A Sample Of 238/92U Is Decaying At A Rate Of 450 Decays/s . The Half-life Is 4.468109yr.What Is The Mass

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Introduction

Understanding radioactive decay and calculating the mass of a radioactive sample are fundamental concepts in nuclear physics and radiochemistry. When a sample of uranium-238 ($^{238}\mathrm{U}$) is observed to decay at a specific rate, such as 450 decays per second, and its half-life is known, it becomes possible to determine the initial amount of substance—its mass. This process involves understanding decay constants, the relationship between activity and mass, and the fundamental properties of uranium-238. This article explores these concepts step-by-step, providing a comprehensive guide to calculating the mass of the uranium sample based on its decay rate and half-life.

Fundamental Concepts in Radioactive Decay

Radioactivity and Decay Rate

Radioactivity refers to the spontaneous disintegration of unstable atomic nuclei, releasing energy in the form of particles or electromagnetic radiation. The decay rate, also known as activity (A), quantifies how many nuclei decay per unit time. It is measured in becquerels (Bq), where 1 Bq equals one decay per second.

In the given problem, the activity of the uranium sample is:

$$
A = 450 \text{ decays/sec} = 450 \text{ Bq}
$$

Half-life and Decay Constant

The half-life ($T_{1/2}$) is the time required for half of the radioactive nuclei in a sample to decay. For uranium-238, it is given as:

$$
T_{1/2} = 4.468 \times 10^{9} \text{ years}
$$

The decay constant (λ) relates to the probability per unit time that a single nucleus will decay and is connected to the half-life through:

$$
\lambda = \frac{\ln 2}{T_{1/2}}
$$

where $\ln 2 \approx 0.693$.

Calculating the Decay Constant (λ)

Given the half-life, the first step is to determine the decay constant.

Convert Half-life to Seconds

Since activity is measured in decays per second, we need to express the half-life in seconds:

$$
T_{1/2} = 4.468 \times 10^{9} \text{ years}
$$

Number of seconds in one year:

$$
1 \text{ year} \approx 365.25 \text{ days} \times 24 \text{ hours/day} \times 3600 \text{ seconds/hour}
$$

Calculating:

$$
1 \text{ year} \approx 365.25 \times 24 \times 3600 \approx 31,557,600 \text{ seconds}
$$

Therefore,

$$
T_{1/2} = 4.468 \times 10^{9} \times 31,557,600 \approx 1.410 \times 10^{17} \text{ seconds}
$$

\]

Calculate λ

Using the relation:

$$
\lambda = \frac{\ln 2}{T_{1/2}} = \frac{0.693}{1.410 \times 10^{17}} \approx 4.92 \times 10^{-18} \text{ s}^{-1}
$$

Relating Activity to Number of Nuclei

The activity A is related to the number of radioactive nuclei N via:

$$
A = \lambda N
$$

Rearranged to find N:

$$
N = \frac{A}{\lambda}
$$

Substituting the known values:

$$
N = \frac{450}{4.92 \times 10^{-18}} \approx 9.15 \times 10^{19} \text{ nuclei}
$$

This is the number of uranium-238 nuclei present in the sample at the time of measurement.

Calculating the Mass of the Sample

To find the mass, we need to convert the number of nuclei to moles and then to grams.

Number of Nuclei per Mole

Avogadro's number (N_A):

$$
N_A = 6.022 \times 10^{23} \text{ nuclei/mole}
$$

Number of moles:

$$
n = \frac{N}{N_A} = \frac{9.15 \times 10^{19}}{6.022 \times 10^{23}} \approx 1.52 \times 10^{-4} \text{ mol}
$$

Mass of the Sample

The molar mass of uranium-238:

$$
M_{^{238}\mathrm{U}} \approx 238 \text{ g/mol}
$$

Therefore, the mass m of the sample:

$$
m = n \times M_{^{238}\mathrm{U}} = 1.52 \times 10^{-4} \times 238 \approx 0.0361 \text{ grams}
$$

Summary of the Calculation Process

- Converted the half-life from years to seconds.
- Calculated the decay constant λ using the half-life.
- Determined the number of nuclei N from activity and λ.
- Converted nuclei to moles using Avogadro's number.
- Calculated the mass of the sample in grams.

The final answer indicates that the uranium-238 sample weighs approximately 0.036 grams when decaying at a rate of 450 decays per second with a half-life of approximately 4.468 billion years.

