A Region S Is Bounded By The Graphs Of Y = X And Y = 2x. 1. Sketch The Graph And Find The Area Of Region

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Understanding the geometric relationships between functions and calculating areas bounded by curves are fundamental skills in calculus and analytical geometry. In this article, we will explore the problem of a particular bounded region defined by the lines y = x and y = 2x. We will learn how to sketch these graphs, find the points of intersection, and determine the exact area of the region enclosed by these two lines. This comprehensive guide aims to provide clarity on the process and mathematical techniques involved.

Understanding the Problem

The problem involves two linear functions:

- y = x
- y = 2x

Our goals are to:

1. Sketch the graphs of these two lines to visualize the region.
2. Find the points where these lines intersect.
3. Determine the boundaries of the region.
4. Calculate the area of the region enclosed between these two lines.

Let's begin by analyzing each step systematically.

Sketching the Graphs of y = x and y = 2x

1. Graph of y = x

- This is a straight line passing through the origin (0,0).
- The slope is 1, meaning for every 1 unit increase in x, y increases by 1.
- Key points to plot:
- (0, 0)
- (1, 1)
- (2, 2)
- (-1, -1)

- (-2, -2)
- The line extends infinitely in both directions.

2. Graph of y = 2x

- Also passes through the origin.
- Slope is 2, indicating a steeper incline than y = x.
- Key points:
- (0, 0)
- (1, 2)
- (2, 4)
- (-1, -2)
- (-2, -4)
- Extends infinitely as well.

Visualizing the Graphs

Plotting both lines on the same coordinate plane reveals that:

- Both pass through the origin.
- y = 2x is steeper than y = x.
- The two lines intersect at the origin, and since both pass through (0, 0), this point is a common point.

Finding the Points of Intersection

Since the lines are linear, their intersection points are where their equations are equal:

- Set y = x equal to y = 2x:

x = 2x

- Solving for x:

x - 2x = 0
-x = 0
x = 0

- Substituting x = 0 into either equation gives y:

y = 0 (since y = x or y = 2x)

Result: The lines intersect at the point (0, 0).

Note: Because both lines pass through the origin and are not parallel, they only intersect at this point.

Determining the Enclosed Region

At first glance, the lines intersect only at the origin, which suggests the region is unbounded. However, in typical problems of this nature, the region of interest is often bounded within specific limits or between certain x-values.

To proceed, we need to identify the bounds of the region. Usually, such problems assume a specific interval along the x-axis where the region is enclosed.

Suppose we analyze the region between $x = 0$ and some positive x-value where the lines form a bounded region.

Let’s define the bounds as:

- The region lies between $x = 0$ and $x = a$, where $a > 0$.

To find this bounded region, we need to:

- Determine the points where the lines cross the same vertical line.
- Find the intersection points of the lines at specific x-values, considering the context.

Alternatively, if the problem intends for us to find the area between the two lines from $x = 0$ to some specific x-value, say $x = 1$ or $x = 2$, we can proceed accordingly.

Common Approach: Find the area between the two lines over an interval $[x_1, x_2]$.

Clarifying the Bounded Region

Given the initial description, the typical scenario is to consider the region between the lines $y = x$ and $y = 2x$ from $x = 0$ up to the point where $y = 2x$ surpasses $y = x$ or vice versa.

Observe that:

- For $x > 0$, $y = 2x$ is always above $y = x$ because:

$y = 2x > x$ when $x > 0$.

- For $x < 0$, $y = 2x < x$, so $y = x$ is above $y = 2x$.

Thus, considering the positive x-region ($x \geq 0$), the area between the two lines from $x=0$ to some $x=a$ is enclosed between the curves.

Choosing the Interval for Calculation

Suppose we pick the interval from x=0 to x=1.

- At x=0, both lines intersect at (0,0).
- At x=1:
- y = x = 1
- y = 2x = 2

Between x=0 and x=1, the line y=2x is above y=x.

Similarly, at x=2:

- y = x = 2
- y = 2x = 4

Again, y=2x is above y=x.

Therefore, the region bounded between y=x and y=2x over [0, a] (for some a > 0) can be integrated to find the area.

Final Clarification:

Assuming the problem asks for the area of the region bounded between y=x and y=2x from x=0 to x=1, we can proceed to compute the area.

