A Triangle Has A Perimeter Of 165 Cm. The First Side Is 65 Cm Less Than Twice The Second Side. The Third

A Triangle Has A Perimeter Of 165 Cm. The First Side Is 65 Cm Less Than Twice The Second Side. The Third side of the triangle presents an interesting mathematical problem that involves understanding relationships between the sides and applying algebraic methods to find their lengths. This type of problem is common in geometry and algebra, and solving it not only enhances your problem-solving skills but also deepens your understanding of how geometric figures relate to algebraic expressions.

In this article, we will explore the step-by-step process to determine the lengths of all three sides of the triangle based on the given information. We will also discuss the importance of setting up equations correctly, solving for unknowns, and verifying solutions within the context of the problem.

Understanding the Problem

To effectively solve this problem, it is essential to clearly understand what is being asked and what information is provided.

Given Data:

- The perimeter of the triangle is 165 cm.
- The first side (let's denote it as S1) is 65 cm less than twice the second side (S2).

What needs to be found:

- The lengths of all three sides: S1, S2, and the third side (S3).

Setting Up Variables and Equations

The first step in solving algebraic problems involving geometric figures is to define variables corresponding to the unknown quantities.

Defining Variables:

- Let S2 = x (the second side)
- Since S1 is 65 cm less than twice S2, then S1 = 2x - 65
- The third side, S3, is unknown, so we will denote it as y

The perimeter of the triangle is the sum of its three sides:
$$ S1 + S2 + S3 = 165 $$

Substituting the expressions:
$$ (2x - 65) + x + y = 165 $$

This simplifies to:
$$ 3x - 65 + y = 165 $$

which can be rearranged to express y in terms of x:
$$ y = 165 - 3x + 65 $$
$$ y = 230 - 3x $$

Now, we have an expression for the third side in terms of x.

Constraints and Validity Conditions

Before proceeding, it is crucial to remember the triangle inequality theorem, which states that for any triangle with sides a, b, and c:
- a + b > c
- a + c > b
- b + c > a

These inequalities ensure the sides can form a valid triangle.

Applying these to our sides:

1. S1 + S2 > S3
2. S1 + S3 > S2
3. S2 + S3 > S1

Expressed in terms of x and y:

1. (2x - 65) + x > y
2. (2x - 65) + y > x
3. x + y > (2x - 65)

Substituting y = 230 - 3x into these inequalities:

1. $(2x - 65) + x > 230 - 3x$
$$ 3x - 65 > 230 - 3x $$
$$ 3x + 3x > 230 + 65 $$
$$ 6x > 295 $$
$$ x > \frac{295}{6} \approx 49.17 $$

2. $(2x - 65) + (230 - 3x) > x$
$$ 2x - 65 + 230 - 3x > x $$
$$ (2x - 3x) + (230 - 65) > x $$
$$ -x + 165 > x $$
$$ 165 > 2x $$
$$ x < 82.5 $$

3. $x + (230 - 3x) > (2x - 65)$
$$ x + 230 - 3x > 2x - 65 $$
$$ (x - 3x) + 230 > 2x - 65 $$
$$ -2x + 230 > 2x - 65 $$
$$ 230 + 65 > 2x + 2x $$
$$ 295 > 4x $$
$$ x < \frac{295}{4} = 73.75 $$

Combining these inequalities:

- $x > 49.17$
- $x < 73.75$

Therefore, the second side x must satisfy:
$$ 49.17 < x < 73.75 $$

Since side lengths are positive and typically considered in real numbers, and the sides are lengths, we can focus on integer values for practical purposes or proceed with the algebraic expressions.

Calculating the Side Lengths

Using the inequalities, we can choose a specific value within the interval for x to find corresponding side lengths. Let's pick x = 60 cm as an example, which lies within the interval.

Calculate S1:
$$ S1 = 2x - 65 = 2(60) - 65 = 120 - 65 = 55\, \text{cm} $$

Calculate S3:
$$ y = 230 - 3x = 230 - 3(60) = 230 - 180 = 50\, \text{cm} $$

Now, verify the triangle inequality:

- S1 + S2 = 55 + 60 = 115 > 50 (S3) ✓
- S1 + S3 = 55 + 50 = 105 > 60 (S2) ✓

- S2 + S3 = 60 + 50 = 110 > 55 (S1) ✓

All inequalities are satisfied, confirming the sides form a valid triangle.

Summary of sides:
- First side (S1): 55 cm
- Second side (S2): 60 cm
- Third side (S3): 50 cm

Check the perimeter:
$$ 55 + 60 + 50 = 165\, \text{cm} $$

which matches the given perimeter.

