(b) Find A Line In The Plane X+y+z=3 That Is Perpendicular To The Line With Parametric Form T2,t/3,134t.

(b) Find A Line In The Plane X + Y + Z = 3 That Is Perpendicular To The Line With Parametric Form T2, t/3, 134t.

Introduction

Finding a line in a plane that is perpendicular to a given line is a fundamental problem in three-dimensional analytic geometry. This task requires understanding the properties of lines, planes, and their directional vectors, as well as applying vector operations such as dot products to determine perpendicularity. In this article, we will explore how to identify a line lying in the plane $X + Y + Z = 3$ that is perpendicular to a given line expressed in parametric form $T(2, t/3, 134t)$. We will cover the necessary concepts, step-by-step procedures, and detailed calculations to solve this problem efficiently.

Understanding the Problem

The Given Elements

- Plane Equation: $X + Y + Z = 3$
This defines a plane in 3D space. Any point (x, y, z) lying on this plane satisfies this linear equation.

- Line in Parametric Form: $T(2, t/3, 134t)$
This represents a line parameterized by t. The parametric equations are:

$$
\begin{cases}
x = 2 \\
y = \frac{t}{3} \\
z = 134t
\end{cases}
$$

Objective

- Find a line in the plane $X + Y + Z = 3$ that is perpendicular to the given line.

Key Concepts

- Direction Vector of the Given Line: The vector representing the direction of the line is derived from the parametric equations.
- Perpendicularity Condition: Two lines are perpendicular if their directional vectors are orthogonal,

i.e., their dot product is zero.

Step-by-Step Approach

1. Find the Direction Vector of the Given Line

From the parametric equations:

$$
x = 2, \quad y = \frac{t}{3}, \quad z = 134t
$$

The parametric equations can be written as:

$$
\mathbf{r}(t) = (2, 0, 0) + t \cdot \mathbf{d}
$$

where $\mathbf{d}$ is the direction vector. Comparing components:

$$
\mathbf{d} = (0, \tfrac{1}{3}, 134)
$$

(Note: The point $(2, 0, 0)$ is a point on the line when $t = 0$.)

Important: The direction vector of the line is $\mathbf{d} = (0, \frac{1}{3}, 134)$.

2. Simplify the Direction Vector

Since scalar multiples of vectors represent the same direction, it is convenient to clear fractions for easier calculations. Multiply the vector by 3:

$$
\mathbf{d} = (0, 1, 402)
$$

This scaled vector maintains the same direction, which simplifies dot product calculations.

3. Find a Direction Vector for the Desired Line

To find a line in the plane $X + Y + Z = 3$ that is perpendicular to the given line, the following must be true:

- The line must lie entirely within the plane, i.e., any point on the line must satisfy the plane equation.
- The direction vector of this line, say $\mathbf{d}_l$, must be perpendicular to the direction vector $\mathbf{d} = (0, 1, 402)$.

Therefore, the condition is:

$$
\mathbf{d}_l \cdot \mathbf{d} = 0
$$

4. Find a Direction Vector $\mathbf{d}_l$ Perpendicular to $\mathbf{d}$

Let $\mathbf{d}_l = (a, b, c)$. The dot product condition:

$$
(a, b, c) \cdot (0, 1, 402) = 0
$$

which simplifies to:

$$
a \times 0 + b \times 1 + c \times 402 = 0
$$

$$
b + 402c = 0
$$

Thus,

$$
b = -402c
$$

Constructing the Line in the Plane

1. Find a Point in the Plane

We need a point (x_0, y_0, z_0) that lies on the plane $X + Y + Z = 3$.

Choose convenient values for x_0 and y_0, then solve for z_0:

$$
x_0 + y_0 + z_0 = 3
$$

For simplicity, pick $x_0 = 0$, $y_0 = 0$:

$$
0 + 0 + z_0 = 3 \Rightarrow z_0 = 3
$$

So, one point on the plane is $(0, 0, 3)$.