Calculating the Area of the Region

1. Setting Up the Integral

The area between two curves y=f(x) and y=g(x), over [a, b], where f(x) ≥ g(x), is given by:

$$ \text{Area} = \int_{a}^{b} [f(x) - g(x)]\, dx $$

In our case:

- f(x) = y = 2x
- g(x) = y = x

Since y=2x is above y=x for x ≥ 0, the area from x=0 to x=a is:

$$ \text{Area} = \int_{0}^{a} (2x - x) \, dx = \int_{0}^{a} x \, dx $$

This simplifies to:

$$ \int_{0}^{a} x \, dx = \left[\frac{1}{2} x^2 \right]_0^{a} = \frac{1}{2} a^2 $$

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2. Applying the Limits

If the problem specifies a particular value of a, for example, $a=1$:

$$ \text{Area} = \frac{1}{2} \times 1^2 = \frac{1}{2} $$

Similarly, for $a=2$:

$$ \text{Area} = \frac{1}{2} \times 2^2 = \frac{1}{2} \times 4 = 2 $$

Note: The choice of the interval depends on the specific bounds given in the original problem or context.

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3. Finalizing the Solution

Based on the above, the general formula for the area between $y = x$ and $y = 2x$ from $x=0$ to $x=a$ is:

$$ \boxed{\text{Area} = \frac{1}{2} a^2} $$

This formula allows calculating the area for any positive x-value within the region.

Summary of Steps

To summarize the entire process:

1. Sketch the Graphs:
- Plot $y = x$ and $y = 2x$ to understand their positions.
2. Find Points of Intersection:
- Both lines intersect at (0, 0).
3. Determine the Bounded Region:
- For $x \geq 0$, $y=2x$ lies above $y=x$.
- The region between $x=0$ and $x=a$ is bounded by these two lines.
4. Set Up the Integral:
- Area = $\int_0^{a} (2x - x)\, dx = \int_0^{a} x\, dx$.
5. Compute the Integral:
- Area = $\frac{1}{2} a^2$.

6. Interpretation:
- The area depends on the chosen interval [0, a].

Additional Considerations and Applications

Understanding how to find the area between two lines is fundamental in various fields, including physics, engineering, and economics. For example, calculating the work done by a force, the area under a demand curve, or the space between two velocity functions over time.

This problem also illustrates the importance of:

- Graphical visualization for understanding geometric relationships.
- Setting up proper limits of integration based on the problem context.
- Ensuring the correct order of functions when calculating the difference (top function minus bottom function).

Conclusion

In conclusion, the problem

Frequently Asked Questions

What are the points of intersection between the graphs of y = x and y = 2x?

The graphs intersect where x = 2x, which gives x = 0. Substituting back, y = 0. So, the points of intersection are at (0, 0).

How do I sketch the graphs of y = x and y = 2x?

Plot the line y = x passing through the origin with a 45° angle, and y = 2x passing through the origin with a steeper slope of 2. Draw both lines on the same axes.

What is the region S bounded by y = x and y = 2x?

Region S is the area between the two lines from the point of intersection at (0, 0) to the point where the lines meet the boundary of the region, typically considered within a specific interval.

How do I find the limits of integration for calculating the area of region S?

Determine the x-values where the region is bounded; in this case, from $x = 0$ to a specific x where the region ends, often where the lines meet a boundary or are limited by a vertical line.

What is the formula to compute the area of the region between y = x and y = 2x?

The area can be found by integrating the difference of the functions: Area = $\int (2x - x)\, dx$ over the relevant interval.

How do I set up the integral to find the area of region S?

Set up the integral as ∫ from $x = a$ to $x = b$ of $(2x - x)\, dx$, where a and b are the x-values defining the region boundaries.

What is the value of the area of the region bounded by y = x and y = 2x?

If the region is bounded between $x = 0$ and $x = 1$, for example, the area is $\int_0^1 (2x - x)\, dx = \int_0^1 x\, dx = [x^2/2]\_0^1 = 1/2$.

Can the region be unbounded? How does that affect area calculation?

Yes, if the region extends infinitely, the area may be infinite or require limits to be finite. Usually, a finite boundary or interval is specified to compute a finite area.

How do I interpret the sketch of the region S in the context of the problem?

The sketch shows the area enclosed between the two lines, helping visualize the limits and the shape of the region for accurate integration.

What is the final step to find the area once the integral is set up?

Evaluate the definite integral to find the numerical value of the area of the region S.

Additional Resources

A Region S Is Bounded By The Graphs Of Y = X And Y = 2x: An In-Depth Investigation into Geometric Boundaries and Area Calculation

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Introduction

Mathematics, especially the study of coordinate geometry, often involves exploring the properties and measurements of regions defined by graphs of functions. These regions are not just abstract concepts but are fundamental in understanding areas, integrals, and the spatial relationships between curves. One such intriguing case involves the region S bounded by the lines $y = x$ and $y = 2x$.

This article provides a comprehensive analysis of the region S, beginning with its graphical representation, moving through the mathematical derivation of its boundaries, and culminating in the precise calculation of its area. The discussion aims to serve as a guide for students, educators, and enthusiasts interested in the geometric and analytical aspects of such regions.