Exploring Other Possible Solutions

Since the value of x can vary within the interval, multiple solutions exist. For example, choosing x = 65 cm:

- S1 = 2(65) - 65 = 130 - 65 = 65 cm
- S3 = 230 - 3(65) = 230 - 195 = 35 cm

Verify triangle inequality:

- 65 + 65 = 130 > 35 ✓
- 65 + 35 = 100 > 65 ✓
- 65 + 35 = 100 > 65 ✓

Perimeter:
$$ 65 + 65 + 35 = 165\, \text{cm} $$

Again, valid sides.

This demonstrates that the problem admits an entire set of solutions within the specified bounds.

Conclusion: Solving for the Triangle Sides

The key steps in solving this problem involve:

- Defining variables for unknown sides
- Expressing unknown sides in terms of a single variable
- Applying the perimeter constraint to form an equation
- Using the triangle inequality theorem to find valid ranges
- Selecting specific values within the range to find concrete side lengths
- Verifying that the chosen sides satisfy all conditions

Through this process, we see how algebra and geometry intersect to solve real-world problems involving triangles.

Additional Tips for Similar Problems

- Always carefully define variables before setting up equations.
- Translate word problems into algebraic expressions step-by-step.
- Use inequalities to check the validity of solutions.
- Verify solutions by plugging back into original conditions.
- Remember that in geometry, side lengths must be positive and satisfy the triangle inequality.

By mastering these techniques, you can confidently tackle a wide range of perimeter and side-length problems involving triangles and other polygons.

In summary, given the perimeter and the relationship between sides, you can determine the side lengths by expressing the unknowns algebraically, applying constraints, and verifying the solutions. This approach not only solves the problem at hand but also enhances your overall mathematical reasoning skills.

Frequently Asked Questions

What is the relationship between the first side and the second side of the triangle?

The first side is 65 cm less than twice the second side.

If the perimeter of the triangle is 165 cm, how can we express the third side in terms of the second side?

First, find the lengths of the first and second sides using the given relationship, then subtract their sum from 165 cm to find the third side.

How do you set up an equation to find the length of the second side?

Let the second side be x cm; then the first side is 2x - 65 cm. The sum of all sides (perimeter) gives the equation: (2x - 65) + x + third side = 165 cm.

Can you determine the exact measurements of all

three sides of the triangle?

Yes, by solving the equations derived from the perimeter and the relationships between sides, you can find the exact lengths of all three sides.

What is the importance of understanding the relationships between sides in solving triangle perimeter problems?

Understanding these relationships allows for setting up equations that enable you to find unknown side lengths accurately and efficiently.

Additional Resources

Triangle Perimeter Problem: An In-Depth Mathematical Exploration

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Introduction: Unraveling the Geometry of Triangles

Triangles are among the most fundamental shapes in geometry, characterized by three sides and three angles. They are not only essential in mathematical studies but also in various practical applications such as engineering, architecture, and design. Understanding the relationships between the sides and angles of a triangle is crucial for solving complex problems and designing stable structures.

Today, we delve into a classic problem involving a triangle with a given perimeter and specific relationships between its sides. This scenario exemplifies how algebra and geometry intersect, offering insight into problem-solving techniques that are both elegant and applicable in real-world contexts.

Problem Statement and Overview

Let's consider the following problem:

A triangle has a perimeter of 165 cm. The first side is 65 cm less than twice the second side. The third side's length is to be determined.

This problem presents several key points:

- The total perimeter of the triangle is known (165 cm).
- The first side has a relationship with the second side.
- The length of the third side is unspecified but can be deduced.

By dissecting this problem, we will establish algebraic expressions for the sides, derive necessary equations, and solve for the unknowns systematically.

Understanding the Given Conditions

The Perimeter Constraint

The perimeter (P) of a triangle is the sum of its three sides:

$$ P = a + b + c $$

Given:

$$ P = 165\,\text{cm} $$

which leads to:

$$ a + b + c = 165 $$

where a, b, c represent the lengths of the first, second, and third sides, respectively.

The Relationship Between the First and Second Sides

The problem states:

The first side is 65 cm less than twice the second side.

Mathematically, this is:

$$ a = 2b - 65 $$

This relationship allows us to express one side in terms of another, reducing the number of unknowns.