2. Express the Equation of the Line

The line passing through point $(0, 0, 3)$ with direction vector $\mathbf{d}_l = (a, b, c)$, where $b = -402 c$, can be written parametrically as:

$$
\begin{cases}
x = 0 + a t \\
y = 0 + b t = -402 c t \\
z = 3 + c t
\end{cases}
$$

where t is a real parameter.

3. Ensure the Line Lies in the Plane

Since the line is defined with direction vector and passing through a point on the plane, it automatically lies within the plane as long as its points satisfy the plane equation.

Check for any point on the line:

$$
x(t) + y(t) + z(t) = (a t) + (-402 c t) + (3 + c t) = a t - 402 c t + 3 + c t
$$

Simplify:

$$
a t - 402 c t + c t + 3 = (a + (-402 c) + c) t + 3 = (a - 401 c) t + 3
$$

To ensure all points on the line lie in the plane $X + Y + Z = 3$, this expression must be equal to 3 for all values of t. Therefore:

$$
(a - 401 c) t + 3 = 3
$$

which implies:

$$
(a - 401 c) t = 0
$$

for all t, which can only be true if:

$$
a - 401 c = 0 \Rightarrow a = 401 c
$$

Final Parameterization of the Desired Line

Summarizing the constraints:

- $a = 401 c$
- $b = -402 c$
- c is arbitrary (non-zero for a non-trivial direction vector)

Choose $c = 1$ for simplicity:

$$
a = 401, \quad b = -402, \quad c = 1
$$

Thus, the direction vector:

$$
\mathbf{d}_l = (401, -402, 1)
$$

and the point on the line:

$$
(0, 0, 3)
$$

The parametric equations of the line are:

$$
\begin{cases}
x = 0 + 401 t = 401 t \\
y = 0 - 402 t = -402 t \\
z = 3 + t
\end{cases}
$$

which can be written as:

$$
\boxed{
\text{Line } L: \quad
\begin{cases}
x = 401 t \\
y = -402 t \\
z = 3 + t
\end{cases}
}
$$

Verification

1. Check if the line lies in the plane $X + Y + Z = 3$:

$$
x + y + z = 401 t - 402 t + (3 + t) = (401 t - 402 t + t) + 3 = (0) + 3 = 3
$$

Indeed, every point on the line satisfies the plane equation.

2. Check perpendicularity to the given line:

Dot product of the direction vectors:

$$
\mathbf{d}_l = (401, -402, 1)
$$

$$
\mathbf{d} = (0, 1, 402)
$$

Calculate:

$$
(401)(0) + (-402)(1) + (1)(402) = 0 - 402 + 402 = 0
$$

Since the dot product is zero, the lines are perpendicular.

Summary

- The line passing through $(0, 0, 3)$ with direction vector $(401, -402, 1)$ lies in the plane $X + Y + Z = 3$ and is perpendicular to the given line \(T(2, t/3, 134t)\

Frequently Asked Questions

How do you find a line in the plane x + y + z = 3 that is perpendicular to a given line in parametric form?

To find such a line, first identify the direction vector of the given line, then determine a line in the plane with a direction vector perpendicular to that, and ensure the line lies within the plane.

What is the direction vector of the given line T = (2, t/3, 13/4

t)?

The direction vector of the line is obtained from the parametric equations: for y = t/3, the coefficient of t is 1/3; for z = 13/4 t, the coefficient is 13/4; and for x = 2, which is constant, indicating no change. Therefore, the direction vector is (0, 1/3, 13/4).

How do I find a line in the plane x + y + z = 3 that is perpendicular to the given line?

First, find the direction vector of the given line. Then, find a direction vector in the plane that is perpendicular to this vector. Use this perpendicular vector to define the direction of your required line, ensuring it passes through a point in the plane.

What point can I use on the plane x + y + z = 3 to define the new line?

Any point satisfying the equation x + y + z = 3 can be used. For simplicity, choose a convenient point such as (0, 0, 3) or (3, 0, 0).

How do I verify that the line in the plane is perpendicular to the given line?

Check that the dot product of the direction vector of the line in the plane and the direction vector of the given line is zero, indicating perpendicularity.

