(i) A Random Variable X Follows Binomial Distribution With 12 Trails. (a) Find The Value Of P Given That

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When dealing with binomial distributions, one of the most common problems involves determining the probability of success, denoted as p, given certain information about the random variable X. In this context, suppose that a random variable X follows a binomial distribution with 12 trials, and we are tasked with finding the value of p given specific conditions such as the probability of a certain number of successes or the expected value. Understanding how to approach this problem is essential for students and professionals working in statistics, data analysis, and related fields.

Understanding the Binomial Distribution

What Is a Binomial Distribution?

The binomial distribution models the number of successes in a fixed number of independent trials, each with the same probability of success. It is characterized by two parameters:

- The number of trials, denoted as n (in this case, 12).
- The probability of success in a single trial, denoted as p.

The probability mass function (PMF) for a binomial distribution is given by:
$$ P(X = k) = \binom{n}{k} p^k (1-p)^{n-k} $$
where k is the number of successes.

Key Properties of Binomial Distribution

- The expected value (mean): $\mu = n p$
- Variance: $\sigma^2 = n p (1 - p)$
- The distribution is discrete and symmetric when p = 0.5.

Problem Statement: Finding p Given Certain Conditions

Suppose that:
- The random variable $X \sim \text{Binomial}(n=12, p)$
- We are given some information about X (for example, the probability of a specific number of successes or the expected number of successes)
- Our goal is to determine the value of p based on this information

Example Scenario:
For instance, if it is given that the probability of getting exactly 3 successes is a certain value, or that the expected number of successes is known, we can set up an equation to solve for p.

Methods to Find the Value of p

Using the Probability of a Specific Number of Successes

Suppose you know:
$$P(X = k) = p_k$$
for some k and p_k (a specific probability). You can then use the binomial PMF:
$$p_k = \binom{12}{k} p^k (1-p)^{12 - k}$$
and solve for p.

Example:
If $P(X=3) = 0.22$, then:
$$0.22 = \binom{12}{3} p^3 (1-p)^9$$
which simplifies to:
$$0.22 = 220 \times p^3 (1-p)^9$$
and solving for p involves numerical methods or iterative solutions such as the Newton-Raphson method.

Using the Expected Value of X

The expected value of X for a binomial distribution is:
$$E[X] = n p$$
If you are given the average number of successes, say $\bar{x}$, then:
$$p = \frac{\bar{x}}{n}$$
This approach is straightforward and often used when the mean is provided.

Example:
If the average number of successes in repeated experiments is 4.8:

$$ p = \frac{4.8}{12} = 0.4 $$

Using the Cumulative Probability

In some instances, you might be given the probability of X being less than or equal to a certain value:

$$ P(X \leq k) = p_{c} $$

You can then use binomial tables or statistical software to find p that satisfies this cumulative probability.

Practical Steps to Find p in the Given Scenario

Step 1: Identify the Given Data

- Is the probability of a specific number of successes provided?
- Is the expected number of successes given?
- Is the cumulative probability provided?

Step 2: Choose the Appropriate Method

- Use the probability mass function if a specific success probability is given.
- Use the mean if the average number of successes is provided.
- Use cumulative probabilities when applicable.

Step 3: Set Up the Equation

- For specific probabilities: $$ p_k = \binom{n}{k} p^k (1-p)^{n-k} $$
- For mean: $$ p = \frac{\text{mean}}{n} $$
- For cumulative probabilities: Use statistical software or binomial tables to find p.

Step 4: Solve for p

- Algebraic manipulation for simple cases.
- Numerical methods for complex equations.

Example Problem and Solution

Problem:
Suppose that in 12 trials, the probability of getting exactly 4 successes is 0.15. Find the value of p.

Solution:
Given:
$$ P(X=4) = 0.15 $$
Using the binomial PMF:
$$ 0.15 = \binom{12}{4} p^4 (1-p)^8 $$
Calculate $\binom{12}{4}$:
$$ \binom{12}{4} = 495 $$
So:
$$ 0.15 = 495 \times p^4 (1-p)^8 $$
Divide both sides by 495:
$$ \frac{0.15}{495} \approx 0.000303 $$
Then:
$$ p^4 (1-p)^8 = 0.000303 $$
This equation cannot be solved algebraically easily; instead, use numerical methods or software (like calculator, Excel, R, or Python) to approximate p.