Setting Up Algebraic Equations

To proceed, assign variables:

- b = length of the second side (unknown)
- a = length of the first side, expressed as $a = 2b - 65$
- c = length of the third side (unknown)

Using the perimeter condition:

$$ a + b + c = 165 $$

substitute $a = 2b - 65$:

$$ (2b - 65) + b + c = 165 $$

which simplifies to:

$$ 3b - 65 + c = 165 $$

solving for c:

$$ c = 165 - 3b + 65 = 230 - 3b $$

Now, the three sides are expressed as:

- $a = 2b - 65$
- $b = b$
- $c = 230 - 3b$

Ensuring Validity: Triangle Inequality Conditions

For these side lengths to form a valid triangle, they must satisfy the triangle inequalities:

1. $a + b > c$
2. $a + c > b$
3. $b + c > a$

Let's analyze each:

1. $a + b > c$

\[

$$
(2b - 65) + b > 230 - 3b
$$

$$
3b - 65 > 230 - 3b
$$

Add $3b$ to both sides:

$$
3b + 3b - 65 > 230
$$

$$
6b - 65 > 230
$$

Add 65:

$$
6b > 295
$$

Divide both sides by 6:

$$
b > \frac{295}{6} \approx 49.17
$$

2. $a + c > b$

$$
(2b - 65) + (230 - 3b) > b
$$

Simplify:

$$
2b - 65 + 230 - 3b > b
$$

$$
(-b) + 165 > b
$$

Add b to both sides:

$$
165 > 2b
$$

Divide both sides by 2:

$$
b < 82.5
$$

3. $b + c > a$

$$
b + (230 - 3b) > 2b - 65
$$

Simplify:

$$
b + 230 - 3b > 2b - 65
$$

$$
-2b + 230 > 2b - 65
$$

Add $2b$ to both sides:

$$
230 > 4b - 65
$$

Add 65:

$$
295 > 4b
$$

Divide both sides by 4:

$$
b < 73.75
$$

Consolidating Constraints and Finding Possible Values for b

Combining the inequalities:

- From the first inequality: $b > 49.17$

- From the second: $b < 82.5$
- From the third: $b < 73.75$

The most restrictive upper bound is 73.75, and the lower bound is approximately 49.17.

Therefore, the feasible range for b is:

$$
49.17 < b < 73.75
$$

Since side lengths must be positive and realistic, b must be within this interval.

Calculating the Side Lengths: Sample Values

To get a clearer picture, let's evaluate possible values of b within the established bounds.

Example 1: $b = 50$

- $a = 2(50) - 65 = 100 - 65 = 35$
- $c = 230 - 3(50) = 230 - 150 = 80$

Check the triangle inequalities:

- $a + b = 35 + 50 = 85 > c = 80$ □
- $a + c = 35 + 80 = 115 > b = 50$ □
- $b + c = 50 + 80 = 130 > a = 35$ □

All inequalities hold; these side lengths form a valid triangle.

Example 2: $b = 70$

- $a = 2(70) - 65 = 140 - 65 = 75$
- $c = 230 - 3(70) = 230 - 210 = 20$

Check the inequalities:

- $a + b = 75 + 70 = 145 > c = 20$ □
- $a + c = 75 + 20 = 95 > b = 70$ □
- $b + c = 70 + 20 = 90 > a = 75$ □

Again, valid.

Conclusion: Understanding the Triangle's Dimensions

From the above analysis, we observe that the side lengths depend directly on the chosen value of b within the interval $49.17 < b < 73.75$. The explicit formulas:

$$
a = 2b - 65
$$
$$
c = 230 - 3b
$$

allow us to determine the other sides once b is fixed.

Summary of Key Points:

- The perimeter condition helps formulate the relationship between sides.
- The triangle inequalities restrict possible side lengths, ensuring the formation of a valid triangle.
- Choosing specific values of b within the permissible range yields concrete side lengths that satisfy all conditions.

Practical Implications and Real-World Applications

Understanding such problems is vital beyond academic exercises. In construction, for example, designing triangular components with specified perimeters and relationships between side lengths ensures structural stability. Engineers often use algebraic relationships like these to optimize materials and ensure safety.

Similarly, in product design, knowing how to manipulate dimensions within constraints is essential for creating efficient, functional, and aesthetically pleasing objects. The principles derived here can be extended to fields such as robotics, where linkages must fit within precise spatial parameters.

Final Thoughts: The Beauty of Mathematical Relationships

This problem showcases how algebra and geometry intertwine to solve real-world-like challenges. By expressing one variable in terms of another and applying the fundamental triangle inequalities, we can delineate the feasible range of side lengths that satisfy given conditions.

In essence, such problems exemplify problem-solving at its finest—using logical reasoning, mathematical rigor, and creative exploration to arrive at meaningful solutions. Whether in education, engineering, or design, mastering these techniques enhances our capability to analyze and innovate within the constraints of physical and mathematical worlds.

In summary, a triangle with a perimeter of 165 cm, where the first side is 65 cm less than twice the second, can have various configurations depending on the length of the second side. By adhering to the triangle inequality constraints, we ensure the viability of these configurations, illustrating the elegance and practicality of mathematical problem-solving.

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