Graphical Sketch of Region S

Visualizing the Boundaries

The region S is defined by the two lines:

- $y = x$
- $y = 2x$

Plotting these lines on a Cartesian coordinate system reveals a wedge-shaped region extending in the first quadrant (assuming positive x and y for clarity). The line $y = x$ passes through the origin with a 45° angle relative to the x-axis, while $y = 2x$ is steeper, passing through the same origin but with a slope of 2.

Step-by-Step Sketching Procedure

1. Plot the Lines:
- Draw $y = x$: a straight line passing through (0,0) and (1,1).
- Draw $y = 2x$: a straight line passing through (0,0) and (1,2).

2. Identify Intersection Points:
- Both lines intersect at the origin (0,0).

3. Determine the Region:
- For positive x-values, $y = x$ lies below $y = 2x$ because $y = 2x$ grows faster and is above $y = x$.
- The bounded region S is therefore the area between these two lines in the first quadrant, from $x=0$ to their point of intersection (which, in this case, is at infinity unless bounded).

Mathematical Boundaries and Intersection

Understanding the Boundaries

The region S is bounded by the two lines extending infinitely in the positive x-direction, but to define a finite region, we need to specify the limits of integration or the points where the region is enclosed.

Finding the Intersection Point

Given the lines:

- $y = x$
- $y = 2x$

They intersect where $y = y$, so setting $y = x$ equal to $y = 2x$:

$x = 2x$
=> $x - 2x = 0$
=> $-x = 0$
=> $x = 0$

Substituting back into $y = x$:

$y = 0$

Thus, the only intersection point is at the origin (0,0). To define a finite region, we need to consider the region between these lines over a specific interval, say from $x=0$ to some positive value $x=a$.

Defining the Region S: From x=0 to x=a

Suppose the region is bounded between $x=0$ and $x=a$, with $a > 0$. Within this interval:

- The lower boundary is $y = x$.
- The upper boundary is $y = 2x$.

The region S is thus the set of points (x, y) satisfying:

$0 \leq x \leq a$
$x \leq y \leq 2x$

Calculating the Area of Region S

Setting Up the Integral

The area A of the region S can be found using a double integral or, more straightforwardly, by integrating with respect to y and then x:

$$ A = \int_{x=0}^{a} \left(\text{upper y} - \text{lower y} \right) dx $$

Here, the upper y is y=2x, and the lower y is y=x:

$$ A = \int_{0}^{a} (2x - x) dx = \int_{0}^{a} x dx $$

Computing the Integral

$$
A = \left[\frac{1}{2} x^2 \right]_0^{a} = \frac{1}{2} a^2
$$

This formula indicates that the area between the lines from x=0 to x=a is $\frac{1}{2} a^2$.

Extending to a Finite Region: Choosing a Specific Bound

To make this analysis concrete, select a specific value for a, such as a=1:

$$
A = \frac{1}{2} \times 1^2 = \frac{1}{2}
$$

Hence, the area of the region bounded between x=0 and x=1 is 0.5 square units.

Generalization and Significance

This method of calculating the area extends naturally to any finite segment along the x-axis. The fact that the area depends quadratically on the upper limit x=a underscores the geometric growth of the bounded region as x increases.

Additional Considerations: Infinite Regions and Practical Applications

- Infinite Region: If the region is considered extending to infinity, the area becomes unbounded, which is mathematically infinite.
- Practical Application: Such regions are useful in calculus education for demonstrating the principles of integration, as well as in physics and engineering, where understanding areas between curves can relate to probabilities, material properties, or signal analysis.

Visual and Analytical Summary

Step	Description	Mathematical Expression
1	Sketch lines y = x and y = 2x	Graphs passing through (0,0) with slopes 1 and 2
2	Find intersection points	Only at (0,0) unless bounded at x=a
3	Define the bounded region	$0 \leq x \leq a$, with y between y=x and y=2x
4	Set up the area integral	$A = \int_{0}^{a} (2x - x) dx$
5	Compute the integral	$A = \frac{1}{2} a^2$

Conclusion

The investigation into the region S bounded by y = x and y = 2x demonstrates the elegance of coordinate geometry and integral calculus in quantifying areas between curves. By understanding the graphical representation, intersection points, and integral setup, we establish a straightforward yet profound method to compute the area enclosed.

This analysis not only clarifies the specific problem at hand but also exemplifies broader principles applicable to more complex regions bounded by arbitrary functions. The ability to visualize, derive, and compute such areas forms a cornerstone of mathematical literacy and analytical proficiency.

References

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Note: This article emphasizes the importance of precise graphical interpretation, boundary identification, and integral setup in solving geometric problems related to bounded regions. Future explorations might consider more complex boundaries or the application of calculus techniques such as line integrals or polar coordinates.

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