Using software:
- Set up the equation and iterate over possible p values to find the one that satisfies the equation.
- For instance, in Python, one can write a small script to find p numerically.

Conclusion

Determining the value of p in a binomial distribution when given specific information about the distribution involves understanding the key properties and applying the appropriate mathematical tools. Whether using the probability mass function, the expected value, or cumulative probabilities, the approach depends on the data provided. In real-world applications, statistical software greatly simplifies the process, especially for equations that lack closed-form solutions.

Summary of key points:

- Identify the given data: probability of successes, mean, or cumulative probability.
- Choose the appropriate method based on the data.
- Set up the binomial probability equation.

- Solve for p using algebraic manipulation or numerical methods.

By mastering these steps, students and practitioners can efficiently find the unknown probability p for binomial distributions with 12 trials or any other number of trials, enhancing their understanding of probabilistic models and their applications in various fields.

Frequently Asked Questions

A random variable X follows a binomial distribution with 12 trials. If the probability of success p is unknown, and the expected value E[X] is 6, what is the value of p?

Since E[X] = n p, and E[X] = 6 with n = 12, then p = 6 / 12 = 0.5.

Given a binomial distribution with 12 trials, if the probability of success p is 0.4, what is the probability that exactly 5 successes occur?

Using the binomial probability formula: P(X=5) = C(12,5) (0.4)^5 (0.6)^7, which calculates to approximately 0.206.

In a binomial experiment with 12 trials, if the probability of success p is 0.7, what is the probability of getting at most 3 successes?

Calculate P(X ≤ 3) = sum of P(X=0), P(X=1), P(X=2), and P(X=3) using the binomial formula with p=0.7 and n=12.

A binomial random variable X with 12 trials has a probability of success p. If P(X ≥ 9) = 0.05, how can we estimate p?

This involves solving P(X ≥ 9) = sum from x=9 to 12 of C(12,x)p^x(1-p)^(12-x)=0.05. Numerical methods or binomial tables are used to estimate p, approximately around 0.85.

If a binomial distribution with 12 trials has a

success probability p, and the probability of exactly 4 successes is 0.2, how can we find p?

Use the binomial probability formula: $P(X=4) = C(12,4) p^4 (1-p)^8 = 0.2$. Solve this equation numerically for p, which is approximately 0.33.

Additional Resources

A Random Variable X Follows Binomial Distribution With 12 Trails

Understanding the binomial distribution is fundamental in probability theory and statistics, especially when analyzing experiments with fixed numbers of independent trials, each with the same probability of success. When a random variable X follows a binomial distribution with 12 trials, it signifies that the experiment being modeled has exactly 12 independent and identical attempts, each resulting in either success or failure. The binomial distribution is characterized by two parameters: the number of trials (n) and the probability of success in a single trial (p). In this context, with n = 12, the focus often turns to determining the probability of a specific number of successes, which hinges critically on knowing or estimating p.

Understanding the Binomial Distribution

The binomial distribution models the number of successes in a fixed number of independent Bernoulli trials, where each trial has the same probability of success p. The probability mass function (PMF) for a binomial random variable X, representing the number of successes, is expressed as:

$$ P(X = k) = \binom{n}{k} p^k (1 - p)^{n - k} $$

where:
- n = total number of trials,
- k = number of successes ($0 \leq k \leq n$),
- p = probability of success on each trial.

In our case, with n = 12, the distribution provides a complete picture of the likelihood of observing any specific number of successes, from 0 to 12.

Why Is Determining p Important?

Knowing the probability of success p is essential for several reasons:

- Prediction: Estimating the likelihood of a certain number of successes.
- Decision-making: Determining the best course of action based on success probabilities.
- Parameter Estimation: Inferring p from observed data using maximum likelihood or other statistical methods.