What is the step-by-step process to find the perpendicular line in the plane?

1. Find the direction vector of the given line.
2. Determine a vector in the plane perpendicular to this direction vector.
3. Select a point in the plane.
4. Write the parametric equations of the new line using this point and the perpendicular direction vector.

Can you give an example of the perpendicular line in the plane for this problem?

Yes. The direction vector of the given line is (0, 1/3, 13/4). A vector perpendicular to this could be, for example, (13/4, 0, -1/3). Using point (0, 0, 3), the line can be parametrized as:
x = 0 + 13/4 t,
y = 0,
z = 3 - 1/3 t.

How do I confirm that the constructed line is in the plane x +

y + z = 3?

Substitute the parametric equations into x + y + z and verify if the sum equals 3 for all t. If it does, the line lies entirely within the plane.

What is the significance of the vectors being perpendicular in this problem?

Perpendicular vectors indicate that the lines are orthogonal, satisfying the condition that the line in the plane is perpendicular to the given line, which is essential for the problem's geometric relationship.

Additional Resources

Understanding the Problem: Finding a Line in the Plane X + Y + Z = 3 That Is Perpendicular to a Given Parametric Line

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Introduction

In the realm of three-dimensional analytic geometry, one common problem involves finding lines that satisfy certain geometric constraints. A typical scenario is to find a line within a specified plane that is perpendicular to another given line in space. This problem not only tests your understanding of vectors and their geometric interpretations but also your proficiency in applying parametric equations and plane equations.

Specifically, the problem at hand asks:

Find a line in the plane $x + y + z = 3$ that is perpendicular to the line with parametric form $\mathbf{T}(t) = \left(t, \frac{1}{3}, 134t \right)$.

This task involves several key steps:
- Understanding the parametric form of the given line
- Analyzing the geometric properties of the plane
- Determining the direction vector of the sought line that satisfies perpendicularity
- Ensuring the line lies within the plane

In this comprehensive review, we will analyze each component systematically, explore the mathematical tools involved, and walk through the solution process step-by-step.

---

1. Geometric Foundations

1.1 The Plane $x + y + z = 3$

- Equation of the Plane: The plane is given by the linear equation $x + y + z = 3$.
- Normal Vector: The vector perpendicular (normal) to this plane is derived directly from the

coefficients of x, y, z, which yields:

$$
\mathbf{n} = \langle 1, 1, 1 \rangle
$$

- Properties:
- The plane contains infinitely many lines.
- Any line lying entirely within this plane must satisfy the plane's equation for all points on the line.

1.2 The Given Line $\mathbf{T}(t) = (t, \frac{1}{3}, 134t)$

- Parametric form:

$$
\mathbf{T}(t) = (x(t), y(t), z(t)) = (t, \frac{1}{3}, 134t)
$$

- Direction Vector of the line:

$$
\mathbf{d} = \frac{d}{dt} \mathbf{T}(t) = \langle 1, 0, 134 \rangle
$$

- Geometric significance:
- It is a straight line extending infinitely in both directions.
- The line's direction vector indicates that as t varies, x and z change linearly, while y remains constant at $\frac{1}{3}$.

---

2. The Goal: Find a Line in the Plane That Is Perpendicular to the Given Line

This involves two main conditions:

- The line must lie within the plane $x + y + z = 3$.
- The line's direction vector must be perpendicular to the direction vector of the given line.

---

3. Mathematical Approach

3.1 Conditions for the Required Line

Suppose the desired line has:

- A point $\mathbf{P}_0 = (x_0, y_0, z_0)$ lying in the plane $x + y + z = 3$,
- A direction vector $\mathbf{v}$.

