(k = 1.5)$ (since each dimension is increased by 50%, or 1.5 times)

- The area scales by:

$$1.5^2 = 2.25$$

- Therefore:

$$A_{\text{new}} = A_0 \times 2.25 = 60 \times 2.25 = 135 \text{ sq ft}$$

This confirms our earlier calculation and emphasizes the quadratic relationship between linear scaling and area.

Implications and Practical Applications

Understanding how the area scales with dimension changes has numerous practical implications:

- Design and Architecture: When enlarging a room or building plan proportionally, knowing that the area increases quadratically helps in estimating materials, costs, and space utilization.
- Manufacturing: Scaling parts or products while maintaining proportions requires awareness of how surface area (or material needed) increases disproportionately.
- Landscaping and Agriculture: Expanding plots proportionally involves calculating increased surface area for planting, irrigation, or fencing.
- Mathematical Education: This problem serves as an excellent example to teach concepts of scale factors, area calculation, and quadratic relationships.

Additional Considerations and Variations

While the core problem involves uniform scaling, several variations can deepen understanding:

1. Increasing Dimensions by Different Percentages

Suppose only the length increases by 50%, and the width remains the same:

- New Length:

$$10 \times 1.5 = 15 \text{ ft}$$

- Width remains:

$$6 \text{ ft}$$

- New Area:

$$15 \times 6 = 90 \text{ sq ft}$$

Or, if the width increases by 50% while the length stays the same:

- Length remains:

$$10 \text{ ft}$$

- New Width:

$$6 \times 1.5 = 9 \text{ ft}$$

- New Area:

$$10 \times 9 = 90 \text{ sq ft}$$

These variations illustrate how partial scaling affects the total area differently.

2. Decreasing Dimensions

If dimensions are decreased by 50%:

- New Length:

$$10 \times 0.5 = 5 \text{ ft}$$

- New Width:

$$6 \times 0.5 = 3 \text{ ft}$$

- New Area:

$$5 \times 3 = 15 \text{ sq ft}$$

Demonstrating how reductions proportionally decrease the area.

3. Non-Uniform Scaling Ratios

Scaling dimensions by different factors (e.g., length by 1.2 and width by 1.7) affects the area by the product of these factors:

$$A_{\text{new}} = L \times W \times (1.2 \times 1.7)$$

This flexibility allows for customized scaling in design and engineering.

Mathematical Summary and Key Takeaways

- Initial dimensions: 10 ft by 6 ft
- Initial area: 60 sq ft
- Dimension increase: 50% (or a factor of 1.5)
- New dimensions: 15 ft by 9 ft
- New area: 135 sq ft
- Scale factor for area: $(1.5)^2 = 2.25$
- Area after scaling: $60 \times 2.25 = 135 \text{ sq ft}$

The core concept is that linear increases in dimensions lead to quadratic increases in area. This fundamental geometric principle is vital in multiple disciplines.

Final Thoughts and Broader Context

The problem of scaling a rectangle's dimensions by a certain percentage provides a window into the more extensive realm of geometric transformations. It showcases the importance of understanding proportionality and how simple percentage increases can have significant effects on surface area and volume.

In real-world situations, such as expanding a room, enlarging a billboard, or designing scalable models, grasping this quadratic relationship ensures accurate calculations and effective planning. It also serves as a mathematical foundation for more complex concepts involving surface area, volume, and dimensional analysis.

By mastering these principles, students, professionals, and enthusiasts can develop a more intuitive understanding of how size modifications influence overall dimensions, leading to better decision-making and innovative design solutions.

In Summary:

- Increasing each linear dimension by 50% results in the rectangle's area increasing by 125%, from 60 sq ft to 135 sq ft.
- The key mathematical principle is that when dimensions are scaled by a factor (k) , the area scales by (k^2) .
- This problem illustrates the importance of proportional reasoning and quadratic relationships in geometry.

Understanding these concepts lays a robust foundation for exploring more advanced topics in mathematics and their practical applications across various fields.

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