Often, p is unknown and must be estimated from data or deduced from additional information provided in the problem statement.

Given the Problem: Find the Value of p

Suppose the problem states:

Given that X ~ Binomial(n=12, p= ?), and some condition or data is provided (e.g., the probability of observing a certain number of successes), find the value of p.

For example, the problem might specify:

- The probability of exactly 4 successes is 0.2.
- The probability of at most 3 successes is 0.1.
- The expected number of successes is known.

Let's explore a typical scenario to illustrate how to find p.

Scenario 1: Given a Specific Probability

Suppose the problem states:

Given that P(X = 4) = 0.2, find p.

Step 1: Write the PMF for X=4

$$ P(X=4) = \binom{12}{4} p^4 (1 - p)^{8} = 0.2 $$

Step 2: Calculate the binomial coefficient

$$ \binom{12}{4} = \frac{12!}{4! \times 8!} = 495 $$

Step 3: Set up the equation

$$ 495 \times p^4 (1 - p)^8 = 0.2 $$

Step 4: Solve for p

This is a nonlinear equation in p and typically requires numerical methods or trial-and-error to approximate p. Using tools like a graphing calculator, Excel, or computational software, one might find:

- For $p \approx 0.3$, the value approximates the given probability.
- For $p \approx 0.35$, the probability is close to 0.2.

Through iterative approximation, you might find:

$$ p \approx 0.33 $$

which satisfies the equation approximately.

Scenario 2: Using the Expected Value

Suppose the problem states:

The expected number of successes is 4.8. Find p.

Step 1: Recall the expectation for a binomial distribution

$$ E[X] = n \times p $$

Step 2: Solve for p

$$ p = \frac{E[X]}{n} = \frac{4.8}{12} = 0.4 $$

This straightforward approach demonstrates how expectations can directly determine p.

Methods for Estimating p

When data is available, statisticians often estimate p using observed data:

- Maximum Likelihood Estimation (MLE):
Given observed successes k in n trials, the MLE for p is:

$$ \hat{p} = \frac{k}{n} $$

- Method of Moments:
Equates the sample mean to the theoretical mean:

$$\bar{x} = n p \Rightarrow p = \frac{\bar{x}}{n}$$

These methods are simple and effective when large samples are involved.

Features and Pros/Cons of the Binomial Distribution

Features:

- Discrete distribution: Models count data, like the number of successes.
- Parameter simplicity: Fully described by n and p.
- Versatile: Applies in various fields like quality control, medical testing, and survey sampling.

Pros:

- Easy to compute and understand.
- Well-established theoretical properties.
- Suitable for modeling binary outcomes.

Cons:

- Assumes independence of trials.
- Assumes constant probability p across trials.
- Not suitable for overdispersed data or dependent trials.

Applications of Binomial Distribution

The binomial distribution is widely used in scenarios such as:
- Quality control (defective items in a batch),
- Medical tests (probability of disease detection),
- Marketing (success rate of campaigns),
- Election polling (number of favorable responses),
- Biological studies (number of organisms with a particular trait).

Its versatility stems from its straightforward assumptions and mathematical tractability.

Conclusion

The binomial distribution with 12 trials provides a powerful framework for understanding binary outcome processes in fixed trials. Determining the probability of success p is central to leveraging this model effectively. Depending on the information provided—whether a specific probability, expected value, or observed data—various methods can be employed to estimate p. While the model’s simplicity and clarity are advantageous, recognizing its assumptions and limitations is vital for accurate application. Whether for academic, research, or practical purposes, mastering the process of finding p within the binomial framework enhances one's ability to interpret and analyze binomially distributed data effectively.

In summary:

- The binomial distribution is characterized by parameters n and p.
- Given specific data or conditions, p can be estimated or calculated directly.
- Multiple methods exist for estimating p, depending on the data.
- The binomial distribution's features make it suitable for many real-world problems, but its assumptions should be carefully considered.
- Accurate estimation of p enables precise probability calculations, vital for decision-making and statistical inference.

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