The line's parametric form is:

$$
\mathbf{L}(s) = \mathbf{P}_0 + s \mathbf{v}
$$

Conditions:

1. Point in the plane:

$$
x_0 + y_0 + z_0 = 3
$$

2. Line lies within the plane:

For the entire line to be within the plane, all points on the line must satisfy the plane's equation. Since the line extends in the direction $\mathbf{v}$:

$$
(x_0 + s v_x) + (y_0 + s v_y) + (z_0 + s v_z) = 3
$$

Which simplifies to:

$$
(x_0 + y_0 + z_0) + s (v_x + v_y + v_z) = 3
$$

For the line to be completely contained in the plane, the coefficient of s must be zero:

$$
v_x + v_y + v_z = 0
$$

3. Perpendicularity to the given line:

The direction vectors $\mathbf{v}$ of the sought line and $\mathbf{d} = \langle 1, 0, 134 \rangle$ of the given line must satisfy:

$$
\mathbf{v} \cdot \mathbf{d} = 0
$$

---

4. Solving for the Direction Vector $\mathbf{v}$

From the conditions:

- $v_x + v_y + v_z = 0$,
- $v_x + 0 + 134 v_z = 0$ (perpendicularity condition).

Let's analyze these equations systematically.

4.1 Express v_x in terms of v_z:

From the second condition:

$$
v_x + 134 v_z = 0 \Rightarrow v_x = -134 v_z
$$

4.2 Express v_y in terms of v_z:

Using the first condition:

$$
v_x + v_y + v_z = 0
$$

Substitute $v_x = -134 v_z$:

$$
-134 v_z + v_y + v_z = 0 \Rightarrow v_y = 134 v_z - v_z = 133 v_z
$$

Thus, the general form for $\mathbf{v}$:

$$
\boxed{
\mathbf{v} = \langle v_x, v_y, v_z \rangle = v_z \langle -134, 133, 1 \rangle
}
$$

Since the direction vector is determined up to a scalar multiple, choose $v_z = 1$ for simplicity:

$$
\mathbf{v} = \langle -134, 133, 1 \rangle
$$

---

5. Finding a Point on the Line in the Plane

The line must:

- Lie within the plane $x + y + z = 3$,
- Have a point $\mathbf{P}_0 = (x_0, y_0, z_0)$ satisfying:

$$
x_0 + y_0 + z_0 = 3
$$

Since the line has direction $\mathbf{v} = \langle -134, 133, 1 \rangle$, the parametric form becomes:

$$
\mathbf{L}(s) = (x_0, y_0, z_0) + s \langle -134, 133, 1 \rangle
$$

Any point on this line must satisfy the plane equation:

$$
(x_0 - 134 s) + (y_0 + 133 s) + (z_0 + s) = 3
$$

Simplify:

$$
x_0 + y_0 + z_0 + s (-134 + 133 + 1) = 3
$$

Note that:

$$
-134 + 133 + 1 = 0
$$

Therefore:

$$
x_0 + y_0 + z_0 = 3
$$

which is consistent with the initial condition for $\mathbf{P}_0$. Thus, any point $\mathbf{P}_0$ satisfying $x_0 + y_0 + z_0 = 3$ will work.

---

6. Explicit Construction of the Line

Choose specific values for x_0, y_0, z_0 satisfying the plane equation. For simplicity:

- Let $x_0 = 0$,
- $y_0 = 0$,
- Then,

$$
z_0 = 3 - x_0 - y_0 = 3
$$

Thus, a point on the line is $(0,0,3)$.

Parametric equations of the required line:

$$
\boxed{
\begin{cases}
x(s) = 0 - 134 s = -134 s \\
y(s) = 0 + 133 s = 133 s \\
z(s) = 3 + s
\end{cases}
}
$$

This line lies within the plane $x + y + z = 3$ because:

$$
x(s) + y(s) + z(s) = -134 s + 133 s + (3 + s) = (-134 s + 133 s + s) + 3 = 0 + 3 = 3
$$

which confirms the point is within the plane for all s.

---

7. Summary and Final Result

The line in the plane \(x + y + z = 3

[B Find A Line In The Plane X Y Z 3 That Is Perpendicular To The Line With Parametric Form T2 T 3 134t]

B Find A Line In The Plane X Y Z 3 That Is Perpendicular To The Line With Parametric Form T2 T 3 134t

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