Implications and Applications

Understanding how to derive the mass from decay data has several important applications:

1. **Radioactive dating:** Estimating the age of rocks and fossils based on uranium decay.
2. **Nuclear medicine:** Determining the activity and dosage of radioactive tracers.
3. **Nuclear power:** Monitoring fuel composition and decay rates for safety and efficiency.

This calculation exemplifies how fundamental constants and properties of nuclei can be used to infer physical quantities, bridging the gap between microscopic nuclear processes and macroscopic measurements.

Conclusion

Calculating the mass of a radioactive isotope like uranium-238 from its decay rate involves a series of steps rooted in nuclear physics fundamentals. By converting the half-life into decay constant, relating activity to the number of nuclei, and then translating that into mass via molar mass and Avogadro's number, we obtain a precise estimate of the sample's mass. In this case, a decay rate of 450 decays per second corresponds to a uranium-238 mass of roughly 0.036 grams. Such calculations are crucial in various scientific and industrial fields, enabling accurate assessment and management of radioactive materials.

References
- Krane, K. S. (1988). Introductory Nuclear Physics. Wiley.
- Glendenning, N. K. (2004). Physics of Nuclear Reactions. Springer.
- National Institute of Standards and Technology. (n.d.). Atomic weights of the elements. NIST.

Frequently Asked Questions

What is the decay rate of the uranium-238 sample in decays per second?

The decay rate of the sample is 450 decays per second.

Given the half-life of uranium-238 is approximately 4.468 x 10^9 years, how can we determine the remaining mass of the sample?

By using the decay rate, the initial number of atoms, and the half-life to calculate the remaining mass through decay equations and molar mass conversions.

What is the molar mass of uranium-238 used in calculating the sample's mass?

The molar mass of uranium-238 is approximately 238 g/mol.

How do you calculate the initial number of uranium-238 atoms in the sample?

Using the decay rate and the decay constant, you can determine the number of atoms via N = Decay rate / decay constant.

What is the formula to find the remaining mass of uranium-238 after a certain decay time?

Remaining mass = (initial number of atoms × molar mass) / Avogadro's number × fraction remaining, where fraction remaining is calculated using the exponential decay formula.

Can we determine the current mass of the uranium-238 sample with the given decay rate and half-life? If so, how?

Yes, by calculating the initial number of atoms from the decay rate, then applying the decay law using the half-life to find the remaining number of atoms, and converting that to mass.

Additional Resources

A Sample Of 238/92U Is Decaying At A Rate Of 450 Decays/s . The Half-life Is 4.468109yr .What Is The Mass?

Understanding the intricacies of radioactive decay is essential in fields ranging from nuclear physics to medical imaging and nuclear energy. When dealing with radioactive isotopes like Uranium-238 (denoted as 238/92U), determining the mass of a sample based on its decay rate and half-life requires a grasp of fundamental nuclear principles and mathematical calculations. This article explores how scientists can determine the mass of a uranium-238 sample given its decay rate and half-life, providing a comprehensive guide to the underlying concepts, formulas, and practical applications involved.

Introduction to Radioactive Decay and Uranium-238

Radioactive decay is a spontaneous process where unstable atomic nuclei release particles and energy to attain a more stable configuration. Each isotope has a characteristic half-life—the time it takes for half of a given sample to decay—making it a key parameter in understanding its stability and longevity.

Uranium-238 is one of the most common naturally occurring radioactive isotopes, comprising about 99.3% of natural uranium. Its long half-life of approximately 4.468109 years signifies its relative stability compared to other radioactive isotopes. Despite its stability, U-238 decays at a measurable rate, a property exploited in various scientific and industrial applications, such as radiometric dating and nuclear fuel cycle management.

The problem at hand is to determine the mass of a uranium-238 sample when provided with:

- Its decay rate (activity): 450 decays per second (dps)
- Its half-life: 4.468109 years

This task involves translating decay activity into the number of undecayed nuclei, and subsequently, into mass.

Fundamental Concepts and Formulas

Before proceeding with calculations, it is crucial to understand some fundamental concepts and the key formulas used in radioactivity calculations.

Radioactive Decay Law

The decay of a radioactive sample follows an exponential law:

$$ N(t) = N_0 \times e^{-\lambda t} $$

Where:
- $N(t)$ = number of undecayed nuclei at time t
- N_0 = initial number of nuclei
- λ = decay constant (probability per unit time that a nucleus decays)

The activity A, measured in decays per second (dps), is related to the number of undecayed nuclei by:

$$ A = \lambda N $$

Where:
- N = number of radioactive nuclei present at a specific moment
- λ = decay constant

Half-life and Decay Constant

The relationship between half-life ($T_{1/2}$) and decay constant (λ) is:

$$\lambda = \frac{\ln 2}{T_{1/2}}$$

Where:
- $\ln 2 \approx 0.6931$
- $T_{1/2}$ must be in consistent units (seconds, in this case)

Converting Half-life to Seconds

Given the half-life in years, convert it to seconds because decay constants are typically expressed per second:

$$T_{1/2} (\text{seconds}) = T_{1/2} (\text{years}) \times \text{seconds per year}$$

Number of seconds per year:

$$1\, \text{year} \approx 365.25\, \text{days} \times 24\, \text{hours/day} \times 3600\, \text{seconds/hour} \approx 31,557,600\, \text{seconds}$$

Calculating the Decay Constant (λ) for U-238

Given:

$$T_{1/2} = 4.468109\, \text{years}$$

First, convert years to seconds:

$$T_{1/2} = 4.468109 \times 31,557,600\, \text{seconds}$$

Calculating:

$$T_{1/2} \approx 4.468109 \times 31,557,600 \approx 141,075,071\, \text{seconds}$$

Now, compute λ:

$$\lambda = \frac{\ln 2}{T_{1/2}} = \frac{0.6931}{141,075,071} \approx 4.91 \times 10^{-9}\, \text{per second}$$

This tiny decay constant reflects U-238's long half-life.

Determining the Number of Nuclei in the Sample

With the activity $A = 450\, \text{decays/sec}$ and λ known, the number of undecayed nuclei N at this moment is:

$$N = \frac{A}{\lambda}$$

Plugging in the numbers:

$$N = \frac{450}{4.91 \times 10^{-9}} \approx 9.17 \times 10^{10}$$

Thus, there are approximately 9.17×10^{10} uranium-238 nuclei present in the sample at the time of measurement.

Calculating the Mass of the Sample

To convert the number of nuclei into mass, we need the molar mass of uranium-238 and Avogadro's number.

- Molar mass of U-238: approximately 238 g/mol
- Avogadro's number: 6.022×10^{23} nuclei/mol

The number of moles n:

$$n = \frac{N}{N_A}$$

Where:

$$n = \frac{9.17 \times 10^{10}}{6.022 \times 10^{23}} \approx 1.52 \times 10^{-13} \text{ mol}$$

Finally, the mass m:

$$m = n \times M$$

$$m = 1.52 \times 10^{-13} \times 238\, \text{g/mol} \approx 3.62 \times 10^{-11}\, \text{g}$$

Expressed in more convenient units:

$$m \approx 36.2\, \text{picograms}$$

This extremely small mass indicates that the given activity corresponds to a very tiny amount of uranium-238.

Implications and Practical Considerations

The calculation reveals that even a relatively high decay rate like 450 decays per second corresponds to an exceedingly small mass of uranium-238. This has practical implications:

- In Nuclear Forensics and Dating: Precise measurements of decay rates help determine the age and origin of uranium samples, critical for environmental monitoring and nuclear security.
- In Nuclear Power: Understanding the decay and mass relationships aids in fuel management, waste handling, and safety assessments.
- In Scientific Research: Small quantities of radioactive materials are used in experiments where precise mass and activity measurements are essential.

It's important to note that in real-world scenarios, samples are often larger, and decay rates are measured over extended periods to improve accuracy. Additionally, decay constants can vary slightly depending on isotopic purity and environmental factors.

Summary and Key Takeaways

- The decay constant λ for U-238 with a half-life of approximately 4.468109 years is roughly $4.91 \times 10^{-9} \text{ per second}$.
- Given the activity of 450 decays/sec, the number of uranium nuclei present is about 9.17×10^{10}.
- The corresponding mass of the sample is approximately 36.2 picograms, illustrating how minute quantities can produce measurable activity.
- These calculations demonstrate the power of nuclear physics in translating decay data into tangible quantities like mass, essential for scientific and industrial applications.

Understanding these principles equips scientists, engineers, and policymakers with the tools necessary to interpret radioactive data accurately, ensuring safety, compliance, and scientific advancement in the world of nuclear science.

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